Midterm 2023 Phys 505

5) Goal: Compute the magnetic moment of the $\boldsymbol{\rho^+}$ meson in the simplest constituent quark model, and compare to the Dirac point particle value:

$\boldsymbol{\mu = gS\frac{Q}{e}\frac{e\hbar}{2mc} \overset{g=2}{=} S\frac{Q}{e}\frac{e\hbar}{mc}}$

Given:

The quark content of the $\boldsymbol{\rho^+}$ is $\boldsymbol{u\bar{d}}$.

The $\boldsymbol{\rho^+}$ has spin $\boldsymbol{S = 1}$ and electric charge $\boldsymbol{Q = e}$

$\boldsymbol{u}$ has $\boldsymbol{S = 1/2}$ and $\boldsymbol{Q = 2e/3}$

$\boldsymbol{\bar{d}}$ has $\boldsymbol{S = 1/2}$ and $\boldsymbol{Q = e/3}$

Steps:

The $\boldsymbol{\rho}$ wavefunction with spin projection $\uparrow$ is:

$\boldsymbol{|\rho,\ m = +1\rangle = -|u\uparrow \bar{d}\uparrow\rangle}$

a) There is no need to symmetrize or antisymmetrize under exchange. Why not? I.e., do $\boldsymbol{u}$ and $\boldsymbol{\bar{d}}$ form an isospin doublet of identical particles, or are they distinguishable particles?

They are distinguishable particles, and do not form an isospin doublet. This is why Wong does not consider antisymmeterizing wf's of mesons composed of constituent fermions, because they are distinguishable.

b) Compute, summing over constituents,

$$\boldsymbol{\mu_\rho = \langle \rho,\ m = +1|\mu_z|\rho,\ m = +1\rangle = \langle \rho,\ m = +1|\sum_{i=1}^{2}\mu_{z\,i}|\rho,\ m = +1\rangle}$$

in terms of $\boldsymbol{\mu_u}$ and $\boldsymbol{\mu_{\bar{d}}}$

$\boldsymbol{\mu_\rho = \mu_u + \mu_d}$

Assume further that:

the quark constituent masses in this system are $\boldsymbol{m_u = m_{\bar{d}} = m_\rho/2}$, and $\boldsymbol{\mu_u}$ and $\boldsymbol{\mu_{\bar{d}}}$ are given by the Dirac point particle expression above.

c) Write $\boldsymbol{\mu_\rho}$ in terms of $\boldsymbol{m_\rho}$ in this constituent quark model.

d) Compare to the value for $\boldsymbol{\mu_\rho}$ if $\boldsymbol{\rho^+}$ were a Dirac point particle.

The spins, masses, and fractional charges work out so $\boldsymbol{\mu_\rho}$ in the constituent quark model is the same as the Dirac point particle expression above.

In lecture a lattice QCD calculation was mentioned to be in agreement with g=2.1±0.5 (though with large error), and a detailed calculation had very small additional corrections, but there is no measurement to test theory.

JB did not find this an intuitive result, expecting changes to point-like Dirac μ from the underlying structure. The comparative richness of the baryon wf– the need for antisymmetry under consituent identical fermion exchange– produces nontrivial cancellations in baryon magnetic moment contributions.