Holographic Superconductors

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Abstract

We give an outline of how a gravitational dual of a superconductor can be obtained using holographic constructions. We show that a gravitational system with an AdS black hole can shed light on the theoretical understanding of the high T_c superconductivity experimentally observed in many condensed matter systems. We also introduce the AdS/CFT correspondence as a useful computational tool to study condensed matter systems with strong couplings.

1 Introduction

The AdS/CFT duality states that certain quantum field theories known as conformal field theories (CFT) *without* gravity in d spacetime dimensions are related to gravitational systems in $d+1$ spacetime dimensions. It is also a strong-weak duality – meaning strongly coupled systems can be described by weakly coupled "dual" systems using the correspondence. That means one can perform computations on one side of the duality that are hard or impossible to do on the other side. This is great – because in many condensed matter systems, the coupling constants characterizing the strengths of various interactions are not small. As a result, the usual methods of perturbation theory break down. If such a strongly-coupled system has a gravitational dual, it is often possible to perform various calculations on the gravity side. Then knowing how different quantities are related to each other by the duality, the behavior of the system under consideration can be understood. This is precisely the reason why the AdS/CFT correspondence, even though originally introduced in the context of string theory [1], has found numerous applications in condensed matter systems such as quantum phase transitions, non-fermi liquids, strange metals, hydrodynamics etc. [2, 3].

One particular application of the AdS/CFT correspondence in condensed matter systems is to understand the behavior of high T_C superconductors. In this note, we shall construct a gravitational dual of a superconductor and the high T_c behaviors will follow naturally from the model. The subject of holographic superconductors is a vast one and have been studied extensively over the last decade [4, 5, 6]. We shall make no attempt to cover the vast details of the subject. Rather, we shall give a brief overview motivated from a condensed matter perspective. In particular, we shall not give the details of the holographic computations, for which, we refer the reader to the excellent reviews [7, 8].

The organization of this note is as follows. In section 2, we give a lightening review of superconductivity. In section 3, we introduce the AdS/CFT duality as a computational tool for condensed matter theorists. Finally, in section 4, we construct the holographic dual of superconductors and explain various features.

2 Superconductivity

In 1911, Onnes found that the resistivity of Mercury drops to zero when cooled down to 4.2K, marking the discovery of superconductivity. It was later discovered that the electrical conductivity of most metals suddenly drops to zero when their temperature T goes below a critical value T_c. In 1950, Landau and Ginzburg gave a second order phase transition description of superconductivity [9]. The key ingredient of such phase transition is the presence of an *order parameter* ϕ which takes a non-zero value $|\phi| \neq 0$ when $T < T_c$ and $|\phi| = 0$ when $T > T_c$ marking the superconducting and ordinary phases respectively. The density of superconducting electrons is given by $n_s = |\phi|^2$. The free energy in terms of ϕ takes the form

$$F = \alpha(T - T_c)|\phi|^2 + \frac{\beta}{2}|\phi|^4 + ... \qquad (1)$$

where α, β are positive constants. When $T > T_c$, the minimum value of F is at $|\phi| = 0$ and there is no superconductivity. However, when $T < T_c$, the quadratic term is negative and F is minimized at

$$|\phi|^2 = \sqrt{\frac{-2\alpha(T - T_c)}{\beta}}. \tag{2}$$

This is the superconducting state.

A more complete theory of superconductivity is given by the BCS theory [10] where the basic idea is that electron-phonon interaction can cause two electrons with opposite spin to bind and form a composite charged boson known as a Cooper pair. When $T < T_c$, a second order phase transition occurs and the boson condensates creating a superconducting phase. One characteristic feature of the BCS theory is the existence of a gap $E_g = \omega_g = 2\Delta$ required to break a Cooper pair into two free electrons [1]. This gap is related to the critical temperature by $\omega_g \approx 3.5T_c$.

The highest T_c for a BCS superconductor was believed to be $\approx 30K$. However, there exists superconductors such as cuprates [11] with much higher T_c than those of BCS superconductors. This high T_c signals strong electron-phonon coupling hinting towards the expectation that the high-T_c superconductivity can be described by some strongly coupled quantum field theory. In condensed matter theory, there are not many theoretical tools to study strongly coupled theories. As mentioned before, the AdS/CFT correspondence allow us to study strongly coupled field theories. In the subsequent sections, we shall see that we can construct simple a gravitational system within the AdS/CFT framework that can reproduce basic properties of superconductors.

3 AdS/CFT correspondence: a computational toolkit

Conformal Field theories (CFTs) are a subset of quantum field theories invariant under a set of mathematical operations known as the conformal transformations. In particular, it is invariant under scale transformation meaning that the physics looks the same at all length scales. CFTs describe second order phase transitions in statistical mechanics. This is precisely the reason we are relying on CFTs to describe superconductivity on the field theory side. On the other hand, Anti - de Sitter (AdS) spacetime is a spacetime with constant *negative* curvature.[2] The duality relates different states in the CFT (referred as *"boundary" theory*), defined on a d dimensional spacetime, to different geometry of the dual $d+1$ dimensional AdS spacetime (referred as *"bulk" theory*) as shown schematically in figure [1].

[1]We work in the natural unit $c = \hbar = k_B = 1$

[2]In contrast to a space with *positive* curvature e.g. a sphere.

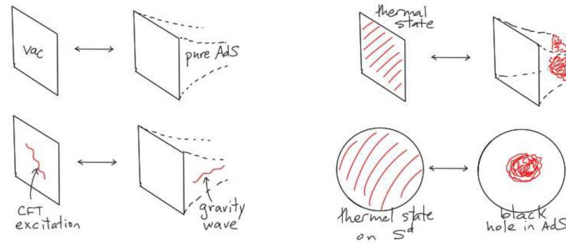

Figure 1: Different states in CFT corresponds to different AdS geometry taken from [12]

Since our intention is to describe a superconducting system where the temperature $T \neq 0$, the dual gravitational system we need to consider must include a black hole as evident from the right panel in figure 1. This is not very surprising – as shown by Hawking, black holes can be considered as thermodynamic objects with a temperature, T known as the *Hawking temperature*, given by $T = \frac{\kappa}{2\pi}$, where κ is the surface gravity. The CFT temperature is dual to this Hawking temperature of the black hole. Other quantities in the bulk and boundary theory related by the duality that will relevant for our discussion of holographic superconductors are [13]:

Boundary (CFT)	Bulk (AdS)
Scalar operator $\mathcal{O}_b$	Scalar field ϕ
Fermionic operator $\mathcal{O}_f$	Dirac field ψ
Global current J_μ	Maxwell field A_a

As mentioned before, as condensed matter theorists, we shall mostly be interested in using the duality as a computational tool. In particular, it can be used to compute various transport coefficients. To make this point clear, let's consider the response of a system where we have an external field $A_0(x)$ coupled to an operator $O(x)$ through the interaction Hamiltonian

$$H_{int} = -\int dx\, O(x,t) A_0(x,t) \tag{3}$$

Using the linear response theory one can prove that the change in expectation value of $O(x)$ due to this coupling with the external field is given by

$$\delta\langle O\rangle = \int dx' G^R(x,x') A_0(x') \tag{4}$$

where $G^R(x,x') = -i\theta(t-t')\langle[O(x), O(x')]\rangle$. The transport coefficient χ is defined as:

$$\chi(\omega) = -\lim_{k\to 0} \frac{1}{\omega} G^R(k,\omega) \tag{5}$$

where $G^R(k,\omega)$ is the fourier transform of $G^R(x,x')$. When the coupling field is a gauge field A_μ and the operator O is electric current density J, we identify the transport coefficient χ to be the dc conductivity σ.

For strongly-coupled quantum field theories, it is often very difficult to compute G^R directly. However, one can compute this quantity easily in the weakly-coupled dual AdS theory using the AdS/CFT correspondence. See [14] for a review.

4 Holographic Superconductor

In this section we shall construct a gravity theory that can describe superconductivity in the dual CFT. To be more precise, we wish to describe a second order phase transition in AdS spacetime – equivalent to the Landau-Ginzberg phase transition described in section 2. Most of the cuprate compounds that exhibit high-T_c superconductivity are two dimensional planar materials. Hence, we take our CFT to be $2+1$ dimensional [3]. As a result, the holographic theory is four dimensional.

As mentioned in the previous section, in order to describe a thermal system, we must include a black hole in the gravitational theory. We would like to measure conductivity using some current density J_μ. So, in the dual theory we require Maxwell field A_a as shown in the table. They key ingredient we need to describe a phase transition is an order parameter in the *bulk* that will condense like the Cooper pair below the critical temperature T_c and will vanish when $T > T_c$. The simplest kind of *holographic superconductor* that can be constructed using the above mentioned ingredients, use a charged scalar field ϕ, the dual of which will play the role of the order parameter in the CFT. The Lagrangian that includes such an Einstein-Maxwell-Complex scalar system is given by:

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F_{\mu v}F^{\mu v} - |\nabla\phi - iqA\phi|^2 - m^2\left|\phi^2\right| \quad (6)$$

Here R is the Ricci scalar of the curved AdS space, L is the AdS radius, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell field strength tensor, A_μ is the gauge field and ϕ is a scalar field with charge q. This ϕ is related to the order parameter $\langle \mathcal{O}_b \rangle$ of the CFT characterizing superconducting phase transition.

The system given by eqn (6) can be solved either numerically or by making various simplifying assumptions such as the "probe limit" [8] . The details of the calculations can be found in [8]. A numerical result of the order parameter $\langle \mathcal{O}_1 \rangle \equiv \langle \mathcal{O}_b \rangle$ of the CFT as a function of T/T_c is given in figure 2. It is clear when $T < T_c$, the order parameter is non-zero and we have the superconducting state. When $T = T_c$ the order parameter vanishes describing the normal state. Hence, our holographic model includes the features of Landau-Ginsburg like second order phase transitions.

[3]Two spatial dimensions plus time

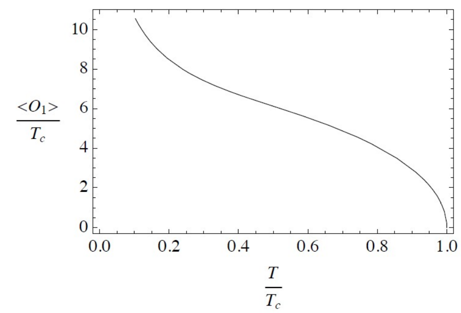

Figure 2: Numerical result of the CFT order parameter $\langle \mathcal{O}_b \rangle$ with $m^2 = \frac{-2}{L^2}$ taken from [15]

Finally, we plot the low temperature optical

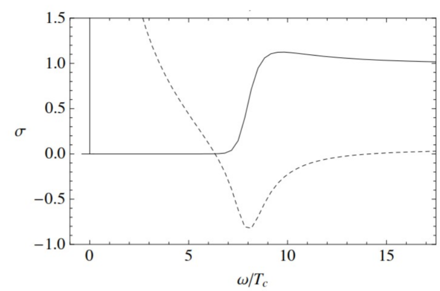

Figure 3: Low temperature limit of optical conductivity $\sigma(\omega)$ as a function of frequency. The solid lines denotes the real part $\Re[\sigma(\omega)]$ and the dashed line denotes the imaginary part $\Im[\sigma(\omega)]$. For the real part, there is a delta function at $\omega = 0$. The figure is taken from [6]

conductivity $\sigma(\omega)$ calculated using the duality in figure 3. Again, the details of the calculations are omitted in this note. Note that these is a delta function when $\omega = 0$ making infinite dc conductivity – as one would expect for a superconductor. The appearance of this delta function can be explained using the Drude model of conductivity. The charge carriers with mass m, density n and charge e in a normal conductor satisfy

$$m\frac{dv}{dt} = eE - \frac{mv}{\tau} \quad (7)$$

where τ is the relaxation time due to scattering. Using $J = env$ and $E = E_0 e^{i\omega t}$ we obtain the following expression for conductivity:

$$\sigma(\omega) = \frac{(ne^2/m)\tau}{1 - i\tau\omega} \quad (8)$$

Thus,

$$\Re[\sigma(\omega)] = \frac{(ne^2/m)\tau}{1+\omega^2\tau^2}, \quad \Im[\sigma(\omega)] = \frac{(ne^2/m)\omega\tau^2}{1+\omega^2\tau^2} \quad (9)$$

For superconductor, the relaxation time $\tau \to \omega$. This implies $\Re[\sigma(\omega)] = \delta(\omega)$ and $\Im[\sigma(\omega)] = 1/\omega$ as seen in the figure.

The AC conductivity $\Re[\sigma(\omega)]$ vanishes for $\omega < \omega_g$, indicating the existence of an energy gap ω_g. When the system is excited with $\omega > \omega_g$, it can create pairs of electrons leading to finite conductivity. From figure 3, we have

$$\frac{\omega_g}{T_c} \approx 8 \tag{10}$$

Comparing this with the BCS conductor energy-gap value 3.5, we can see that we are describing high-T_c superconductors and this value is roughly close to the experimental measurement [16].This is not a surprise because the results are derived using a holographic model. This means that, the strong interactions in the field theory side are automatically taken into account as promised by the AdS/CFT duality.

5 Conclusion

In this note, we gave a very introductory exposure to holographic superconductors. The discussion is far from being complete and we did not give any details of the computations. The basic goal was to make the reader excited about the fact that some computations that are hard to perform in the condensed matter side can be done in a gravitational system. Holographic superconductors are one of the many interesting applications of the AdS/CFT duality in condensed matter physics. Few additional features of the model we described can also be explored (e.g. adding a magnetic field). Even though we followed the same line of reasoning as the Landau-Ginzburg phase transition in our discussion, there are key differences between this and the holographic phase transitions. To get the details oh those and also to know about the various open problems in this field, interested reader should consult the review papers mentioned in this note.

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