5. Compute the Magnetic moment μ of the $\Delta^0(1232)$ resonance:

A wavefunction for the Δ^0 with isospin $t = 3/2$, $t_0 = -1/2$,
and intrinsic spin pointed fully up $S_z = S = 3/2$ is (Wong page 45):

$|\psi_{\text{spin}}\rangle\, |\psi_{\text{isospin}}\rangle = |\uparrow\uparrow\uparrow\rangle\, |ddu + dud + udd\rangle/\sqrt{3}$

a) This part is symmetric under permutation. The $\Delta(1232)$ parity is +, and the spatial part is symmetric. State the permutation symmetry of the color part (no proofs nor calculation needed).

Color wf antisymmetric. (Wong used manifest symmetry of Δ^{++} to show color was necessary for overall antisymmetric wf)

b) Assume L=0 so $\vec{J} = \vec{S}$. Calculate the magnetic moment in terms of quark magnetic moments μ_{up} and μ_{down}

All quark spins are up, so μ operator projects out fully each term.
Result by counting terms, including normalization, is $2\mu_{down} + \mu_{up}$

c) Given known electric charges, and assuming mass $m_{\text{up}} = m_{\text{down}}$, eliminate μ_{up}. (The simplicity of this answer for μ of the Δ^0 is preserved by a published lattice gauge QCD calculation.) Compare to the neutron.

Under assumptions given, $\mu_{up} = -2\mu_{down}$.
Result for μ of Δ^0 is then identically 0, unlike the neutron which shares electric charge 0.
Addenda: This follows naturally from the completely symmetric wf in both spin and isospin, a result of the 'fully stretched' Δ^{++} wf in both spin $Sz = S = 3/2$ and isospin $t_0 = t = 3/2$, permutation symmetries preserved for the $t_0 = -1/2$ partner.

The $t = 1/2$ neutron has a more complex wavefunction and nonzero prediction in this constituent quark model in decent agreement with experiment, because the wf is not 'fully stretched.'

That lattice gauge QCD calculation
(Cloet et al. arXiv:hep-lat/0302008, Phys.Lett.B563:157-164,2003) preserves the vanishing μ, but also inverts whether Δ^+ or the proton has a larger magnetic moment. Particle Data Group 2019 has recent measurements of μ of Δ^{++} and Δ^+ in agreement with constituent quark model, but with 50% uncertainties that do not test the wavefunctions significantly. It's very challenging to measure μ of a wide resonance decaying by strong interaction.

d) List 2 examples of extra physics that could alter this μ computation for the Δ^0 (no proofs nor calculation needed).

Possibilities include:
i) Constituent quark masses not the same.
ii) Mixing with $t = 1/2$ by isospin breaking (electromagnetic, or small strong interaction).
iii) L=2 component (then $S = 1/2$ and thus wf with some spins down would be allowed, still conserving total angular momentum J)