Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

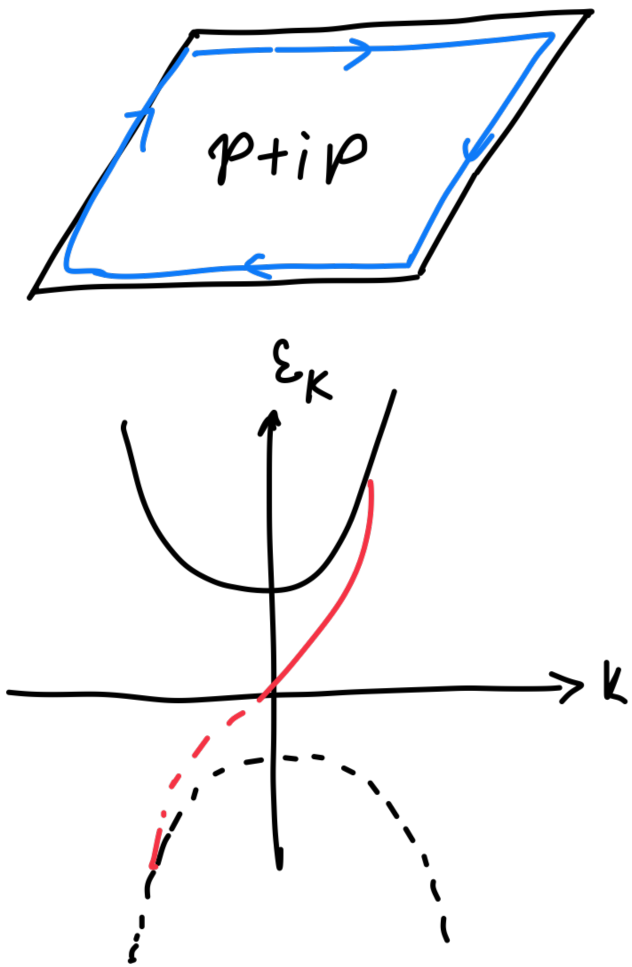
Episode 22: QCPs in TSCs/TSFs II

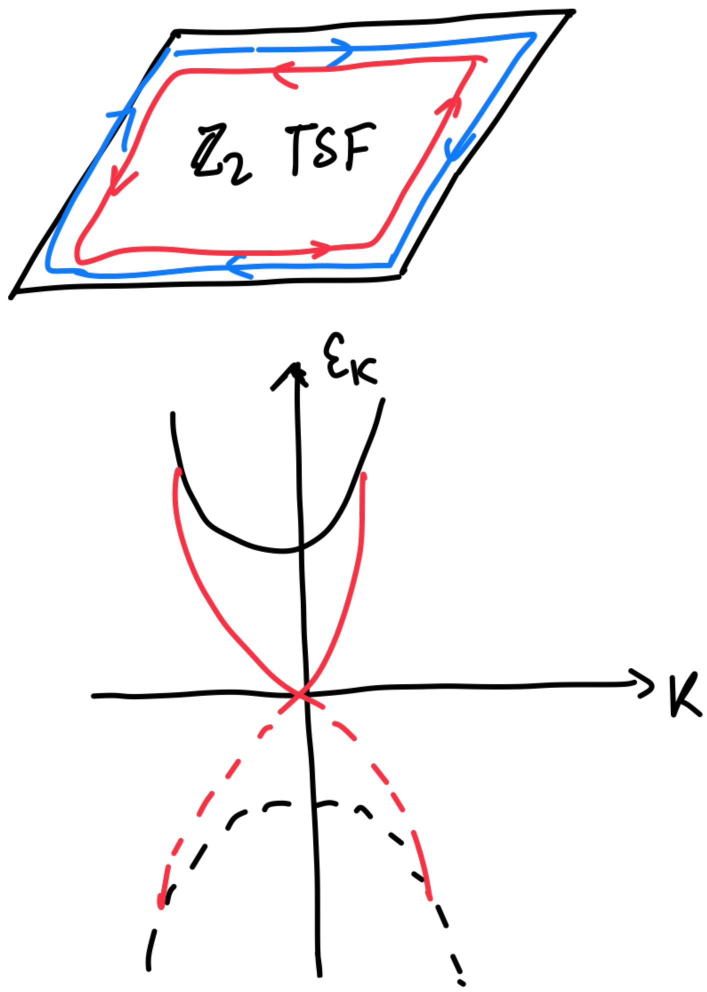
- Topological states are usually characterized by

- I: (non-local) topological order and ground state degeneracy, and sometimes boundaries (examples are strongly interacting FQH, spin liquids with fractionalized excitation; can be related to TQFTs);

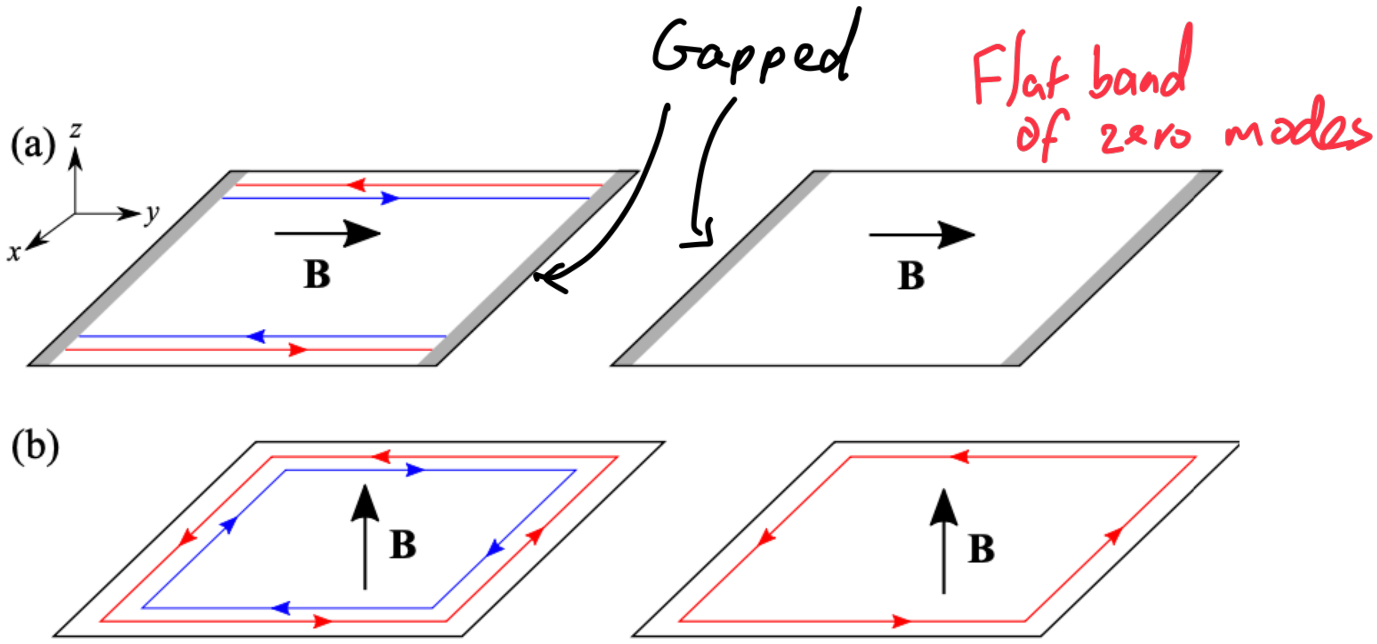
- II: Topological invariants and wavefunction/Hamiltonian manifold topology and boundary states (examples include non-interacting TIs, and interacting TSCs; can be related to TQFTs or QFTs).

Cartoon Picture of edge / boundary states





- Surface states (TRI) in a magnetic field

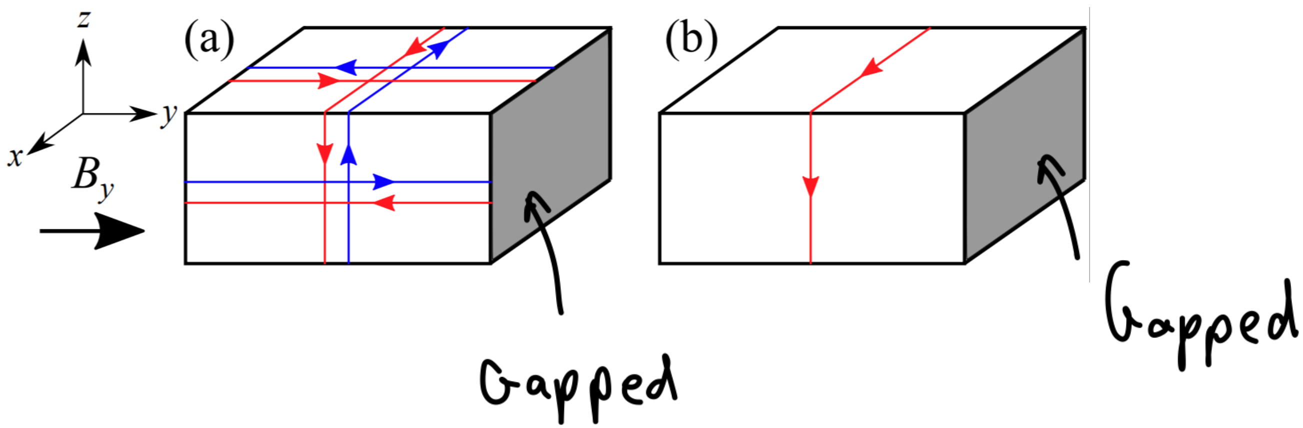


Surface states of TSC

(3D)

TRI

B < B_c B > B_c



- Yang and Zhou, 2020 on topological quantum criticality in TSCs

Topological phase transitions in TSFs/TSCs
(driven by changes of global topologies)

0) Distinct edge/surface states, changes of topological invariants.

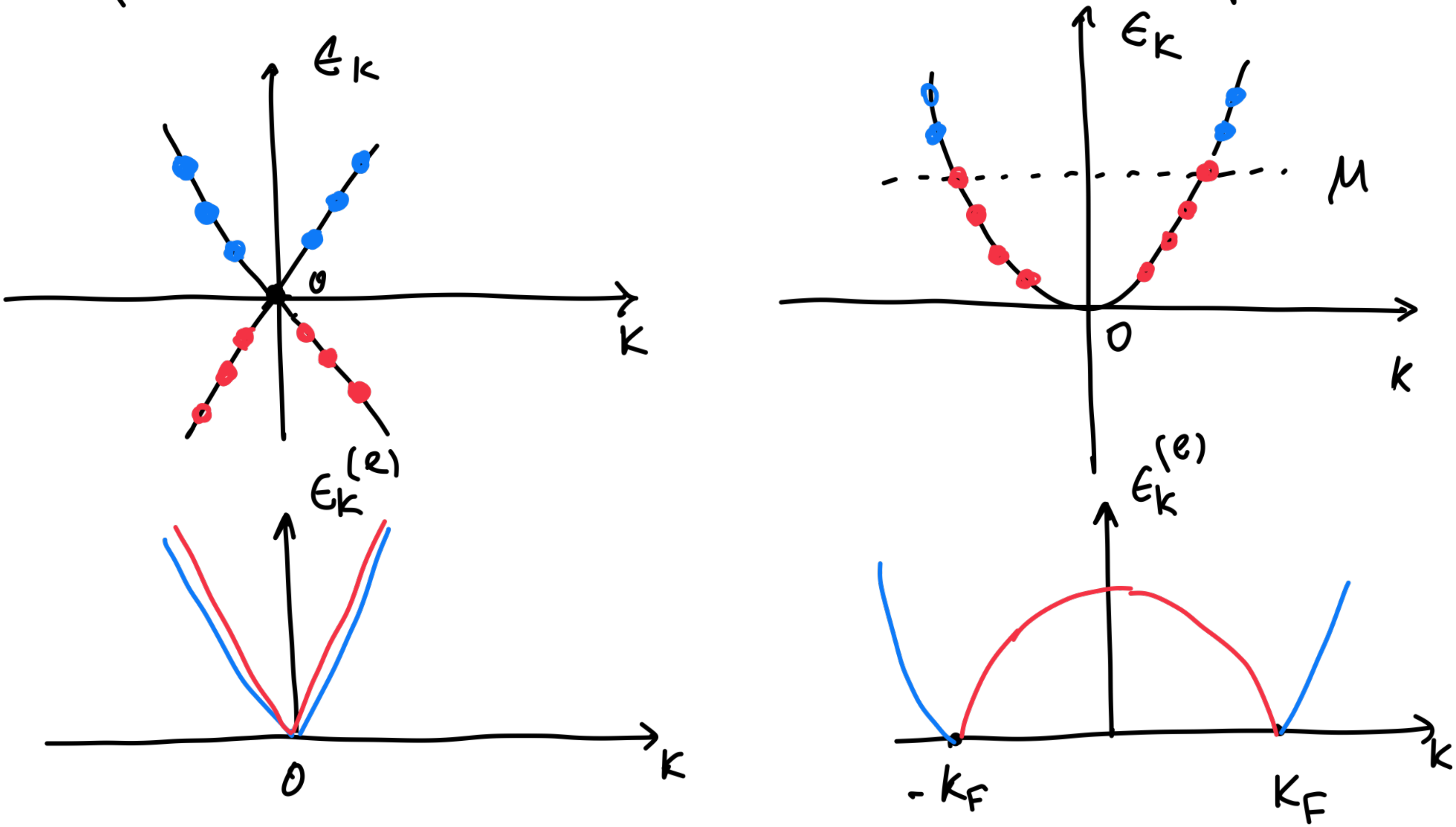
1) Critical point breaks U(1) symmetries, unlike in the standard order-disorder phase transitions;

(i.e. both sides of transitions break the same local symmetries.)

2) Low energy emergent "relativistic" fermions in the bulk with perfect charge-conjugation symmetries (see below).

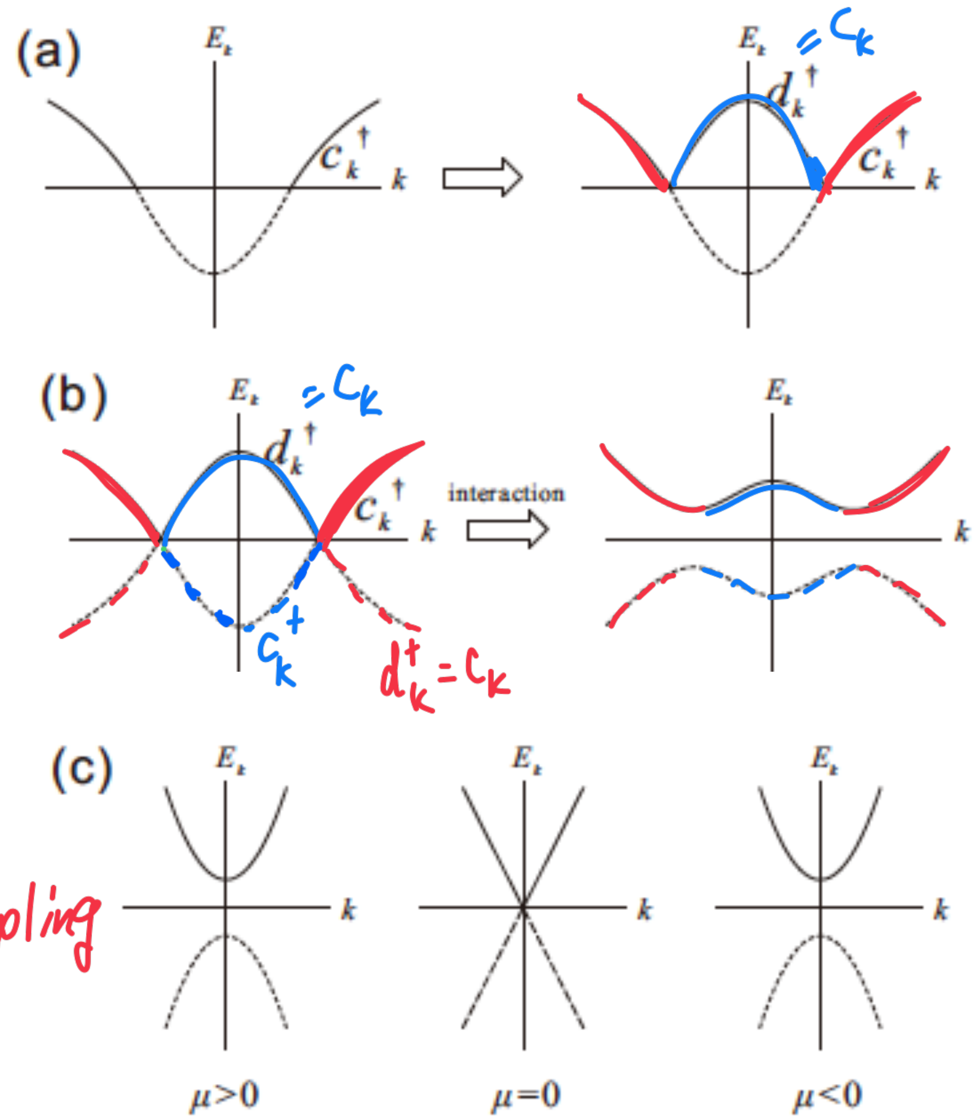
Charge conjugation symmetry Vs particle-hole symmetry

(Dirac) (Fermion Surf)



Excitation spectrum

From Fermi surface to Emergent C-symmetry



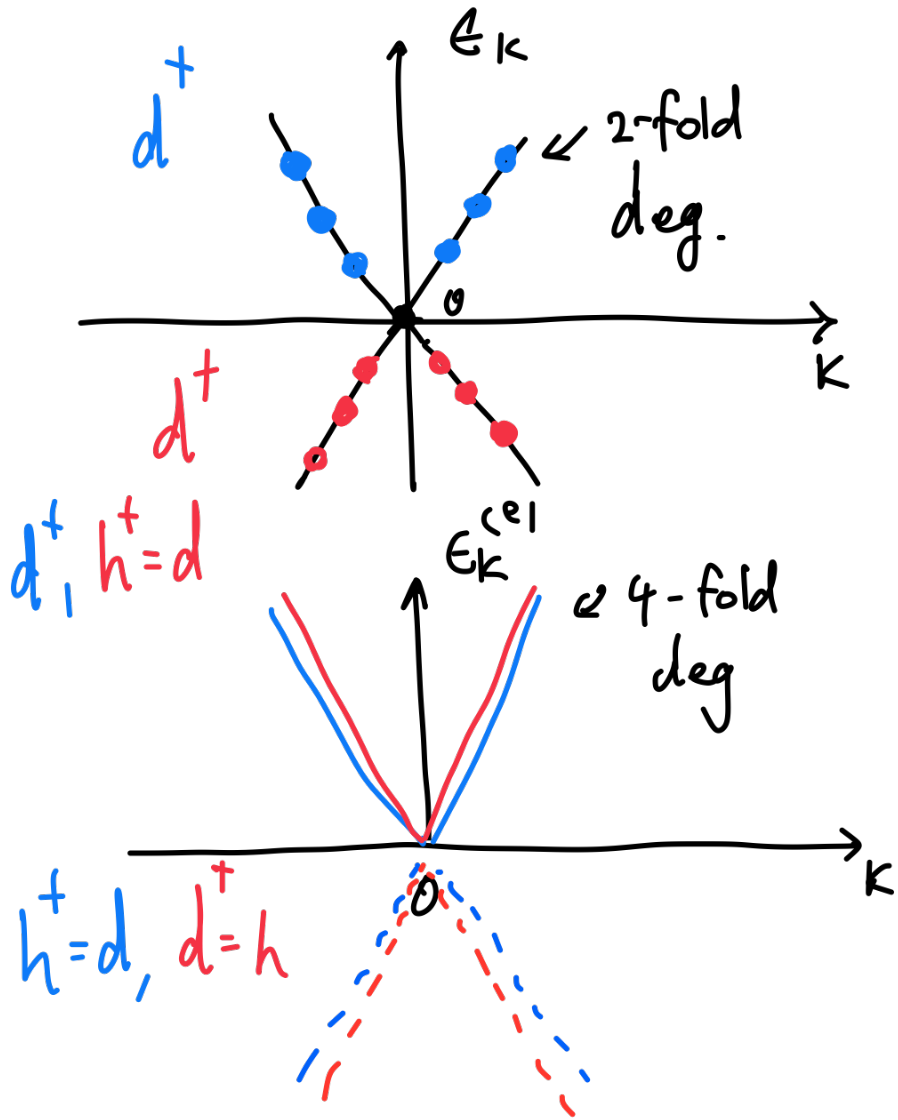
Emergent C-symmetry (induced by U(1) sym. breaking)

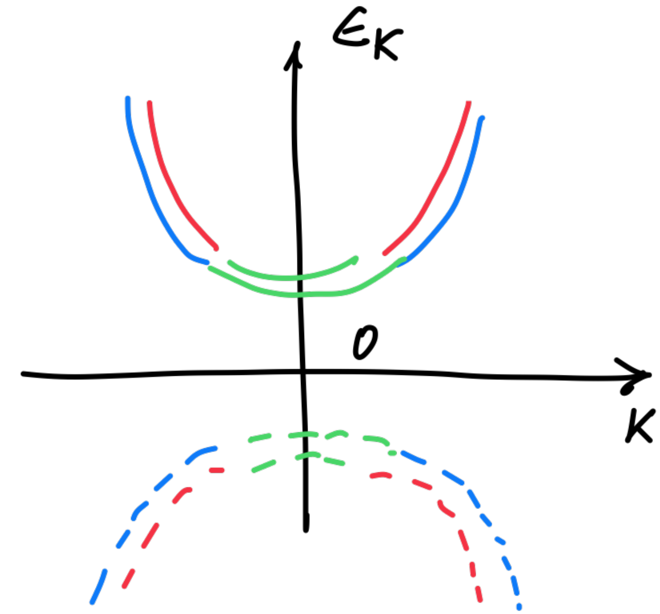
Emergent Lorentz sym. at QCP $\mu=0$

Strong Coupling physics

From Dirac fermions to Majorana fermions

(Real fermion)





$$\chi_1 = \frac{1}{\sqrt{2}}\left(d^{+} + d\right)$$

$$\chi_2 = \frac{1}{i\sqrt{2}}\left(d^{+} - d\right)$$

Topological quantum criticality in TSF/TSCs (driven by changes of global topologies)

How to characterize beyond-Landau-paradigm Transitions between states breaking the same symmetries or with the same local order parameters but different topologies? ?

Cusps near Topological Phase Transitions: Signatures of Majorana fermions and interactions with fluctuations,Yang, Jiang, Zhou, Phys. Rev. ***B*** *100, 054508 (2019).*

Quantum Criticality in Topological Superconductors: Surface criticality, Thermal properties, and Lifshitz Majorana field, Yang, Zhou, ArXiv: arXiv:2012.11143.

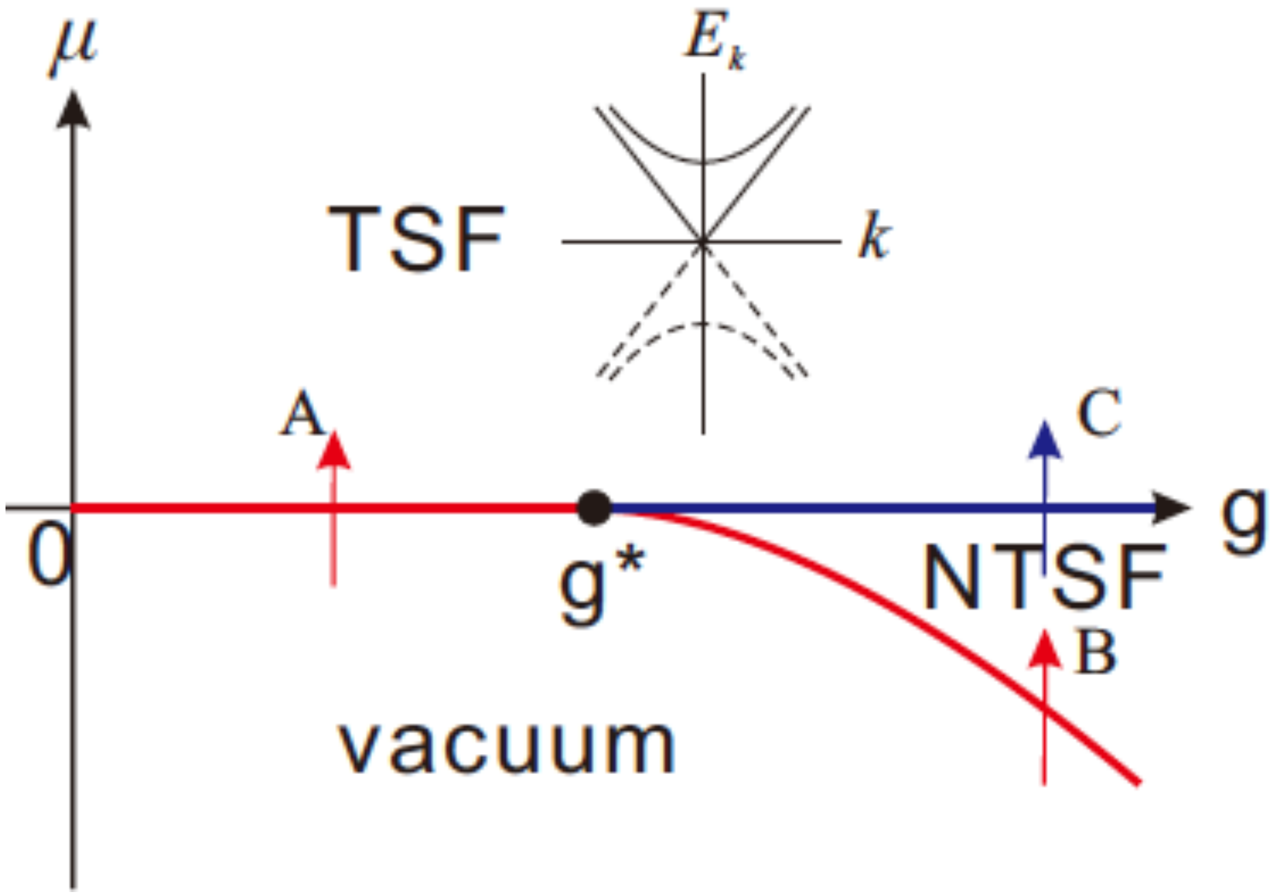
Q1,2,3:

- What are appropriate Effective Field Theories for topological phase transitions in SFs?

- Exists an analogue of strong coupling Wilson-Fisher fixed point/universality ? What is “the upper critical dimension”?

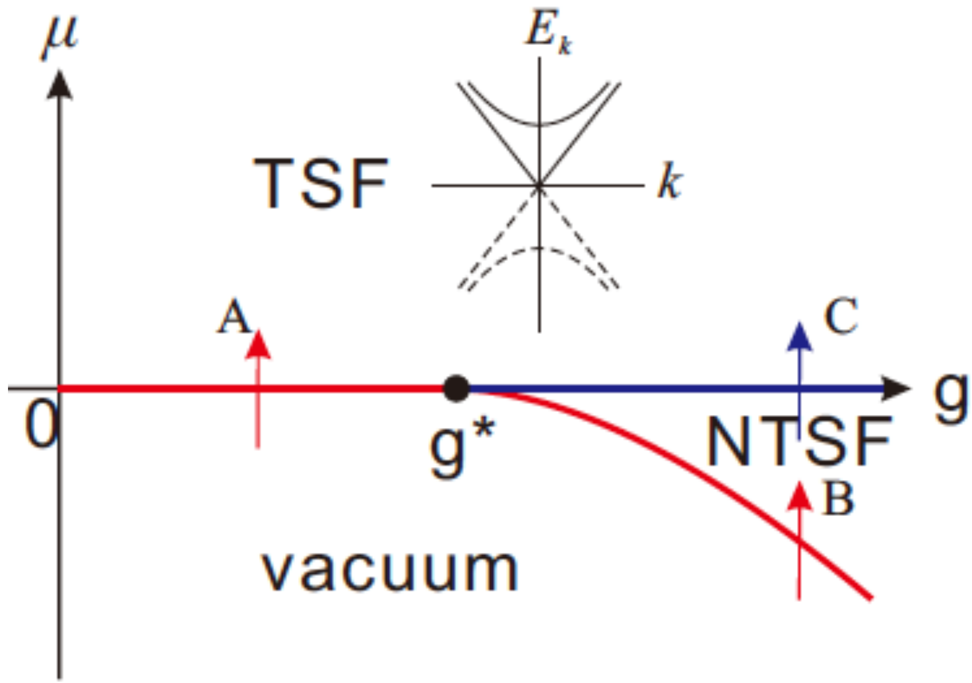
- *What are the bulk signatures of topological phase transitions?*

Example: Phase diagram of p+ip spinless SF



A: Free Fermion; B: Wilson-Fisher/Free boson; C: Majorana class (two component)
g*: QCP of SO(2,1) CFT.

Yang, Jiang and FZ, 2019



1) QCPs in TSCs (d> or =1) with/without T-symmetries belong to *the Majorana fermion universality class.*

2) *It is of (d+1)th order in d-dimension, d=1,2,3.*

3) Robust bulk signatures (i.e. compressibility anomalies)

Generic Model near QCP ($\mu \sim 0$)

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_b + \mathcal{L}_{mb}$$

$$\mathcal{L}_m = v_F \chi^T \left(\Gamma_\alpha \frac{\nabla_\alpha}{i}\right) \chi + \chi^T \mu \Gamma_0 \chi \,, \quad \alpha = x, y, z$$

$$\mathcal{L}_b = \sum_{l=1}^{N'} \varphi_l \left(\partial_\tau^2 - c^2 \nabla_\alpha^2\right) \varphi_l + m_l^2 \varphi_l^2$$

$$\mathcal{L}_{\mu b} = g_{\mu b} \sum_{l=1}^{N'} \varphi_l \; \chi^T \Pi^l \chi$$

$$\Gamma_\alpha = \Gamma_\alpha^T \,, \quad \Gamma_\alpha = \Gamma_\alpha^* \,; \quad \Gamma_0 = -\Gamma_0^T = -\Gamma_0^*$$

$N \times N$ ($\uparrow$ Γ_α)

$$\Pi^l = -(\Pi^l)^T$$

Minimal Model

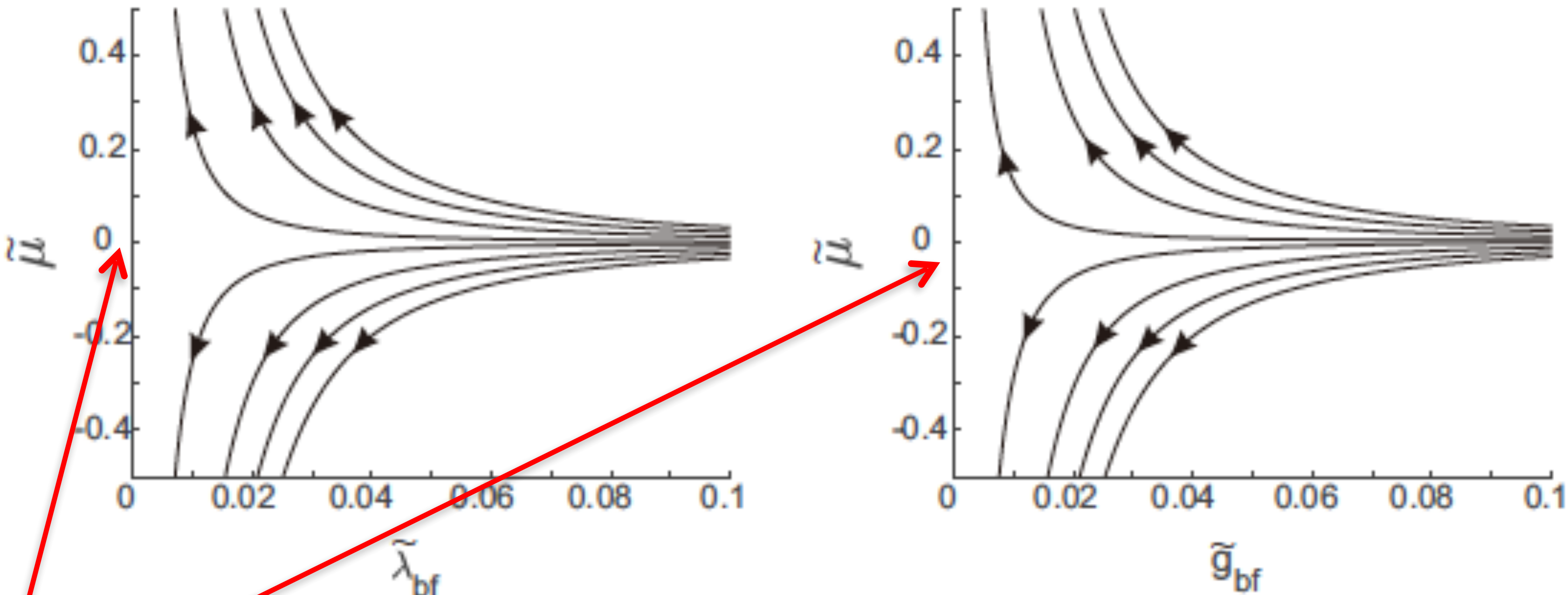
TRB TSC/TSF

$N = 2, \ M' = 1$

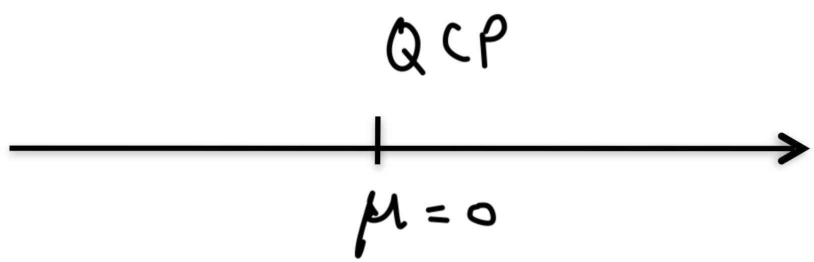
TRS TSC/TSF

$N = 4, \ M' = 1$

Chemical potential



Fixed points



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$$\mathcal{L}_m = v_F \chi^T \left(\Gamma_\alpha \frac{\nabla_\alpha}{i} \right) \chi + \chi^T \mu \Gamma_0 \chi , \quad \alpha = x, y, z$$

$$\mathcal{L}_b = \varphi \left(\partial_\tau^2 - \nabla_\alpha^2 \right) \varphi + m^2 \varphi^2$$

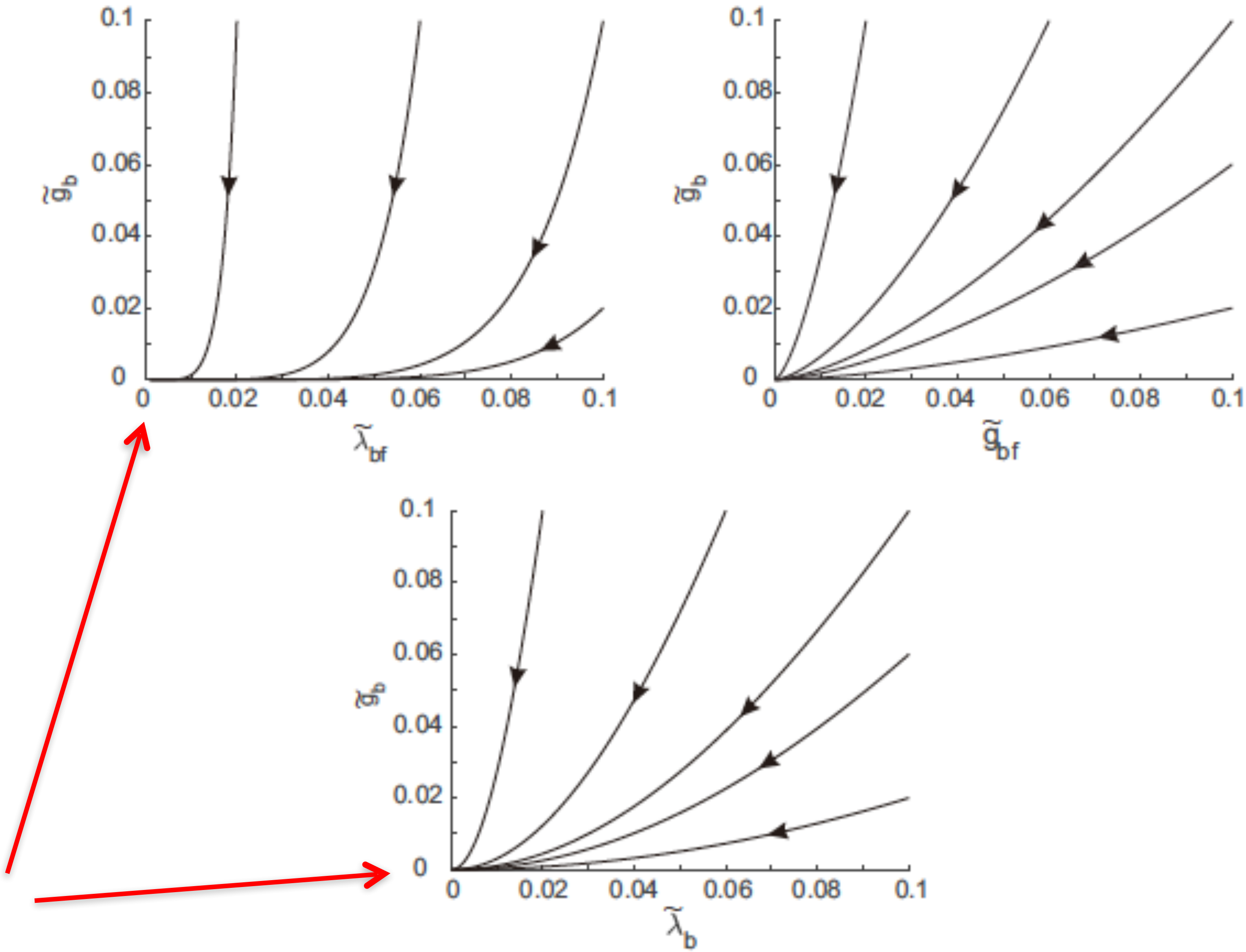
$$\mathcal{L}_{mb} = g_{mb} \varphi \, \chi^T \Pi^\ell \chi$$

$$\Gamma_\alpha = \Gamma_\alpha^T , \quad \Gamma_\alpha = \Gamma_\alpha^* ; \quad \Gamma_0 = -\Gamma_0^T = -\Gamma_0^*$$

$\nearrow$ $N = 2, 4$

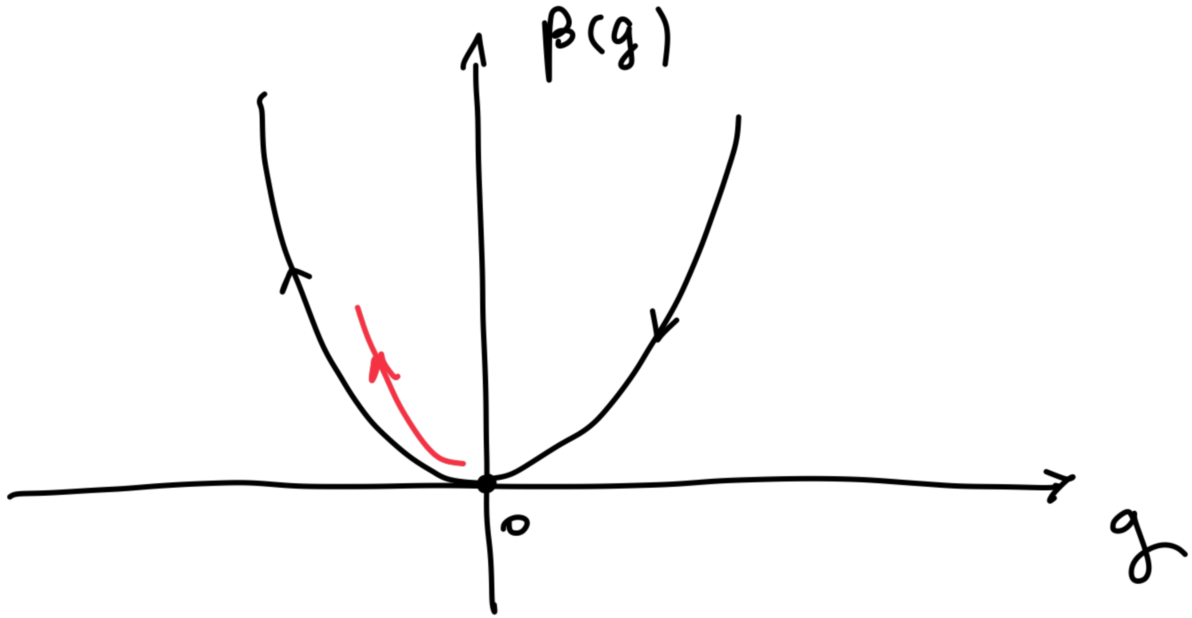
$$\Pi^\ell = -(\Pi^\ell)^T$$

Interactions renormalized away

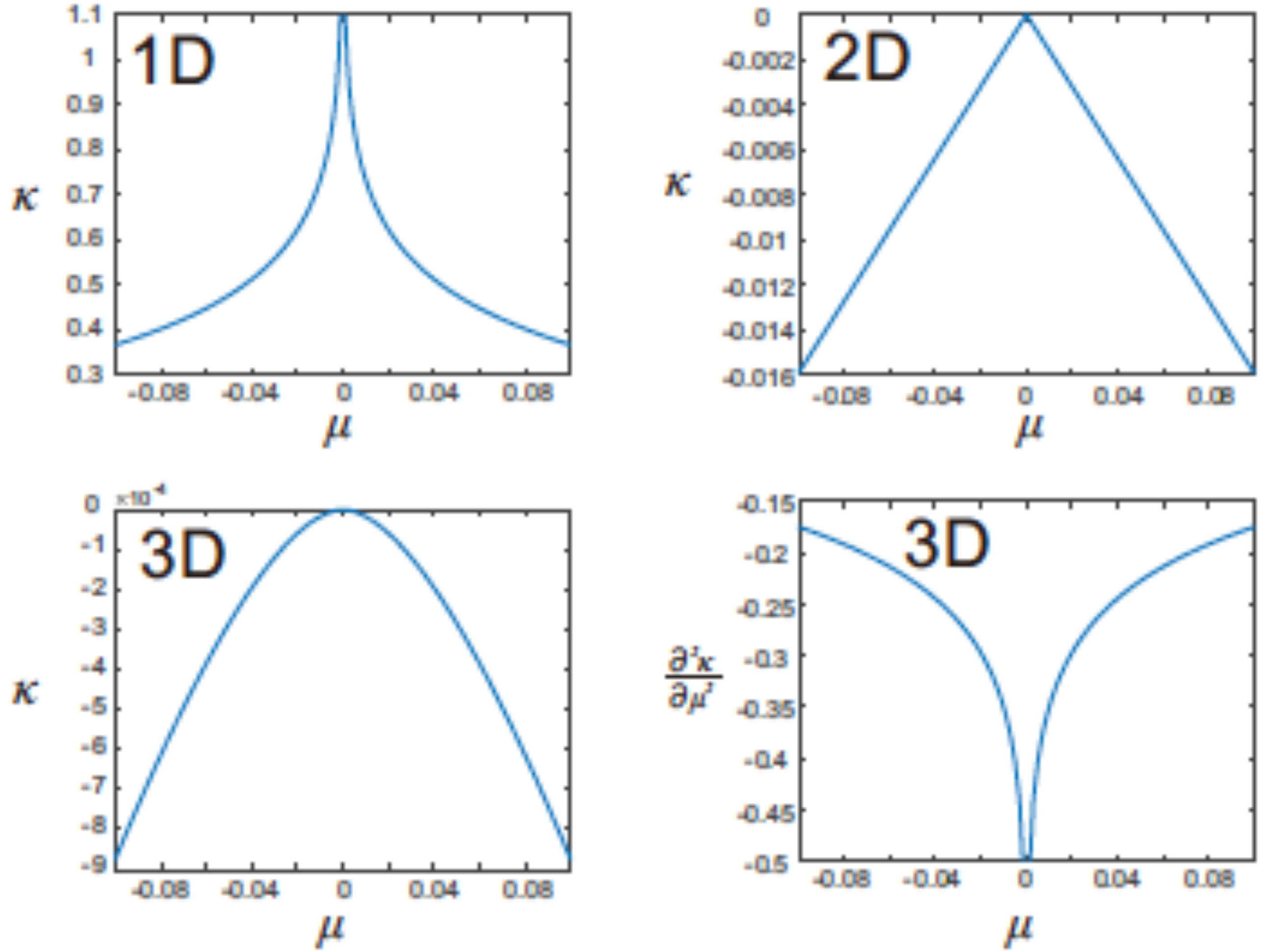


Only Fixed point: a free Majorana Field in d=2,3 SFs with/without TRS

$\mathbb{Z}_2$ $d=1$ Marginal



(d+1)th order topological P.T.
(compressibility versus chemical potential)



Yang, Jiang and FZ, 2019.

Open questions:

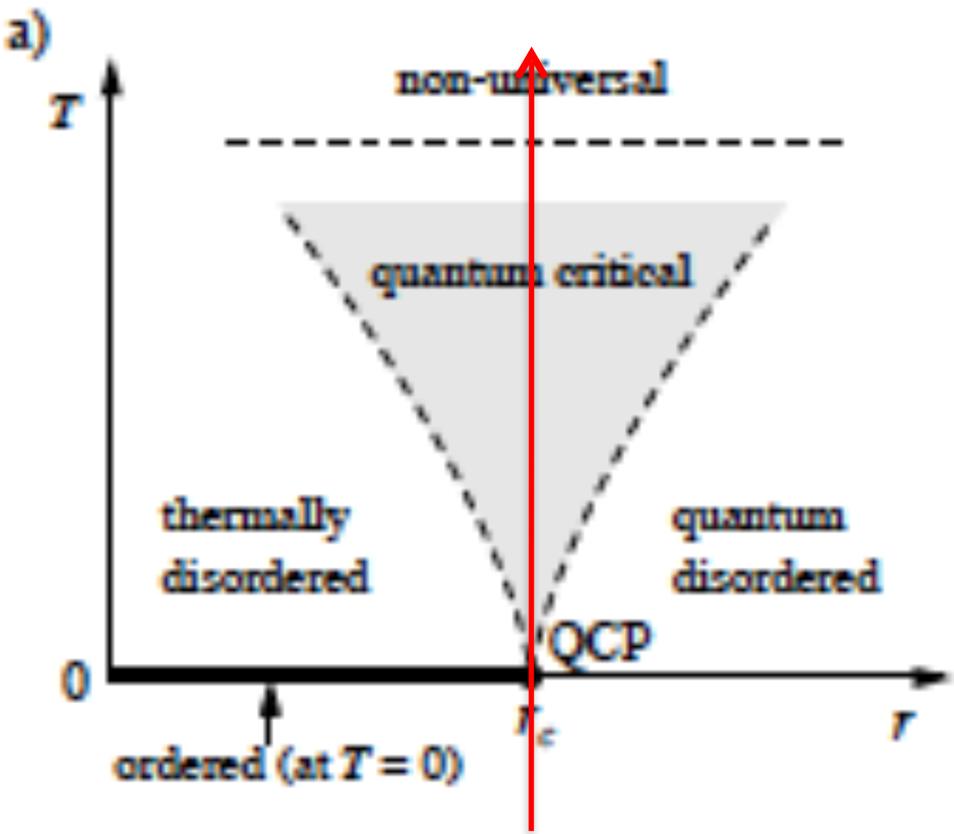
1) Are there finite temperature topological phase transitions in 2D and/or 3D?

2) Can there be TPTs characterized by CFT of majorana fermions with higher emergent symmetries?

3) Can one classify/identify other possible EFTs for TPTs based on the classification theory?

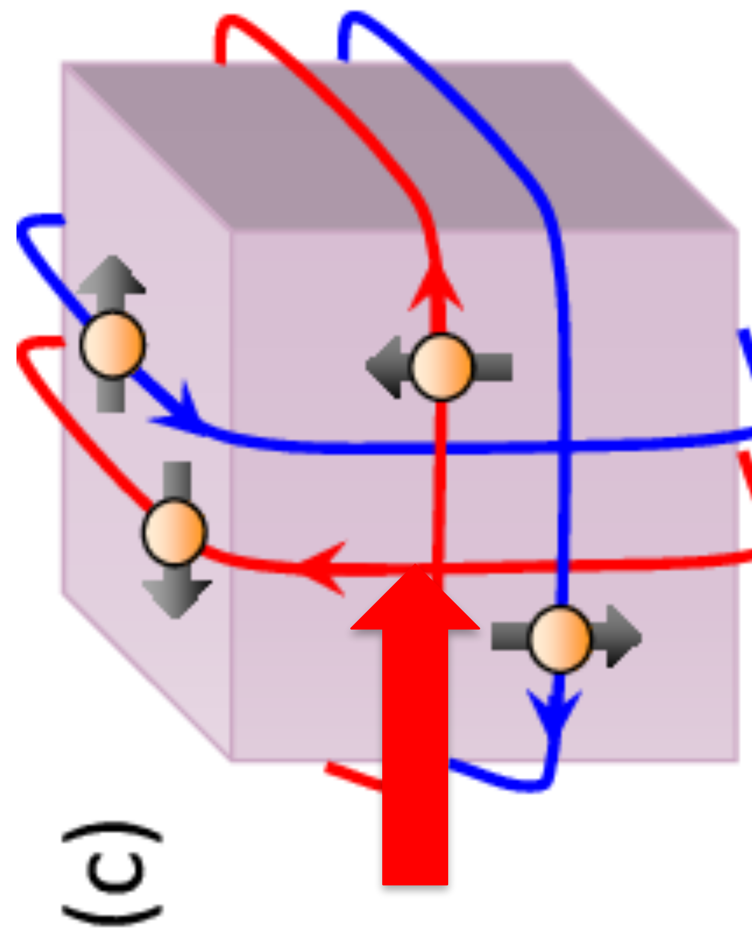
(General classification of phases, see Chen, Gu, Liu and Wen, Phys. Rev.B 87, 155114 (2013).)

Finite temperature CFT if there
is an emergent Lorentz invariance.



Topological QCP/Topological Matter

(Bulk/boundary ?)



Towards Higher Emergent Symmetry TPT ?

$$\mathcal{L} = \mathcal{L}_\sigma + \mathcal{L}_\Psi + \mathcal{L}_{\Psi\sigma} + \mathcal{L}^{SSB}_{\{\Psi,\sigma\}};$$

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma(\partial_\tau^2 + v_b^2\nabla^2 - m^2)\sigma + g_b\sigma^4;$$

$$\mathcal{L}_\Psi = \frac{1}{2}\overline{\Psi}[\partial_\tau - iv_f(\tau_z\partial_x - \tau_x\partial_y) + \mu\tau_y]\Psi;$$

$$\mathcal{L}_{\Psi\sigma} = -g_{bf}\sigma^2\overline{\Psi}\tau_y\Psi;$$

$$\mathcal{L}^{SSB}_{\{\Psi,\sigma\}} = -\lambda_{bf}\sigma\overline{\Psi}\tau_y\Psi + \lambda_b\sigma^3.$$

Fei, Klenbanov et al., 2017 argues SUSY CFT (3+1) > d > (1+1)

Lorentz symmetry and SUSY need to Emerge to characterize multi-critical topological phase transitions (upper critical d=3+1).

Yang, Jiang and FZ, 2019.

Compressibility anomalies near TPT: Majorana-Fermion universality class

$$\kappa_{\mathrm{NA}}^{2\mathrm{D}} = -\frac{|\mu|}{2\pi v_f^2}\left(1 - \frac{3\lambda_{bf}^2\Lambda}{2\pi^2 m^2 v_f}\right).$$

$$\kappa_{\mathrm{NA}}^{3\mathrm{D}} = \frac{3\mu^2}{4\pi^2 v_f^3}\left(1 - \frac{\lambda_{bf,3\mathrm{D}}^2\Lambda_c^2}{2\pi^2 m^2 v_f}\right)\ln\frac{|\mu|}{v_f^2}$$

Yang, Jiang and FZ, 2019.

1D: strongly interacting “Majorana fermi Liquid”

$$\Omega_{\mathrm{NA}}^{1\mathrm{D}} = \frac{\mu^2}{4\pi v_f}\left(1 + \frac{\lambda_{bf}^2}{2\pi m^2 v_f}\right)\ln\frac{|\mu|}{v_f^2} + \frac{\lambda_{bf}^2 \mu^2}{4\pi^2 m^2 v_f^2}\left(\ln\frac{|\mu|}{v_f \Lambda_c}\right)^2 + \text{higher order terms}$$