Hamiltonian Dynamics
Problem Set #1

1. Write down the Lagrangian for a double pendulum restricted to move under gravity in a vertical plane. A mass m_1 is connected by a light rod of length l_1 to a fixed support and a mass m_2 is connected to m_1 with a rod of length l_2. Use as generalized coordinates the angle θ_1 and θ_2 of the rods with the vertical. Find the generalized momenta associated with these coordinates.

2. A bead of mass m can slide without friction along a horizontal circular hoop of radius r

   $$(x - x_0)^2 + y^2 = r^2$$

   The x component of the center of the hoop undergoes forced harmonic motion

   $$x_0 = a*\sin(w*t)$$

   Write down the equation of motion for the bead using the angle θ as generalized coordinate.

   $$x - x_0 = r*\cos\theta; \quad y = r*\sin\theta$$

3. The principle of least action can be extended to "Lagrangians" that contain higher time derivatives than first of the generalized coordinate q_i. Show that, if

   $$S = \mathrm{Int}_a^b \,[dt\, L(q_i, q_i', q_i'')] = \text{extremum}$$

   subject to fixed values of q_i at the ends, the corresponding Euler-Lagrange equation becomes

   $$d^2/dt^2(\partial L/\partial q_i'') - d/dt(\partial L/\partial q_i') + \partial L/\partial q_i = 0$$

   Apply this to the equation of motion for

   $$L = - mqq''/2 - q^2$$

4. A particle moves in the (x,y)–plane with speed proportional to the square of its distance from the origin. What is the minimum-time path between the points (1,1) and (0,2).

   Hint 1: It might help to solve this problem using polar coordinates.
   Hint 2: The construction of this problem is similar to that of the Brachristochrone.