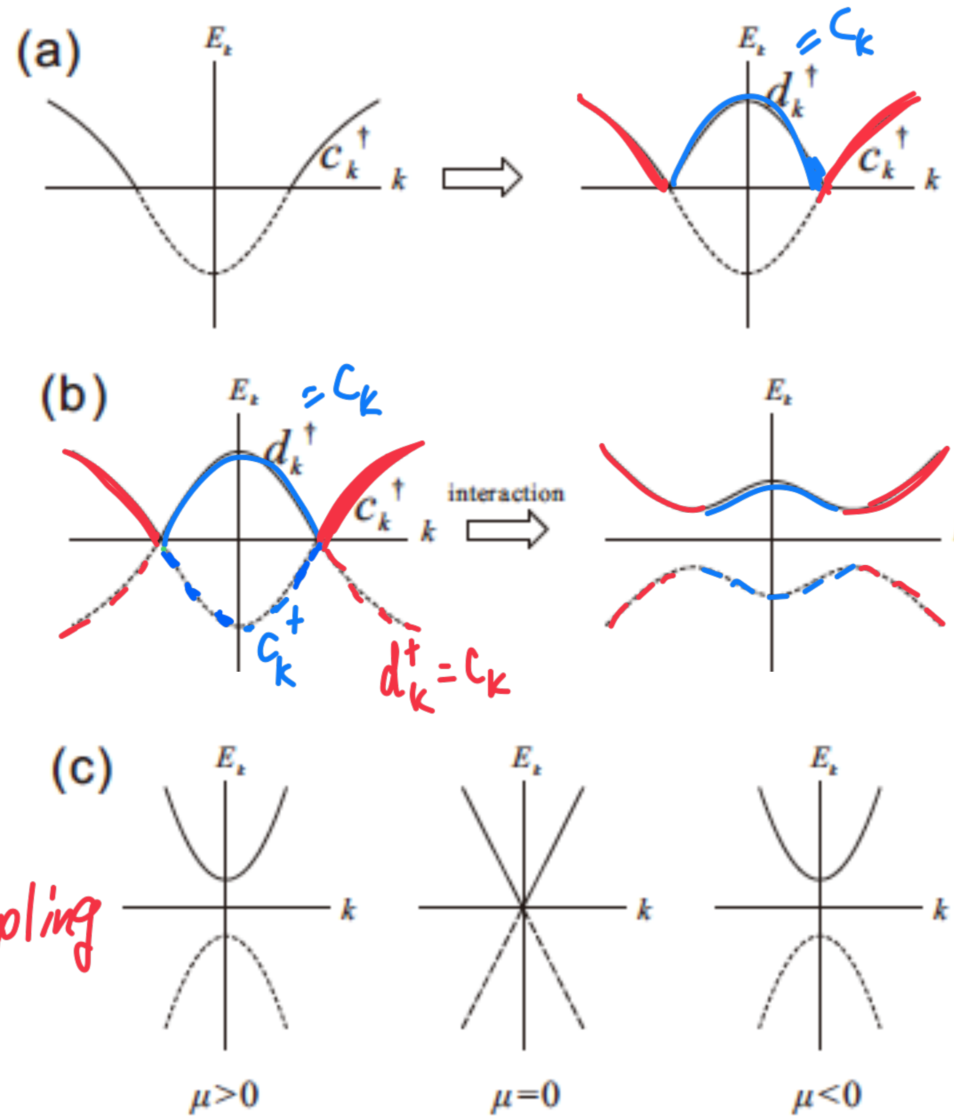
Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode 23: QCPs in TSCs/TSFs II

From Fermi surface to Emergent C-symmetry



Emergent C-symmetry (induced by U(1) sym. breaking)

Strong Coupling physics

Emergent Lorentz sym. at QCP $\mu=0$

Topological quantum criticality in TSF/TSCs (driven by changes of global topologies)

How to characterize beyond-Landau-paradigm Transitions between states breaking the same symmetries or with the same local order parameters but different topologies? ?

*Cusps near Topological Phase Transitions: Signatures of Majorana fermions and interactions with fluctuations,Yang, Jiang, Zhou, Phys. Rev. **B** 100, 054508 (2019).*

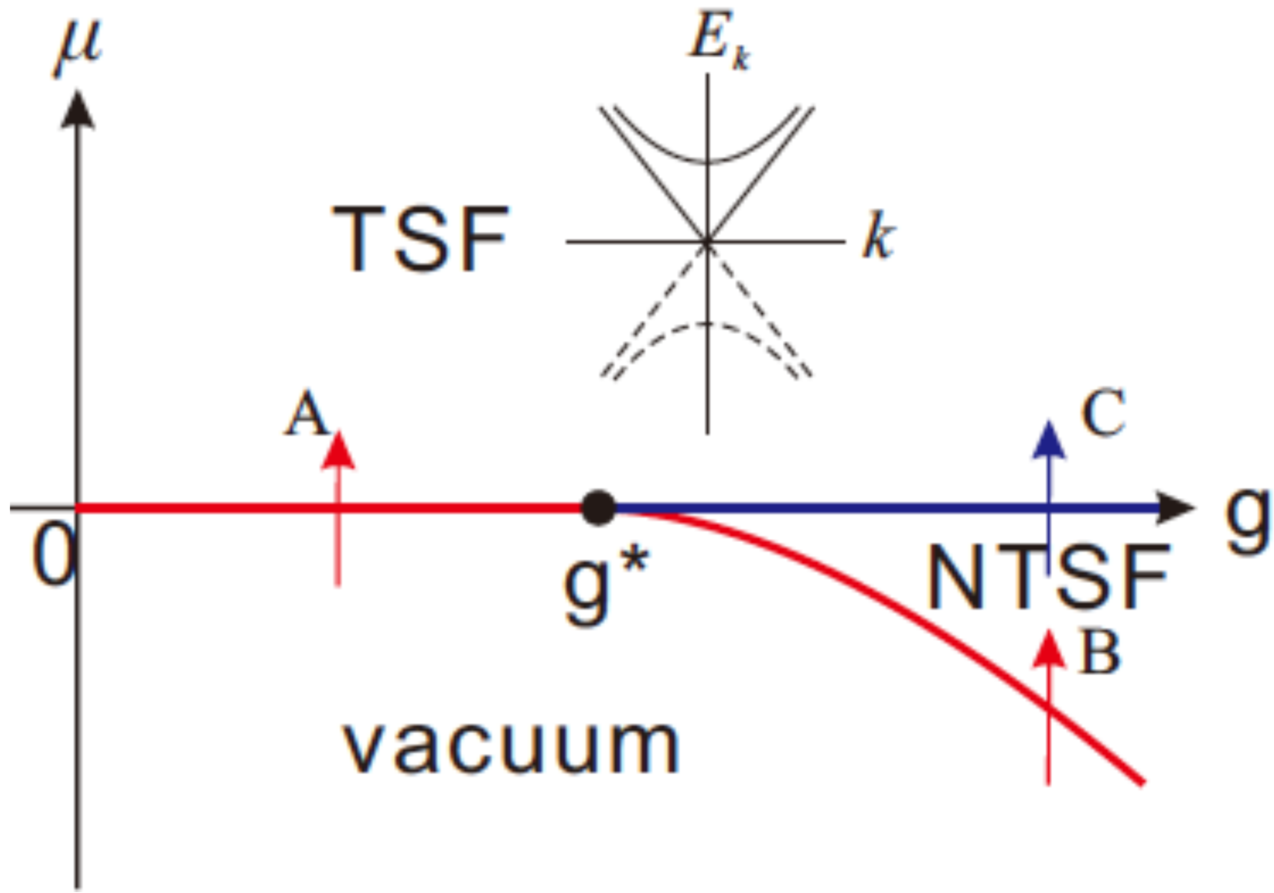
Quantum Criticality in Topological Superconductors: Surface criticality, Thermal properties, and Lifshitz Majorana field, Yang, Zhou, ArXiv: arXiv:2012.11143.

Q1,2,3:

- What are appropriate Effective Field Theories for topological phase transitions in SFs?
- Exists an analogue of strong coupling Wilson-Fisher fixed point/universality ? What is "the upper critical dimension"?
- *What are the bulk signatures of topological phase transitions in addition to boundary-state deformation?*

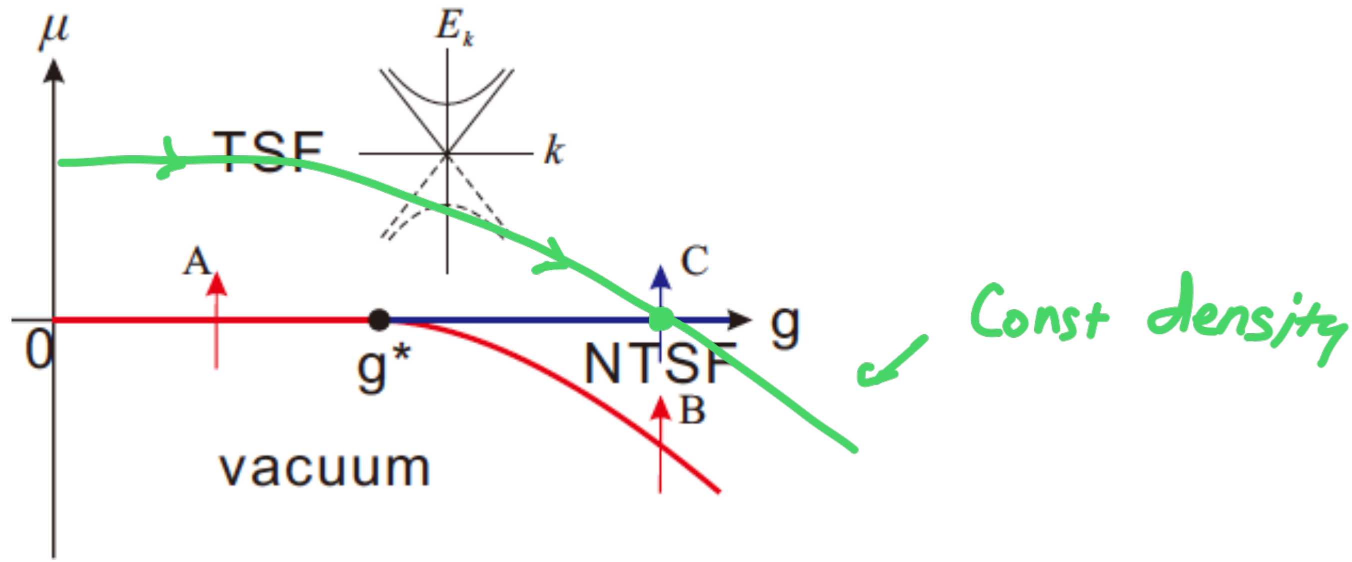
- Topological QCPs (driven by interactions or chemical potentials)

Example: Phase diagram of TRI TSFs/TSCs

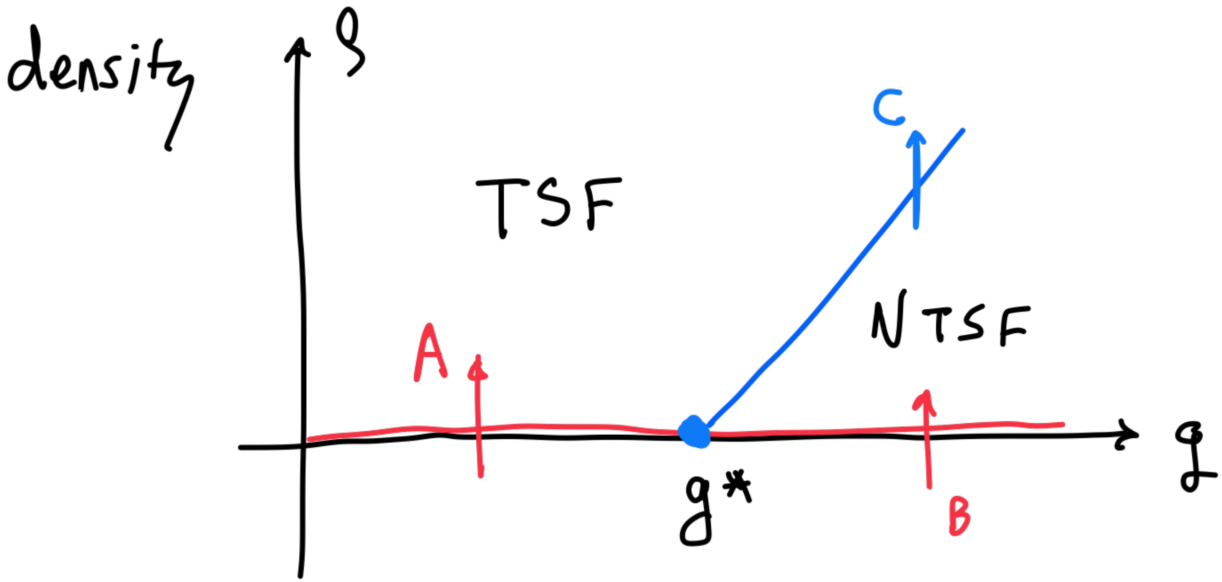


A: Free Fermion; B: Wilson-Fisher/Free boson; C: Majorana class
g*: QCP of SO(2,1) CFT.

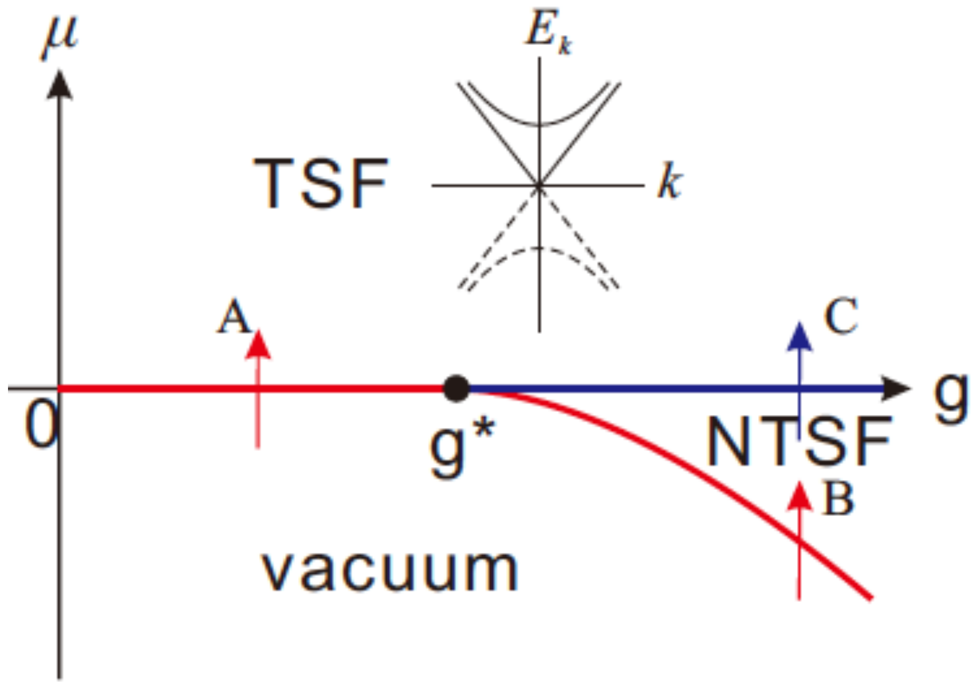
Yang, Jiang and FZ, 2019



μ
E_k
k
TSF
A
C
0
g*
g
NTSF
B
vacuum
Const density



density
ρ
TSF
C
NTSF
A
g*
B
g



1) QCPs in TSCs (d> or =1) with/without T-symmetries belong to *the Majorana fermion universality class.*

2) *It is of (d+1)th order in d-dimension, d=1,2,3.*

3) Robust bulk signatures (i.e. compressibility anomalies)

Generic Model near QCP ($\mu \sim 0$)

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_b + \mathcal{L}_{mb}$$

$$\mathcal{L}_m = v_F \chi^T \left(\Gamma_\alpha \frac{\nabla_\alpha}{i} \right) \chi + \chi^T \mu \Gamma_0 \chi \, , \quad \alpha = x, y, z$$

$$\mathcal{L}_b = \sum_{\ell=1}^{N'} \varphi_\ell \left(\partial_\tau^2 - c^2 \nabla_\alpha^2 \right) \varphi_\ell + m_\ell^2 \varphi_\ell^2$$

$$\mathcal{L}_{mb} = g_{mb} \sum_{\ell=1}^{N'} \varphi_\ell \, \chi^T \Pi^\ell \chi$$

$$\Gamma_\alpha = \Gamma_\alpha^T \, , \quad \Gamma_\alpha = \Gamma_\alpha^* \, ; \quad \Gamma_0 = -\Gamma_0^T = -\Gamma_0^*$$

$2N \times 2N$

$$\Pi^\ell = -\left(\Pi^\ell\right)^T$$

Minimal Model

TRB TSC/TSF

N=1

TRS TSC/TSF

N=2

N>3 Symmetry Protected QCPs

(Requires extra Global Symmetries)

Generic Model near QCP ($\mu \sim 0$)

$$\mathcal{L} = \mathcal{L}_{m} + \mathcal{L}_{b} + \mathcal{L}_{mb}$$

$$\mathcal{L}_{m} = v_F \chi^T \left(\Gamma_\alpha \frac{\nabla_\alpha}{i} \right) \chi + \chi^T \mu \Gamma_0 \chi, \quad \alpha = x, y, z$$

$$\mathcal{L}_{b} = \varphi \left(\partial_\tau^2 - \nabla_\alpha^2 \right) \varphi + m^2 \varphi^2$$

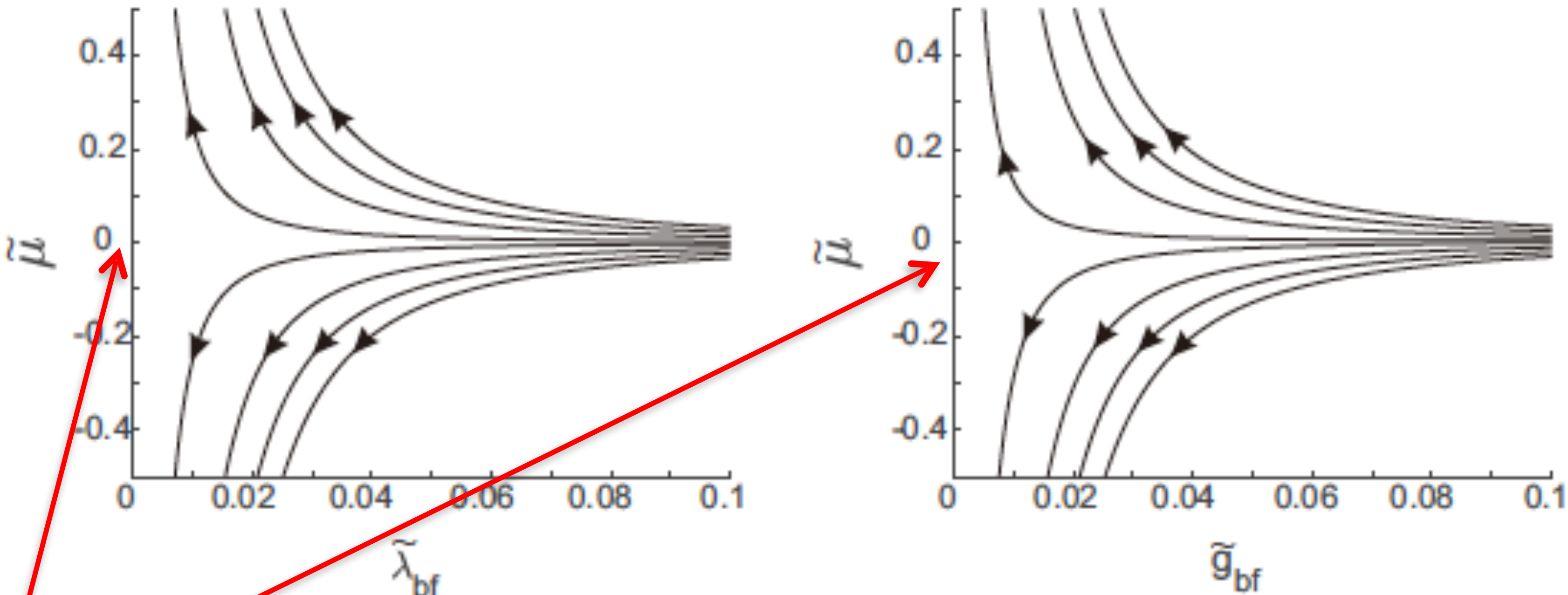
$$\mathcal{L}_{\mu b} = g_{\mu b} \varphi \, \chi^T \Pi^\ell \chi$$

$$\Gamma_\alpha = \Gamma_\alpha^T, \quad \Gamma_\alpha = \Gamma_\alpha^* ; \quad \Gamma_0 = -\Gamma_0^T = -\Gamma_0^*$$

$$\Pi^\ell = -(\Pi^\ell)^T$$

$2N = 2, 4$

Chemical potential

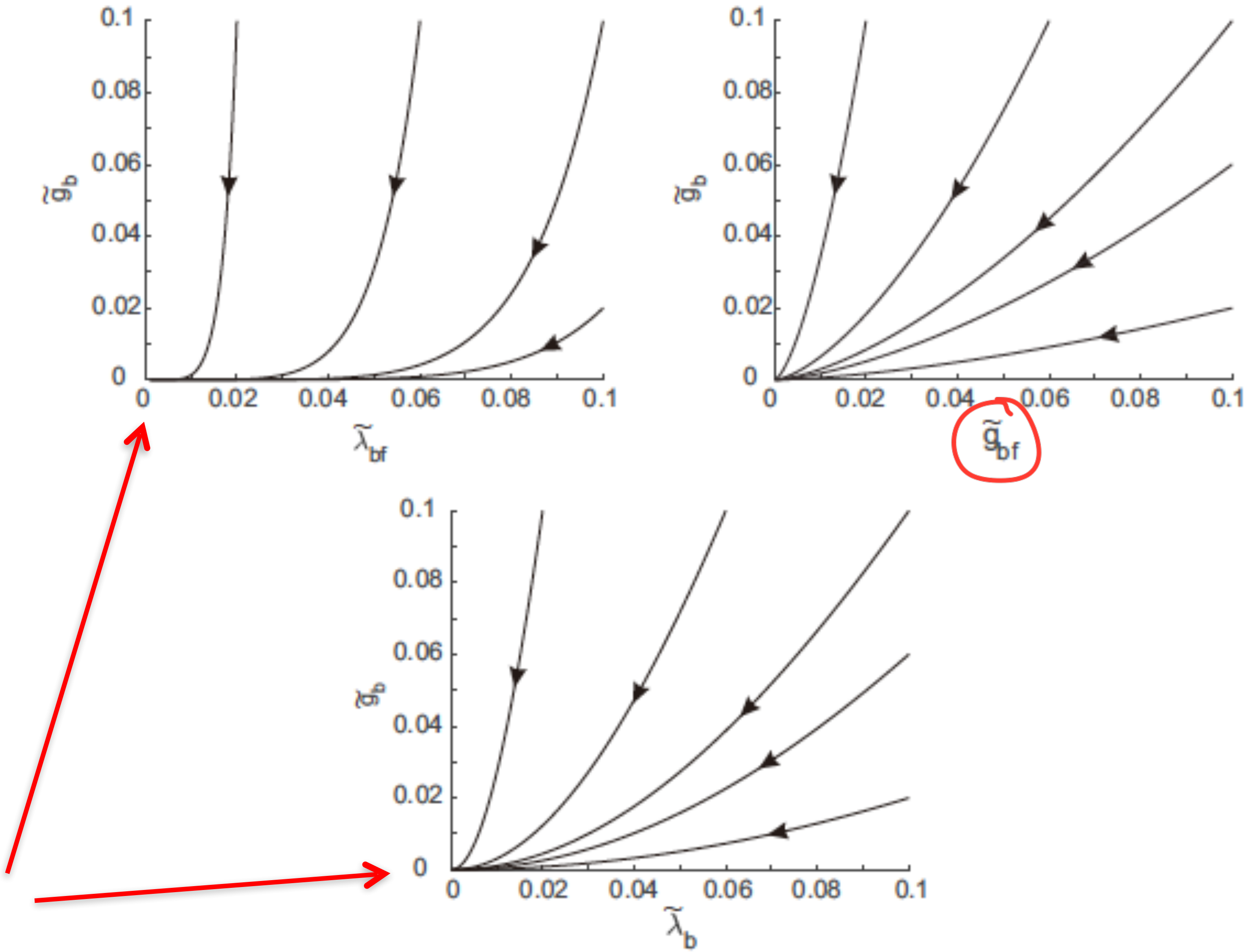


Fixed points

QCP

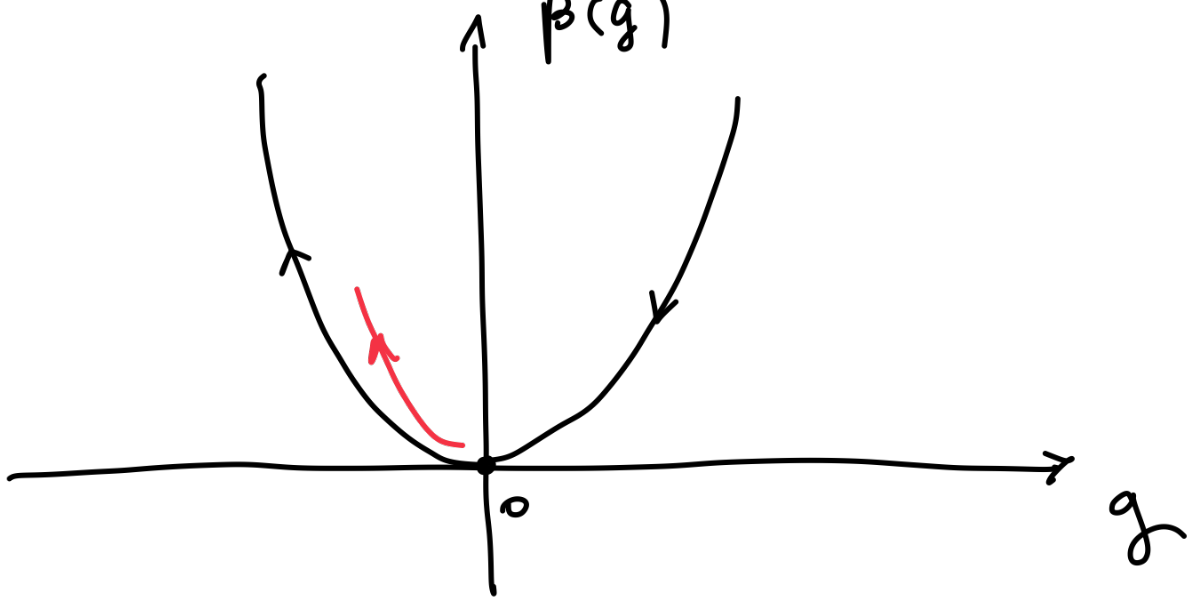
$\mu = 0$

Interactions renormalized away

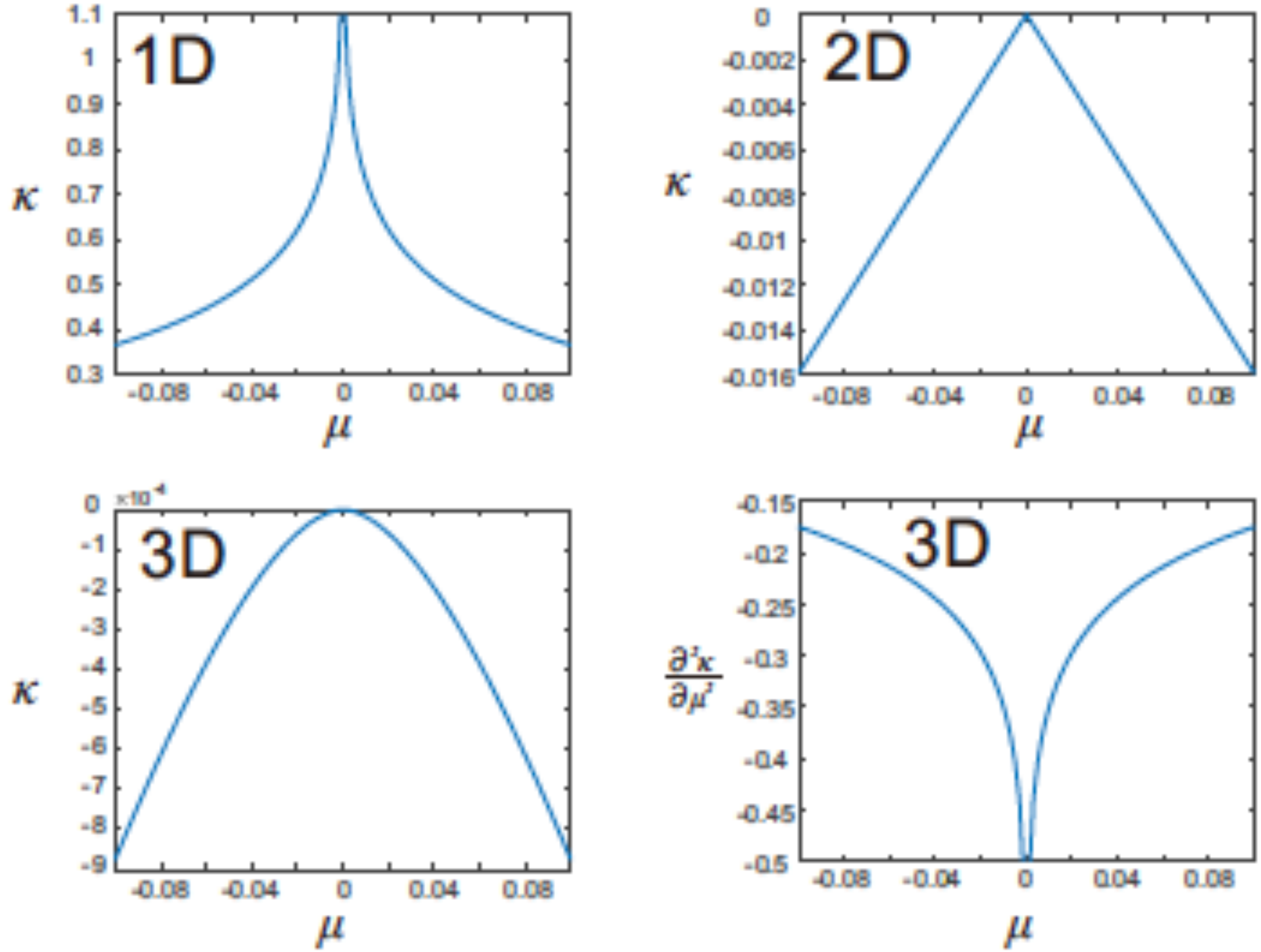


Only Fixed point: a free Majorana Field in d=2,3 SFs with/without TRS

TRI TSC $(\mathbb{Z}_2)$ $d=1$ Marginal



(d+1)th order topological P.T.
(compressibility versus chemical potential)



Yang, Jiang and FZ, 2019.

More questions:

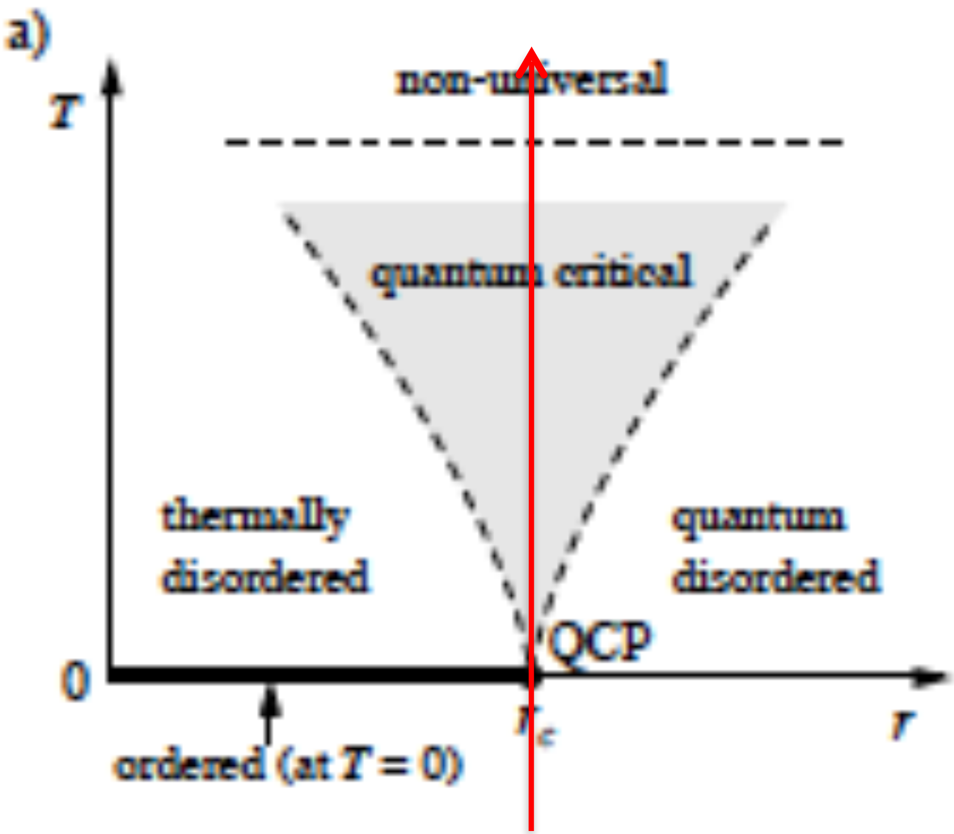
1) Are there finite temperature topological phase transitions in 2D and/or 3D?

2) Can there be TPTs characterized by CFT of majorana fermions with higher emergent symmetries?

3) Can one classify/identify other possible EFTs for TPTs based on the classification theory?

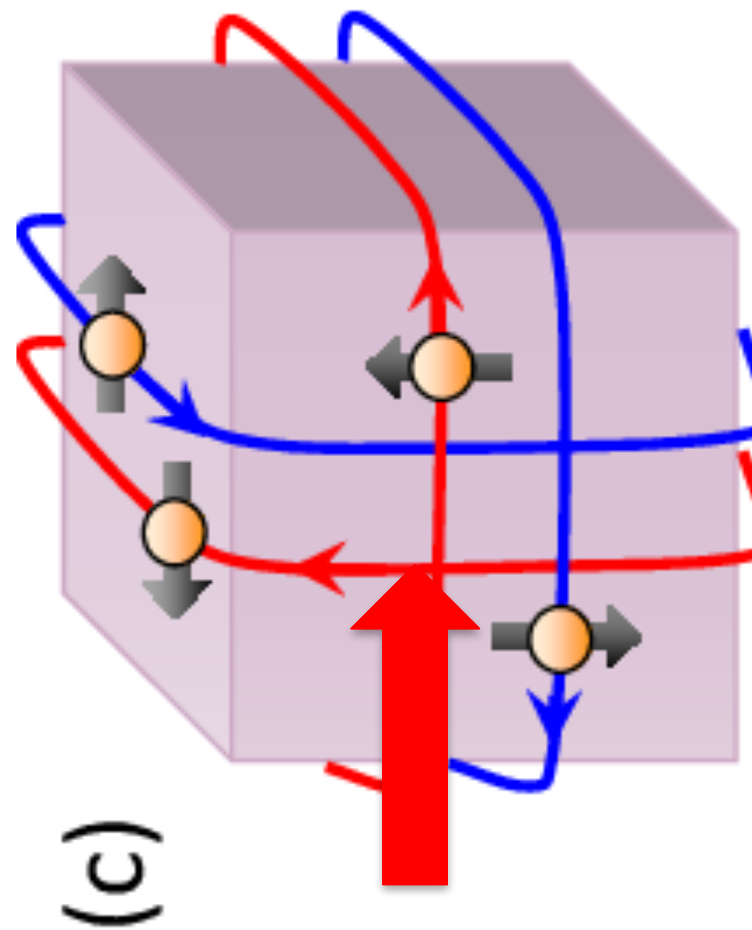
(General classification of phases, see Chen, Gu, Liu and Wen, Phys. Rev.B 87, 155114 (2013).)

Finite temperature CFT if there is an emergent Lorentz invariance.



Topological QCP/Topological Matter

(Bulk/boundary ?)



Towards Higher Emergent Symmetry TPT ?

$$\mathcal{L} = \mathcal{L}_\sigma + \mathcal{L}_\Psi + \mathcal{L}_{\Psi\sigma} + \mathcal{L}^{SSB}_{\{\Psi,\sigma\}};$$

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma(\partial_\tau^2 + v_b^2\nabla^2 - \cancel{m^2})\sigma + g_b\sigma^4;$$

$$\mathcal{L}_\Psi = \frac{1}{2}\overline{\Psi}[\partial_\tau - iv_f(\tau_z\partial_x - \tau_x\partial_y) + \cancel{\mu\tau_y}]\Psi;$$

$$\mathcal{L}_{\Psi\sigma} = -g_{bf}\sigma^2\overline{\Psi}\tau_y\Psi;$$

$$\boxed{\mathcal{L}^{SSB}_{\{\Psi,\sigma\}} = -\lambda_{bf}\sigma\overline{\Psi}\tau_y\Psi + \cancel{\lambda_b\sigma^3}.}$$

Fei, Klenbanov et al., 2017 argues SUSY CFT (3+1) > d > (1+1)

Lorentz symmetry and SUSY need to Emerge to characterize multi-critical topological phase transitions (upper critical d=3+1).

Yang, Jiang and FZ, 2019.

Compressibility anomalies near TPT: Majorana-Fermion universality class

$$\kappa_{\mathrm{NA}}^{2\mathrm{D}} = -\frac{|\mu|}{2\pi v_f^2}\left(1 - \frac{3\lambda_{bf}^2 \Lambda}{2\pi^2 m^2 v_f}\right).$$

$$\kappa_{\mathrm{NA}}^{3\mathrm{D}} = \frac{3\mu^2}{4\pi^2 v_f^3}\left(1 - \frac{\lambda_{bf,3\mathrm{D}}^2 \Lambda_c^2}{2\pi^2 m^2 v_f}\right) \ln \frac{|\mu|}{v_f^2}$$

Yang, Jiang and FZ, 2019.

1D: strongly interacting “Majorana fermi Liquid”

$$\Omega_{\mathrm{NA}}^{1\mathrm{D}} = \frac{\mu^2}{4\pi v_f}\left(1+\frac{\lambda_{bf}^2}{2\pi m^2 v_f}\right)\ln\frac{|\mu|}{v_f^2} + \frac{\lambda_{bf}^2\mu^2}{4\pi^2 m^2 v_f^2}\left(\ln\frac{|\mu|}{v_f\Lambda_c}\right)^2 + \text{higher order terms}$$