Surface Plasmon Polaritons

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Surface plasmon polaritons and the various subclasses of quasi particles have a huge application bases. These particles describe conducting electron oscillations which can propagate on interfaces of noble metals with dielectric media. For curtain frequencies, they couple with light and open a way to sub-wavelength optical circuits. A short mathematical description of surface plasmon polaritons and there localized sub class is given in this paper. Furthermore the light interaction and applications in physics are briefly discussed.

1 Introduction

Surface plasmon polaritons (SPP) can be described as photons coupled to the conducting electron excitation in metals [1], or in other words, surface-charge-density oscillations of the free electrons. The applications of the surface plasmon polaritons are mostly located within nano-optics. Nano-optics describe the scattering of light on sub-wavelength structures [2]. This includes optical communication, chemo- and bio sensors and optical circuits, which have potential applications in the further miniaturization of the modern chip industry [1].
This report is divided into three sections. First, the theoretical description of surface plasmon polaritons, including details on a more classical approach used around elector-dynamics, will be discussed. This will be followed by a description of the special case of the localized surface plasmons (LSP). The next section will cover the interaction of light with surface plasmon polaritons. Lastly, concrete applications, fabrications and measuring methods will be summarized.

2 Description of surface plasmon polaritons

The easiest case one can theoretically imagine is an interface between two media, one of which is a metal that has a frequency dependent complex dielectric function. The upper half-space is described by a dielectric constant, for example a vacuum. This is seen in fig. 1. One also takes all the nomenclature as given in the figure.

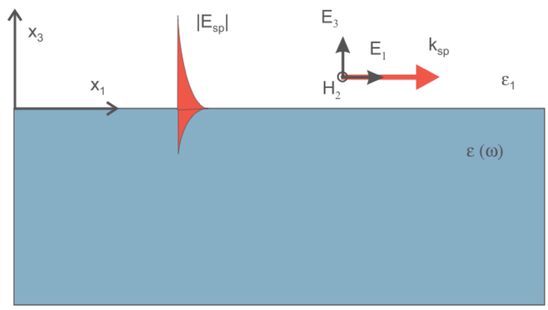

Figure 1: A schematic sketch of surface plasmon polaritons propagation on an interface. Taken out of Fig.1 [1]

The eigenmodes on the interface are then found, characterized by Maxwell's equation, in this case given by the homogeneous wave equation,

$$\nabla \times \nabla \times E(r,\omega) - \frac{\omega^2}{c^2}\epsilon(r,\omega)E(r,\omega) = 0. \quad (1)$$

It should be kept in mind that one is searching for solutions that converges to zero if $|x_3| \to \infty$. Boundary conditions must also be enforced on the interface.
It can be shown that the transverse electric mode does not have a solution in this case [2].

With knowledge of the ansatz of a plane wave propagating along the surface, the dispersion relation for the transverse magnetic mode can be written as follows [1];

$$k_{\mathrm{sp}} = \frac{\omega}{c}\sqrt{\frac{\epsilon_1 \epsilon(\omega)}{\epsilon_1 + \epsilon(\omega)}}. \tag{2}$$

If it is further enforced that the propagation is bounded in both media,

$$\begin{aligned} k^2 &= \frac{\epsilon^2(\omega)}{\epsilon_1 + \epsilon(\omega)} k_0^2 \\ k_1^2 &= \frac{\epsilon_1^2}{\epsilon_1 + \epsilon(\omega)} k_0^2. \end{aligned} \tag{3}$$

When the imaginary part of the dielectric function is neglected, it becomes evident that $\epsilon(\omega) < 0$ has a larger value than ϵ_1, because the electric fields should decay exponentially in both media, so the values for k and k_1 become imaginary. This is, for example, the case for gold in the wavelength range of $1\,\mu$m to 550 nm or up to 350 nm for silver [3]. These kinds of noble metals are often used when one builds devices, and is a direct use of SPP interactions [4].

2.1 Localized surface plasmons

Surface plasmons are a subcategory of surface plasmon polaritons and can be described as a solution of the Laplace equation (electrostatic potential), which is propagating as a wave along a surface [2].

Localized surface plasmons are of particular interest in the field of nano-optics. They describe modes of surface plasmons that oscillates in small structures. Here, the easiest case is taken to be a small metal sphere with radius a (normally in the order of 10 to a view 100 nm) [2].

The Laplace equation is firstly written in spherical coordinates,

$$\begin{aligned} &\frac{1}{r^2 \sin(\theta)}[\sin(\theta)\partial_r r^2 \partial_r + \partial_\theta \sin(\theta)\partial_\theta + \\ &\qquad \frac{1}{\sin(\theta)}\partial_\varphi^2]\Phi = 0, \end{aligned} \tag{4}$$

which has a solution known to be the superposition of

$$\Phi_{l,m} = \left\{ \begin{array}{c} r^l \\ r^{-l-1} \end{array} \right\} \left\{ \begin{array}{c} P_l^m(\cos(\theta)) \\ Q_l^m(\cos(\theta)) \end{array} \right\} \left\{ \begin{array}{c} e^{im\varphi} \\ e^{-im\varphi} \end{array} \right\}, \tag{5}$$

where l, m are integers and P, Q are the Legendre functions of the first and second kind.

If this nanosphere in put in a linear electric field along one axis, such as the x_1 axis, $\Phi_0 = -E_0 x = -E_0 r P_1^0(\cos(\theta))$ can be chosen due to the presence of rotational symmetry. The boundary conditions for $r \to 0$, $r \to \infty$ and the surface of the nanosphere $r = a$ can be used to then obtain the following result for Φ_1 in the sphere and Φ_2 for the rest of the space;

$$\begin{aligned} \Phi_1 &= -E_0 \frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} r\cos\theta \\ \Phi_2 &= -E_0 r\cos\theta + E_0 \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + 2\epsilon_2} \frac{a^3 \cos\theta}{r^2}. \end{aligned} \tag{6}$$

The second term in the potential outside of the sphere can be identified as the scattered field of the nanosphere.

The second thing to notice is the linear field inside the sphere, which is no surprise, because the sphere in our assumptions is smaller than the skin depth of the metal.

The last step to take is going from the static approximation to a plane wave scattering of the electric field on the nanosphere. For this, the scattered cross-section is defined as[2],

$$\sigma = \frac{8\pi k^4 a^6}{3} \left| \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + 2\epsilon_2} \right|^2. \tag{7}$$

This can be plotted for the dielectric functions of gold and silver with different surrounding materials, as seen in figure 2.

It can be seen that for certain wavelengths and surrounding materials, plasmonic resonances are obtained. For gold, these are located near larger wavelengths but are considerably weaker. For silver, the resonances are at shorter wavelengths. This is unsurprising since, in section 2, it was observed that silver has a negative dielectric function for shorter wavelengths. Despite this Gold is used more often in nano-optics, because mono crystalline flaks can be grown more easily.

Furthermore, by changing the geometry or type of wave, such as circular polarized light, the response and wavelength of a device can be tuned. For non-trivial examples like a cross-antenna [5], the Laplace equation cannot be solved analytically so numerical software such as the finite element method is required.

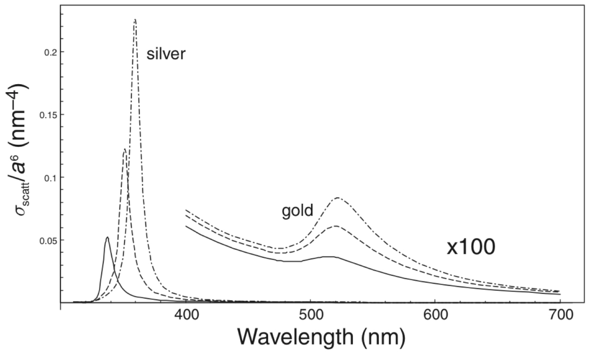

Figure 2: Scattering cross section of a gold or silver nanosphere following excitation with a plane wave, with the surrounding medium being n=1 for a vacuum (solid), n=4/3 for water (dashed), n=3/2 for glass (dash-dotted). Taken out Fig. 12.22 [2]

3 Light surface plasmon polaritons excitation

Momentum and energy conservation must be obeyed in the excitation of single SPPs. Since the free space light dispersion is not crossing the dispersion curve of the SPP (see also figure 4), the SPP must be excited via a different method; frustrated internal reflection. Either a thin slab of metal or air between a higher refractive medium is used and the evanescent wave is aimed so the angle of incidence is below the critical angle causing the wave vector to remain in the higher refractive medium $k = k_0 n$ [2].

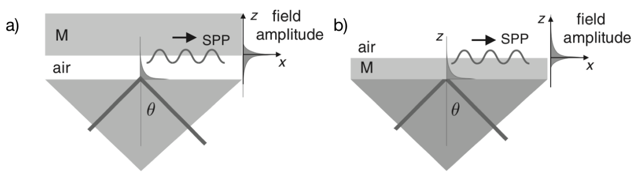

Figure 3: a) is the Otto configuration and b) the Kretschmann configuration. In both cases **M** denotes the metal. The light beam under the critical angle θ in the high refractive medium is shown in dark grey. Taken out Fig. 12.8b-c [2] and added.

There are two common setups: the Otto configuration, which consists of metal, air and a high refractive medium respectively, and the Kretschmann configuration, which consists of air, metal, high refractive medium respectively. In both configurations, the plasmon propagates along the metal-air interface. Both configurations are seen in figure 3.
For both cases, the resonance frequency at the point where the dispersion curve of the surface plasmon polaritons and the refractive light curve intersect can be found, as seen in figure 4.

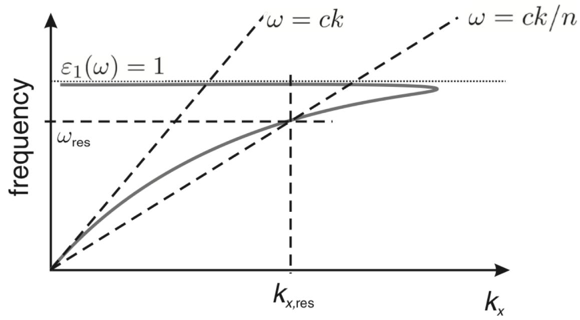

Figure 4: Dispersion relations of light in a vacuum and a high refractive medium. The intersections with dispersion curve of the surface plasmon polaritons show a clear excitation frequency. Taken out Fig. 12.8a [2]

4 Application and Fabrication

The most famous applications of SPP devices are surface enhanced raman spectroscopy and the construction of wave guides for highly integrated photonic circuits [6].
Raman scattering was discovered in 1928 and describes the modulation of optical fields, which get modulated with vibration modes of molecules. The vibration modes are unique to each type of molecule and hence have a fingerprint region which can be used in identification [2]. Normally, enhancement factors can be observed in the order of $10^2 - 10^3$ for the resonance frequencies. When suitable plasmonic devices are used, this enhancement in intensity can be amplified to $10^5 - 10^6$ [6].
One disadvantage is that the surrounding dielectric medium, such as water or air, get contaminated with the molecules under observation.

This change in the dielectric function of the surroundings makes the calculations shown previously, analytically unsolvable. Therefore, numerical methods are required for the design of these structures [2].
Normal optical waveguides are limited by the Abbe limit despite having non localized confinement in the wave guide [6] [7]. Nano-particles and nano-wires can be used to build plasmonic waveguides, although they are usually not long range and experience high damping due to surface roughness of the noble metals. Other devices used in optical wave guides could be rebuilt with the use of SPPs, like the Mach–Zehnder interferometer and beam splitters. In addition, the gap between a nano-wire and the substrate can be used as a capacity like coupler of optical and plasmonic modes [6].
Also of interest are plsamonic-nano-antennas, which are used for photonic electrical interfaces in photonic circuits [2]. They come in antenna-like fashion, such as Yagiuda-, dipol- or crossed-antennas. Because the structures are smaller then the wavelengths, they are not characterized by the inductance, conductivity and capacity, but rather the plasmonic modes. This is unsurprising since the skin depth approximation for such characteristics does not hold at all [2].
To produce such nano-structures, one uses all modern semiconductor growth and etching techniques. A common process is the growth of single crystalline gold flakes and later structuring with ion beam milling [2]. Especially the ion milling with smaller ions such as helium makes it possible to fabricate even smaller structures in the order of a view nm. An example of a crossed nano-antenna is seen in Fig. 5.

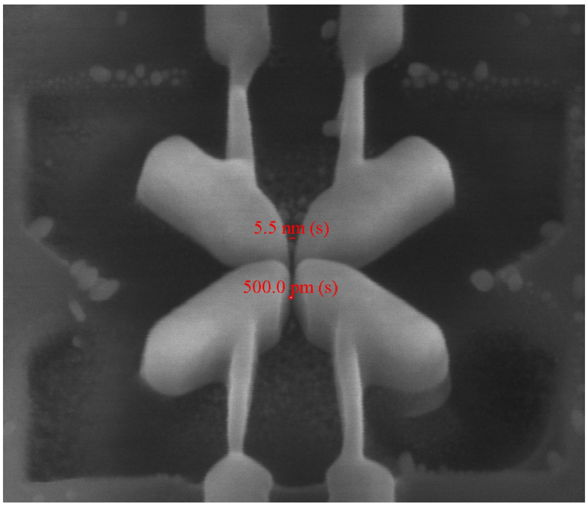

Figure 5: Crossed nano-antenna with a gap of 5 nm [8]

5 Summary

The mathematical description of surface plasmon polaritons is given by Maxwell's equations, which is searching for bounded propagating modes on the interface between two dielectric media. These modes only exist when one media has a high negative part of the dielectric function, such as noble metals like silver and gold.
Furthermore, it was shown that localized surface plasmons could lead to high scattering cross sections and hence are able to couple on external electric fields.
The two most commonly used techniques to excite surface plasmon polaritons were introduced and many related applications such as optical wave guides and nano-antennas were presented. With even better fabrication techniques the field of nano optics and plasmonics should see an increasing amount of attention in the future.

References

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