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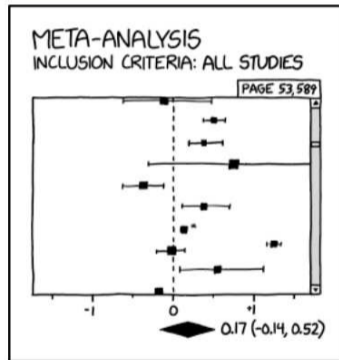
**“Truth loves its limits,
for there it meets the beautiful”
Rabindrinath Tagore, “Fireflies”**

“Good people are key. Be nice.”

**Single-bullet slide, Jan Hall
Nobel Prize talk on frequency combs
APS DAMOP 2006**

**Last 3 lectures are Fri March 31, Wed
April 5, Wed April 12
HW11 on these last 3 fun sym
lectures will be available Wed April 5,
due Mon April 17.**

Final is Friday April 21st at 9:30



Fun Sym

**BAD NEWS: THEY FINALLY DID A META-
ANALYSIS OF ALL OF ~~SCIENCE~~, AND IT
TURNS OUT IT'S NOT SIGNIFICANT.**

E1 is isovector, M1 is mostly isovector, π a Goldstone boson?

Weak Interactions and Nuclei

- Why the weak interaction is weak (at low energies)
- Quark-lepton interaction+QCD induces nucleon-lepton interaction terms:
Conserved Vector Current,
Partially conserved Axial Current
- β decay observables
Lepton long- λ expansion, Fermi function
Allowed,Forbidden Decay; Selection Rules
- Weak quark eigenstates, CKM matrix unitarity

Refs.: Wong 5.5-5.6;

Commins and Bucksbaum “Weak Interactions of Leptons and Quarks” and

Commins “Weak Interactions (Physics) 1st edition.”

Commins’ Notes Ph 250 UCB 1996 (see Canvas “Lecture Notes”)

v2 adds π W slide; v3 adds slide 43 on forbidden β decay

Fun Symmetries:

- \mathcal{P} (complete) : lepton helicity
Decay correlations
- Weak neutral current examples:
Weak interaction between nucleons
- \mathcal{T} (tiny)
CKM phase
Atomic Electric Dipole Moments from:
Nuclear Schiff, magnetic quadrupole,
and from QCD Lagrangian
Nuclear level spacing: Wigner distribution
- $0\nu\beta\beta$ decay intro

E1 transitions are isovector Bohr and Mottleson Vol I p. 44

The **E1 radiative transition strength depends on the matrix element of the electric dipole operator:**

$$\begin{aligned} \mathbf{D} &= \sum_k \mathbf{e}_k \mathbf{z}_k = \mathbf{e} \sum_k \frac{1}{2} (1 - \tau_z(k)) \mathbf{z}_k \\ &= \frac{1}{2} \mathbf{e} \sum_k \mathbf{z}_k - \frac{1}{2} \mathbf{e} \sum_k \tau_z(k) \mathbf{z}_k \end{aligned}$$

1st term depends only on position of c.o.m. of the whole nucleus, no transitions, Thompson scattering

2nd term: z component of a vector in isospin, so $|T_i - T_f| \leq 1 \leq |T_i + T_f|$

Consequence: for N=Z all transitions with $T_f = T_i$ are forbidden.

This selection rule has many phenomenological consequences.

An E1 multipole that would otherwise change $^{12}\text{C} + \alpha \rightarrow ^{16}\text{O}$ is greatly suppressed.

Isvector part of M1 is expected to be about one order larger than isoscalar

magnetic moment (in units of nuclear magnetons) can be expressed in the form

$$\begin{aligned}\mu &= \sum_k (g_s(k) \mathbf{s}_k + g_l(k) \mathbf{l}_k) \\ &= \sum_k \left\{ \frac{1}{2}(1 - \tau_z(k))(g_p \mathbf{s}_k + \mathbf{l}_k) + \frac{1}{2}(1 + \tau_z(k))g_n \mathbf{s}_k \right\} \\ &= \frac{1}{2} \mathbf{I} + 0.38 \sum_k \mathbf{s}_k - \sum_k \tau_z(k) (4.71 \mathbf{s}_k + \frac{1}{2} \mathbf{l}_k)\end{aligned}\quad (1-65)$$

where we have inserted the values $g_p = 5.59$ and $g_n = -3.83$ for the spin g factors for proton and neutron.

The first term in Eq. (1-65) is proportional to the total angular momentum \mathbf{I} and does not contribute to transitions between different states. The second term is a scalar in isospace, but has a coefficient that is an order of magnitude smaller than that for the last term (the isovector part). Thus we expect the isospin selection rules discussed above for $E1$ radiation to be also approximately valid for $M1$ radiation (Morpurgo, 1958). This is confirmed in the examples shown in Fig. 1-8,

The suppression of isoscalar M1's was used to help determine isospin mixing of the 12.71 MeV $1^+; T=0$ level with the 15.11 $1^+; T=1$ Adelberger PL62B 29 (1976)

“...since the π is both a Nambu-Goldstone boson and a $q\bar{q}$ bound state, it holds a unique position in nature” Horn, Roberts JPG 43 2016 073001

Continuous symmetry leaving a Lagrangian invariant \rightarrow spin-0 $m = 0$ boson

Goldstone Salam Weinberg PR 127 965 (1961)

Breaking that symmetry generates a pseudo Nambu Goldstone boson with $m \neq 0$

JB is ignoring subtlety here– the Lagrangian can stay symmetric while a vacuum expectation value of a field can break the symmetry...

- The π can be treated as a Goldstone boson acquiring m from broken chiral symmetry. (Remember chiral symmetry means m_q 's = 0, so q 's then have well-defined handedness.)

Remembering that

$m_\rho \approx 4/5 m_{\text{nucleon}}$, and constituent q 's work for μ_ρ , but $m_\pi \approx 1/7 m_{\text{nucleon}}$

Gell-Mann Oakes Renner PR 175 2195 (1968)

(note this pre-dates QCD)

$$m_\pi^2 \propto \langle \pi | U_{ECSB} | \pi \rangle$$

U_{ECSB} is interactions

breaking chiral symmetry

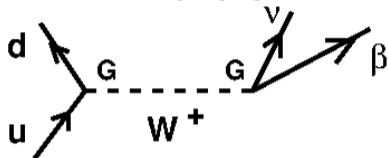
$$(\text{also } m_\pi^2 + 3m_\eta^2 - 4m_K^2 = 0)$$

π as a $q\bar{q}$ bound state of constituent q 's leads instead to $m_\pi \propto \langle \pi | U_{ECSB} | \pi \rangle$ (to get $m_\pi = 135$ MeV needs fine tuning)

- If m_q 's = 0, then for a Goldstone boson π , $m_\pi = 0$, and its interactions also vanish (!) Restoring chiral symmetry in nuclei was a possible solution to the Gamow-Teller strength deficit (below).
- We saw that chiral EFT of N-N interaction is based on \approx chiral symmetry of the q 's. So is chiral perturbation theory, which quantifies a QCD-induced weak decay (below). The axion is another Goldstone boson from \mathcal{T} in QCD (below)

Why the Weak Interaction is weak at low energy

Consider the propagator in the Feynman diagram for W^\pm exchange:



Propagator+vertices: $T \propto \frac{G_W(-g^{\mu\nu} + p^\mu p^\nu / M_W^2)G_W}{p^2 - M_W^2} \xrightarrow{p \ll M_W} \frac{G_W^2}{M_W^2} \Rightarrow$

Rates $\propto \frac{G_W^4}{M_W^4}$. Other physics with G_x, M_x have cross-terms $\propto \frac{G_W^2 G_x^2}{M_W^2 M_x^2}$

So the massive W^+ makes the interaction strength small for β decay with $p \sim \text{MeV}$

At high $p \sim M_W$, the interaction has the same coupling strength as E&M

For nucleons, G can and is different from the quark-lepton couplings

β decay is purely weak \Rightarrow physics at scale $M_W = 80\text{GeV}/c^2$? Sure, if you want to assume an electroweak coupling

Conserved Vector Current hypothesis with Dirac formalism

CVC is sometimes considered more for its consequences than for the physics behind it, so I'm going through the physics assumptions.

- Construct the E&M current for pointlike particles and show its derivative is zero, simply because of conservation of electric charge.
- Consider what happens if the particles are composite, like nucleons. One gets some relations for 'form factors' describing the nucleons, relations necessary to keep this current conserved.
- Hypothesize that the vector part of the weak current should be similarly conserved, and show what that implies for weak interaction physics. I'll use Dirac formalism, because the currents are all relativistic: I'll cite the limited formalism I need as I go along.

The S.M. interaction has W exchange, which at momenta $\ll M_W$ produces this quark-lepton current-current Lagrangian density that is purely 'V-A' (using the opposite-signed convention) (γ_μ reduces to Fermi, $\gamma_\mu\gamma_5$ to Gamow-Teller):

$$L = \frac{G}{\sqrt{2}} J^\mu \bar{J}_\mu^\dagger + h.c. \quad \text{with} \quad J_\mu = J_\mu^{(lep)} + J_\mu^{(had)} \quad \text{and} \quad J_\mu^{(lep)} = \bar{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_{\text{neutrino}}$$

We would really like to just deal with quarks, so that we could write something like:

$$J_\mu^{(had)} = J_\mu^{quark} = \bar{\psi}_d \gamma_\mu (1 + \gamma_5) \psi_u$$

because then everything would be automatically V-A, just like purely leptonic weak interactions (like μ decay).

However, we're stuck with nucleons, composite particles made of quarks. So QCD can 'induce' other terms as it combines quarks into the nucleon wf's.

So we have to go back and construct a general Lorentz vector for the hadrons (to make a bilinear covariant with the Lorentz vector of the leptons), along with an axial Lorentz axial vector for the hadrons (to make a bilinear covariant with the axial current of the leptons).

First we'll back up and do this for the E&M current, which is purely vector. The fact that this vector current is conserved (electric charge is conserved) puts constraints on the composite terms:

First we consider the E&M current, take its divergence, and use Dirac equation

$$(\gamma_\mu \partial_\mu + m)\psi = (\not{p} + m)\psi = 0$$

$\mathbf{J}_\mu = -e\bar{\psi}\gamma_\mu\psi$ (more properly, matrix element $\langle p' | \mathbf{J}_\mu(E\&M) | p \rangle$) for particle with momentum $p \rightarrow p'$

using plane-wave solution to Dirac eq. $\psi = u(p)e^{ip\cdot x}$

$$\begin{aligned} \partial_\mu \mathbf{J}_\mu / (-e) &= \\ \partial_\mu \left[\bar{u}(p_2) \gamma_\mu u(p_1) e^{i(p_1 - p_2) \cdot x} \right] &= \\ = [\bar{u}(p_2) (\not{p}_1 - \not{p}_2)_\mu \gamma_\mu u(p_1)] e^{i(p_1 - p_2) \cdot x} &= \\ = [\bar{u}(p_2) \not{p}_1 u(p_1) - \bar{u}(p_2) \not{p}_2 u(p_1)] e^{i(p_1 - p_2) \cdot x} &= \\ = i(m_1 - m_2) \bar{u}(p_2) u(p_1) e^{i(p_1 - p_2) \cdot x} = 0 & \end{aligned}$$

because $m_1 = m_2$ in E+M interactions

So this E+M current is conserved, so charge is conserved, QED ☺

Electromagnetic current for composite particles

Before we go back to the weak interaction, it is instructive to write down a general electromagnetic current for a composite particle, take its divergence, and set that to zero. We will get a direct prediction about a corresponding term in β decay from CVC.

For composite particles like nucleons, we have to again write the most general Lorentz vector that can be constructed from γ_μ 's and momenta, subject to:

a) momentum conservation means there are only two independent momenta, the difference $k_\mu = (p_2)_\mu - (p_1)_\mu$ and the average $K_\mu = 1/2(p_2 + p_1)_\mu$ of the individual momenta

b) not more than two γ matrices, because three γ 's can be written as one γ with γ_5 :

I , γ_μ , and $\sigma_{\mu\nu} = \frac{1}{2i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$

c) use Dirac eq., i.e. replace \not{p}_1 with im_1 when adjacent to spinors.

Then the most general forms of Lorentz vectors are: γ_μ , $\sigma_{\mu\nu}k_\nu$, k_ν , $\sigma_{\mu\nu}K_\nu$, K_ν .

It turns out that the matrix elements of the last two can be rewritten in terms of the other 3. (Perhaps just reflecting that in the CM frame the total momentum is zero.)

So we can write a general form for the E&M current of a composite particle:

$$\mathbf{J}_\mu/e = \bar{\psi} \left[F_1 \gamma_\mu - \frac{F_2}{2m} \sigma_{\mu\nu} k_\nu + iF_3 k_\mu \right] \psi$$

where F_1, F_2, F_3 are form factors, scalar functions of k^2 . Is this conserved?

$$-\partial_\mu \mathbf{J}_\mu/e = u(\bar{p}_2) \left[F_1 \not{k} - \frac{F_2}{2m} \sigma_{\mu\nu} k_\nu k_\mu + iF_3 k^2 \right] u(p_1) e^{ik \cdot x}$$

The 1st term vanishes as above

The 2nd term is zero independent of F_2 , because $\sigma_{\mu\nu}$ is completely antisymmetric.

$$\sum_{\mu\nu} \sigma_{\mu\nu} k_\nu k_\mu \equiv \sum_{\mu < \nu} [\sigma_{\mu\nu} k_\nu k_\mu + \sigma_{\nu\mu} k_\mu k_\nu] = \sum_{\mu < \nu} [\sigma_{\mu\nu} + \sigma_{\nu\mu}] k_\mu k_\nu = 0$$

The third term is not zero, so for CVC to hold, $F_3(k^2)=0$.

The 2nd term can be related to the magnetic moments, in particular the non-Dirac 'anomalous' magnetic moments, so:

For the proton, $F_1^p(0)=1$, $F_2(p)=\mu_p-1 = 1.793$

For the neutron, $F_1^n(0)=0$, $F_2(n)=\mu_n = -1.913$

Formally setting up isospin-changing operators for a weak 'current':

Recall results from angular momentum algebra: define isospin raising/lowering operators

$$T_{\pm} = T_1 \pm iT_2$$

$$T_{\pm}|T, T_z\rangle = \sqrt{T(T+1) - T_z(T_z \pm 1)}|T, T_z \pm 1\rangle$$

For spin-1/2, $T_z|1/2, \pm 1/2\rangle = \pm 1/2|1/2, \pm 1/2\rangle$

$$T_+|1/2, -1/2\rangle = |1/2, 1/2\rangle$$

Now write the E&M vertex function in terms of isoscalar and isovector parts:

$$e \left(\frac{1}{2} \left[F_1^S \gamma_{\mu} - \frac{F_2^S}{2m} \sigma_{\mu\nu} k_{\nu} \right] + \left[F_1^V \gamma_{\mu} - \frac{F_2^V}{2m} \sigma_{\mu\nu} k_{\nu} \right] T_z \right)$$

$$F_1^S = F_1^{(p)} + F_1^{(n)} = 1 + 0 = 1$$

$$F_1^V = F_1^{(p)} - F_1^{(n)} = 1 + 0 = 1$$

$$F_2^S = F_2^{(p)} + F_2^{(n)} = -0.120$$

$$F_2^V = F_2^{(p)} - F_2^{(n)} = +3.706$$

Now we can finally write the weak vertex function for the hadron part:

$$\frac{g}{2\sqrt{2}} V_{ud} \left[\left(g_V \gamma_\mu - \frac{g_M}{2m} \sigma_{\mu\nu} k_\nu + i g_S k_\mu \right) + \left(g_A \gamma_\mu - \frac{g_T}{2m} \sigma_{\mu\nu} k_\nu + i g_P k_\mu \right) \gamma_5 \right] T_\pm$$

where we have also included the similar axial vector terms, to form the covariant piece with the lepton axial vector current.

The CVC hypothesis includes some bold assertions:

- Vector portion of weak current is conserved, analogous to E&M current
- The two vector weak currents– the β^+ and β^- decay, given by the terms with T_\pm isospin raising/lowering operators– and the isovector part of the electromagnetic current are members of an isotriplet of current operators

This implies:

i) $g_V = F_1^V = 1.00$. Presence of strong interactions has left this term completely untouched \Rightarrow unrenormalized. This has many physics consequences.

ii) $g_M = F_2^V = \mu_p - \mu_n - 1 = 3.70$

This term in the weak current of the nucleon is related to the anomalous magnetic moments of the nucleons, called “Weak magnetism”

iii) $g_S = 0!$ The “induced scalar” term must be zero for CVC to hold

So now our full lepton-nucleon interaction density is (Morita Hyp. Int. 21 143 (1985)):

$$\sqrt{2}L = [V_\lambda + A_\lambda] [\bar{\psi}_e \gamma_\lambda (1 + \gamma_5) \psi_\nu] + [V'_\lambda + A'_\lambda] [\bar{\psi}_\nu \gamma_\lambda (1 + \gamma_5) \psi_e]$$

with explicitly different forms for β^\pm decay:

$$V_\lambda = \bar{\psi}_p \left(g_V \gamma_\lambda + \frac{g_M}{2m} \sigma_{\lambda\rho} k_\rho + i g_S k_\lambda \right) \psi_n \quad A_\lambda = \bar{\psi}_p \gamma_5 \left(g_A \gamma_\lambda + \frac{g_T}{2m} \sigma_{\lambda\rho} k_\rho + i g_P k_\lambda \right) \psi_n$$

$$V'_\lambda = \bar{\psi}_n \left(g_V^* \gamma_\lambda + \frac{g_M^*}{2m} \sigma_{\lambda\rho} k'_\rho - i g_S^* k'_\lambda \right) \psi_p \quad A'_\lambda = \bar{\psi}_n \gamma_5 \left(g_A^* \gamma_\lambda - \frac{g_T^*}{2m} \sigma_{\lambda\rho} k'_\rho + i g_P k'_\lambda \right) \psi_n$$

$$k = k_p - k_n = -k'$$

Yes, the hadron part, because of the QCD-driven “dressing” within the nucleon, is more complicated than the lepton part.

g_S and g_T terms change sign from electron to positron decay. These are therefore odd under charge symmetry. So they vanish in isobaric analog decays to the extent that charge symmetry is good. These are called “2nd-class currents” \rightarrow

There are at least 2 ways to make 2nd-class currents in a quark model:

- Remembering Standard Model has $\bar{u}\gamma_\mu d$ and $\bar{u}\gamma_5 d$ terms only, add derivative terms like $\partial_\mu \bar{u}d$ and $\partial^\nu \bar{u}\sigma_{\mu\nu}\gamma_5 d$

These are not renormalizable, one large reason they were excluded from the Standard Model (Weinberg Phys. Rev. 112 1375 (1958)).

[One perspective is that the Standard Model itself may be an Effective Field Theory good up to some very high energy. Naively, maybe that means renormalizability is not an exact logical requirement. However, deliberately introducing a manifestly unrenormalizable term would still be a very complicated move for the main part of one's basic theory.]

- Introduce a new quantum number in addition to color and flavor! (Feynman famously called this q.n. 'smell'?). You can also interpret this as a second set of quarks (Holstein Treiman PRD 13 3059 (1976)) carrying this quantum number.

A related scenario: recently people consider extra sectors of particles not interacting much with us, but interacting strongly among themselves.

QCD-like symmetries turn out to be a feasible way to generate dark matter.

There are tight constraints from experiment on such scenarios.

- The best experimental limits on 2nd-class currents, from dedicated β decay measurements, allow 2nd-class current effects about an order of magnitude larger than the known ones from charge-symmetry breaking in QCD.

Formal extension from nucleons to nuclei The hadron current we have written is for the spin-1/2 nucleon, where the μ is the only non-Dirac electromagnetic moment.

If you are describing nuclei (or hadrons) with spin $> 1/2$, then higher-rank electromagnetic moments also, by CVC, contribute to the weak vector current. E.g., the electric quadrupole moment produces a component in the weak vector current. Similarly, additional nuclear-structure dependent form factors appear for $J > 1$ in the axial vector current.

Holstein generalizes from nucleons to nuclei and writes decay correlations: Rev. Mod. Phys. 46 789 (1974) erratum 48 673; or “Weak Interactions in Nuclei”.

Nuclei are treated as “elementary particles” and form factors are introduced to include moments and effects from their nonpointlike size.

In isobaric analog decays, the vector current part is given by the measured electromagnetic moments. The g_T term in isobaric analog decays is zero, but in pure Gamow-Teller decay it is not zero, producing a part that depends on a nuclear structure calculation whose accuracy can limit the sensitivity to new physics.

Holstein’s approach considers ‘recoil-order’ terms $\sim (E_\beta/M)^N$ for $N=1,2,3$. Convergence is not guaranteed of such a series.

Behrens&Bühring “Electron Wavefunctions and Nuclear β Decay” has forbidden β decay

Finite nuclear S.M. expressions gain complexity with those corrections (Holstein)

$$l^\mu \langle \beta | V_\mu | \alpha \rangle = \delta_{JJ'} \delta_{MM'} \left(a(q^2) \frac{P \cdot l}{2M} + e(q^2) \frac{q \cdot l}{2M} \right) + ib(q^2) \frac{1}{2M} C_{J'1;J}^{M'k;M} (\mathbf{q} \times \mathbf{l})_k$$

$$+ C_{J'2;J}^{M'k;M} \left[\frac{1}{2M} f(q^2) C_{11;2}^{nn';k} l_n q_{n'} + \frac{1}{(2M)^3} g(q^2) P \cdot l \sqrt{\frac{4\pi}{5}} Y_2^k(\mathbf{q}) \right]$$

$$a = g_V \quad c = \sqrt{3} g_A$$

$$b - a = \sqrt{3} g_M \quad d = \sqrt{3} g_T$$

$$e = g_S \quad h = \sqrt{3} g_P.$$

$$l^\mu \langle \beta | A_\mu | \alpha \rangle = C_{J'1;J}^{M'k;M} \varepsilon_{ijk} \varepsilon_{ij\lambda\eta} \frac{1}{4M} \left[c(q^2) l^\lambda P^\eta - d(q^2) l^\lambda q^\eta \right.$$

$$\left. + \frac{1}{(2M)^2} h(q^2) q^\lambda P^\eta q \cdot l \right]$$

$$+ C_{J'2;J}^{M'k;M} C_{12;2}^{nn';k} l_n \sqrt{\frac{4\pi}{5}} Y_2^{n'}(\mathbf{q}) \frac{1}{(2M)^2} j_2(q^2)$$

$$+ C_{J'3;J}^{M'k;M} C_{12;3}^{nn';k} l_n \sqrt{\frac{4\pi}{5}} Y_2^{n'}(\mathbf{q}) \frac{1}{(2M)^2} j_3(q^2),$$

for decay between isobaric analogs:

$$\langle I, I_z \pm 1 | V_\mu^W | I, I_z \rangle,$$

$$a(0) = [(I \mp I_z)(I \pm I_z + 1)]^{1/2}$$

$$b(0) = a(0) \sqrt{\frac{J+1}{J}} (\mu_\beta - \mu_\alpha)$$

$$e(0) = f(0) = 0$$

$$g(0) = -a(0) \left(\frac{(J+1)(2J+3)}{J(2J-1)} \right)^{1/2} \frac{2M^2}{3} (Q_\beta - Q_\alpha),$$

where $x(q^2) = x_0 + x_1 q^2 \dots$ "form factors" including finite size

Valence nucleon shell-model expressions for G-T, weak mag

This unpaired nucleon expression is incomplete for G-T transitions:
(de-Shalit+Talmi Table 9.1)

THE VALUES OF $\langle \sigma \rangle^2$ FOR SINGLE NUCLEON TRANSITIONS

$j_f \backslash j_i$	$l + \frac{1}{2}$	$l - \frac{1}{2}$
$l + \frac{1}{2}$	$\frac{2l+3}{2l+1} = \frac{j_f+1}{j_f}$	$4 \frac{l+1}{2l+1}$
$l - \frac{1}{2}$	$\frac{4l}{2l+1}$	$\frac{2l-1}{2l+1} = \frac{j_f}{j_f+1}$

but it can lend qualitative understanding for why the G-T/Fermi ratio is so different in n , ^{19}Ne , ^{37}K ... e.g. both μ and G-T transitions are smaller for $d_{3/2}$ proton because the orbital term partly cancels the intrinsic spin term.

Weak magnetism in G-T transitions
(Wang+Hayes PRC 95 064313 (2017):

$$\frac{d\omega}{dE_e} = \frac{G_F^2 \cos^2 \theta_C}{2\pi^3} p_e E_e (E_0 - E_e)^2 F(E_e, Z) g_A^2 |\langle \vec{\Sigma} \rangle|^2$$

$$\times \left(1 + \frac{4}{3} \left[\frac{\mu_\nu + \frac{\langle J_f || \vec{\Lambda} || J_i \rangle}{\langle J_f || \vec{\Sigma} || J_i \rangle}}{2 M_N g_a} \right] (2E_e - m_e^2/E_e - E_0) \right)$$

$$\delta_{LS}^{j_f j_i} = \frac{\langle nl j_f || \vec{\Lambda} || nl j_i \rangle}{\langle nl j_f || \vec{\Sigma} || nl j_i \rangle}$$

$\mu_\nu = 4.7$
(isovector nucleon moment $\mu_p - \mu_n$)

with $j_i = l \mp 1/2$ and $j_f = l \pm 1/2$

$$\delta_{LS}^{--} = -(l+1), \quad \delta_{LS}^{+-} = -1/2,$$

$$\delta_{LS}^{+-} = -1/2, \quad \delta_{LS}^{++} = +l.$$

For reactor ν production, some simple estimates assumed the nucleon contribution $\pm 100\%$.

Weak Magnetism tests

• For isobaric analog decays, the 'weak magnetism' $\frac{g_M}{2m} \sigma_{\mu\nu} k_\nu$ term is directly predicted by CVC by the 'anomalous' magnetic moment difference of the parent and daughter.

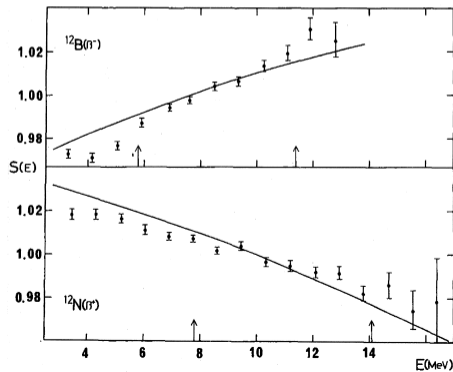
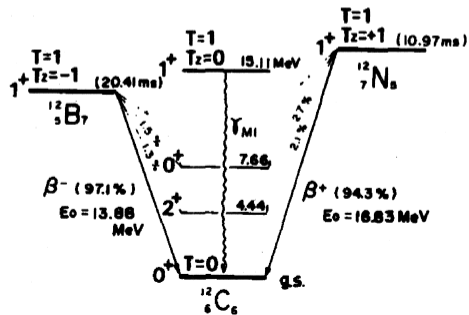
• For 'isospin mirror' Gamow-Teller decays, it is related to the isovector M1 γ -decay strength in the $T_z=0$ nucleus (Gell-Mann PhysRev 111 362 (1958)).

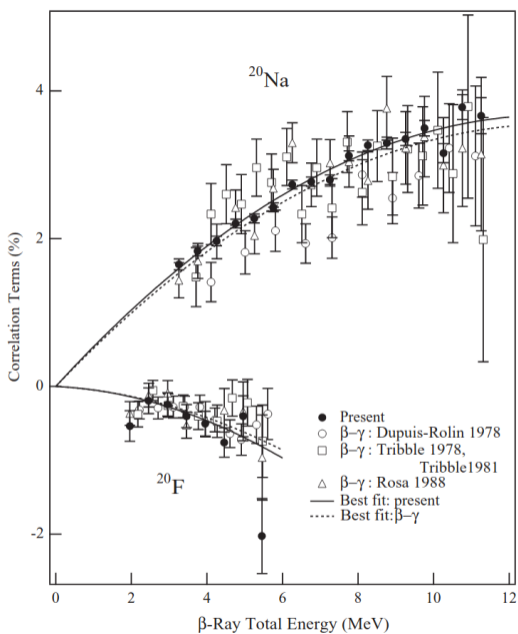
The k -dependence makes 20% distortions in the energy spectrum.

The axial vector $\frac{g_T}{2m} \sigma_{\mu\nu} k_\nu \gamma_5$ term, cancels in the difference unless there is a 2nd-class g_T .

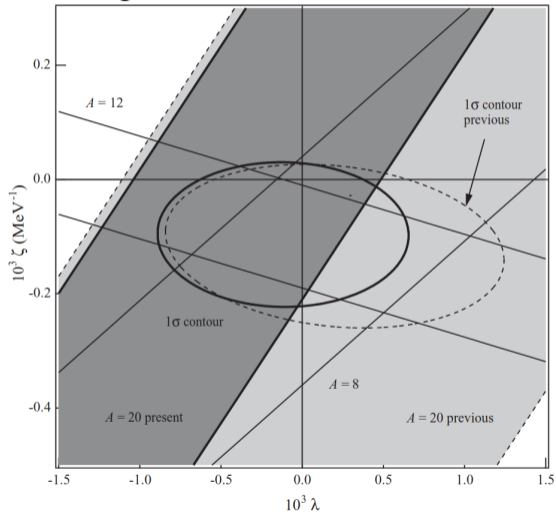
The results are consistent with the CVC prediction

to $\sim 10\%$ of the g_M term

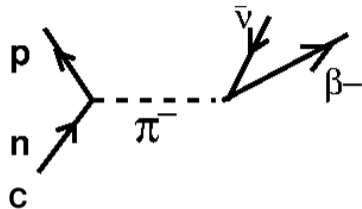
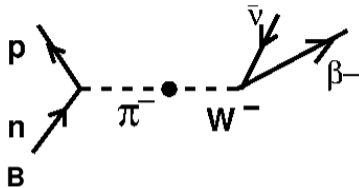
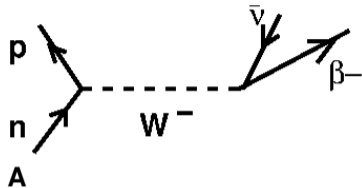




Further weak magnetism test:
 The angular distribution of β 's is isotropic wrt alignment $\langle M_J^2 \rangle$ for Gamow-Teller decay.
 Results agree with CVC to $\sim 5\%$



Sketch of lowest-order calculation of g_P Compare these diagrams:



Because W is short-range, C is same as B

For A (in a pure Gamow-Teller case), transition rate is:

$$T_{fi} = \frac{g}{2\sqrt{2}} V_{ud} \bar{u}_p (g_A \gamma_\mu \gamma_5 + i g_P k_\mu \gamma_5) u_n \frac{1}{m_W^2} \frac{g}{2\sqrt{2}} \bar{u}_e \gamma_\mu (1 + \gamma_5) v_{\nu e}$$

For C:

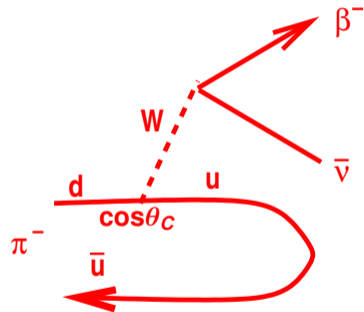
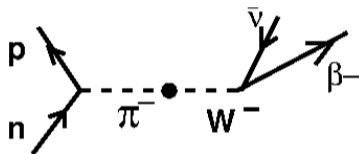
$$T_{fi} = g_{\pi NN} \sqrt{2} \bar{u}_p \gamma_5 u_n \frac{1}{k^2 + m_\pi^2} \frac{G}{\sqrt{2}} i f_\pi k_\mu \bar{u}_e \gamma_\mu (1 + \gamma_5) v_{\nu e} V_{ud}$$

So C is like the g_P part of A; if we declare C responsible for all of it:

$$g_P(k^2) = \frac{g_{\pi NN} \sqrt{2} f_\pi}{k^2 + m_\pi^2}$$

In β decay this is small, but in μ capture it is a large contribution: (in computing the decay lifetime, g_P becomes multiplied by the lepton mass.)

the right-hand half of diagram B is related to semileptonic decay of the pion



$d\bar{u} \rightarrow u\bar{u} + W$ and $u\bar{u}$ vanishes into the vacuum

(Fig. 4.10 Commins and Bucksbaum)

Continuing g_P , weak and strong interactions together:

see Gorringer and Fearing Rev.Mod.Phys. 76 (2004) 1 for chiral perturbation theory
 Further arguments (including PCAC below) give the ‘Goldberger-Treiman’ expression for the QCD-induced ‘pseudoscalar’ coupling:

$$g_P(q^2) = \frac{2m_\mu m_N}{m_\pi^2 - q^2} g_A(0)$$

This is now understood as the first term in an expansion using “chiral perturbation theory”; modern calculations $g_P(-0.88m_\mu^2) = 8.23$.

“Chiral perturbation theory” is a systematic expansion in m_π/m_{nucleon} , guided by similar concepts to chiral EFT’s for NN interaction (small m_{quark} , π ’s as Goldstone bosons from the underlying broken chiral symmetry...). But a calculation, not with free parameters.

History: Experiments in radiative capture on hydrogen: $12.4 \pm 0.9 \pm 0.4$

Experiments as of 2004 in ordinary μ capture on hydrogen: 10.5 ± 1.8

As of 2004, not good enough to help yet: PSI was working on it

This rigorous prediction of low-energy QCD’s effects on weak interaction was not working in 2004. **A more accurate PSI experiment resolved the discrepancy with theory: Andreev Phys Rev Lett 110 012504 (2013) $g_P(-0.88\mu^2) = 8.06 \pm 0.55$.**

Conserved Vector Current and 'Partially Conserved Axial Current': qualitative

One consequence of the conserved 'V' vector current is that the equivalent $g_V=1$. I.e. the interaction between quarks goes directly over to the interaction between nucleons and nuclei because the 'vector current' is conserved.

People looked pretty hard to find a way to find an axial 'A' current that was also conserved (look at Feynman and Gell-Mann PR '57).

Wong writes Eq. 5-52:

$$\sum_{\mu=1}^4 \frac{\partial V_{\mu}}{\partial x_{\mu}} = 0$$

This predicts a relation between the weak coupling constants G_A and G_V , given by :

$$g_A \equiv \frac{G_A}{G_V} = \frac{f_{\pi} g_{\pi N}}{M_N c^2}$$

and then by analogy

$$\sum_{\mu=1}^4 \frac{\partial A_{\mu}}{\partial x_{\mu}} = \text{constant} \phi_{\pi}$$

where ϕ_{π} represents the pion field.

where f_{π} scales π decay and $g_{\pi N}$ can be deduced from π -nucleon scattering. This 'Goldberger-Treiman relation' predicts $|g_A| = 1.31$; experimental value is $g_A = -1.259 \pm 0.004$. This either 'confirms PCAC' or enforces that 'PCAC is a bad name for a poor approximation'.

Lattice QCD is at 1% accuracy for g_A

Note that this is all at momentum transfer $q^2 \sim 0$: the constants are really 'form factors,' functions of momentum.

PCAC in more detail

Axial (hadronic) Current:

$$\mathbf{A}_\mu = -i \frac{g}{2\sqrt{2}} \bar{u}(\mathbf{p}_2) (g_A \gamma_\mu \gamma_5 + i g_P k_\mu \gamma_5) u(\mathbf{p}_1) e^{i(\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{x}}$$

PCAC hypothesis: the non-conservation of this current is due entirely to pions, and \mathbf{A}_μ becomes conserved as m_π goes to 0:

$$\partial_\mu \mathbf{A}_\mu \xrightarrow{m_\pi \rightarrow 0} 0$$

So evaluate the divergence of this current:

$$\partial_\mu \mathbf{A}_\mu =$$

$$\frac{-ig}{2\sqrt{2}} \bar{u}(\mathbf{p}_2) (g_A i \not{p}_1 \gamma_5 - g_A i \not{p}_2 \gamma_5 + g_P k^2 \gamma_5) u(\mathbf{p}_1) e^{i(\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{x}} =$$

using Dirac eq.

$$\frac{-ig}{2\sqrt{2}} \bar{u}(\mathbf{p}_2) (2mg_A + g_P k^2) \gamma_5 u(\mathbf{p}_1) e^{i(\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{x}}$$

By PCAC this vanishes as $m_\pi \rightarrow 0$, so:

$$g_A \xrightarrow{m_\pi \rightarrow 0} \frac{g_P k^2}{2m} = \frac{g_{\pi NN} \sqrt{2} f_\pi}{k^2 + m_\pi^2} \frac{k^2}{2m} = - \frac{g_{\pi NN} \sqrt{2} f_\pi}{2m}$$

the Goldberger-Treiman relation

Summary of hadronic weak current form factors in S.M.

• Exact Predictions of CVC for vector current:

- 1) $g_V=1\dots$: Experimental $0^+ \rightarrow 0^+$ Ft values same to ≈ 0.001 . CKM unitarity has a 0.001 deficit at 2 to 3 σ . ($\pi^+ \rightarrow \pi_0 + \nu + \beta^+$ agrees to 0.005 (PIBETA))
- 2) $g_M=3.70$: Weak magnetism measured to $\approx 5\%$ of its value
- 3) $g_S=0$: Ft , and relative helicity of leptons from β - ν correlation and $\pi \rightarrow e\nu$, show no evidence for scalar term at $C_S < 0.05$ level.

• Estimates from PCAC (Goldberger-Treiman) and similar:

- 1) $g_A = \frac{g_{\pi NN} \sqrt{2} f_\pi}{2m} = -1.32$; Decay of neutron $\Rightarrow -1.26$
- 2) $m_\mu g_P = \frac{g_{\pi NN} \sqrt{2} f_\pi m_\mu}{m_\pi^2} = 9.2$ Including chiral perturbation theory more like 8.0, PSI's μ CAP experiment μ capture on hydrogen agrees well.

Charge Symmetry (G-parity): No 2nd-class currents: $f_3 \approx 0$, $g_2 \approx 0$

(The best tests of this SU(2) symmetry are still in β decay: similar tests in hadronic decays of τ)

• V and A are the dominant known couplings for nuclear β decay. (The most precise a_β measurement in the neutron disagrees badly, suggesting a small Lorentz tensor interaction.) Interesting that a couple of simple surmises determined 6 couplings so well—reasonable to call it an “effective field theory” for the lepton-nucleon weak interaction. See Ando PhysLettB595 250 (2004) for an EFT of neutron β decay including radiative

β decay: Energy release, other basics, orbital angular momentum

$$Q_{\beta^-} = M(Z,N) - M(Z+1,N-1) \text{ using atomic masses}$$

(this is in some sense accidental: the β is created in the nucleus and leaves; if nothing else happens, this would create a negative atomic ion...)

$$Q_{EC} = M(Z,N) - M(Z-1,N-1) - |B.E.(\text{electron})|$$

$$Q_{\beta^+} = M(Z,N) - M(Z+1,N-1) + 2m_e$$

Sometimes EC is allowed energetically when β^+ is not.

Atomic electron overlap with nucleus is greater as one goes heavier; EC \sim 1% at $Z \sim 40$ isotopes where β^+ is allowed, but can be 10's of % at $Z=82$

Ratio is given well by atomic wavefunctions, and has some sensitivity to the weak interaction nature (Brysk and Rose, Rev Mod Phys 30 (1958) 1169)

- Q can vary from 18 keV (t to ^3He) to > 10 MeV

($m_\beta = 0.511$ MeV, so β 's can be relativistic or non-relativistic.)

- electron DeBroglie wavelength: $\lambda = h/p = 2\pi(197 \text{ MeV fm}) / \sqrt{E^2 - m_e^2}$

For kinetic energy 1 MeV, this is 870 fm, much larger than the nucleus.

So the long-wavelength expansion we're about to make is a good one.

Similarly, $\ell = rxp \sim 0.005 \hbar$ is typically small

Fermi's Golden Rule, applied to β Decay to get rates

For now write the transition probability

$$W = \frac{2\pi}{\hbar} |\langle \phi_f(\vec{r}) | H | \phi_i(\vec{r}) \rangle|^2 \rho(E_f)$$

The initial state is simply the parent nucleus at rest:

$$|\phi_i(\vec{r})\rangle = |J_i m_i \vec{r}\rangle$$

The final state consists of 3 particles. Ignoring for now Coulomb effect between the β and final nucleus, this is a product of 3 parts, with plane waves for the leptons:

$$|\phi_k(\vec{r})\rangle = \frac{1}{\sqrt{V}} e^{i\vec{k}_e \cdot \vec{r}} \frac{1}{\sqrt{V}} e^{i\vec{k}_\nu \cdot \vec{r}} |J_f m_f r'\rangle$$

The V's normalize the plane waves. Expand the plane waves in terms of spherical harmonics (we could do this for γ -rays, too: we're about to do a 'long-wavelength expansion'):

$$e^{i\vec{k} \cdot \vec{r}} = \sum_0^\infty \sqrt{4\pi(2\lambda + 1)} i^\lambda j_\lambda(kr) Y_{\lambda 0}(\theta, 0)$$

where $\vec{k} = \vec{k}_e + \vec{k}_\nu$ and θ is the angle between \vec{k} and \vec{r} .

Now we make the long-wavelength expansion:

$$j_\lambda(kr) \xrightarrow{kr \ll 1} \frac{(kr)^\lambda}{(2\lambda + 1)!!}$$

so that the final state wavefunction becomes:

$$|\phi_k(\vec{r})\rangle = \frac{1}{V} \left(1 + i \sqrt{\frac{4\pi}{3}} (kr) Y_{10}(\theta, 0) + \mathcal{O}(k^2 r^2) \right) |J_f m_f r'\rangle$$

Even without the formal weak interaction theory, we can now surmise the form of H , the nuclear part of the β -decay operator.

Neutrons are transformed into protons \Rightarrow the nuclear operator:

1) must be one-body, i.e. only one nucleon is involved at a time;

2) must involve single particle isospin raising/lowering operator τ_\pm (this comes from the 'vector' V 'Fermi' part of 'V-A')

- The axial vector 'A' 'Gamow-Teller' part produces a product of σ and τ_{\pm}

Then we can write the matrix element $\langle \phi_f(\vec{r}) | H | \phi_i(\vec{r}) \rangle =$

$$\frac{1}{V} \langle \mathbf{J}_f m_f r | \sum_{j=1}^A (\mathbf{G}_V \tau_{\pm}(j) + \mathbf{G}_A \vec{\sigma}(j) \tau_{\pm}(j)) \left(1 - i \sqrt{\frac{4\pi}{3}} (kr) Y_{10}(\theta, 0) + \mathcal{O}(k^2 r^2) \right) | \mathbf{J}_i m_i r' \rangle$$

- The Fermi operator does nothing to space/spin. So it only links isobaric analog states, or pieces of isobaric analog states, i.e. states with same wavefunction except proton/neutron interchange.

- This form shows both the allowed terms and some '1st forbidden' terms: these are from the same nuclear operators σ and τ , but including the next order of the lepton long wavelength expansion and thus suppressed. However, the nuclear matrix elements also vary, so some 1st forbidden rates are faster than some G-T. The 1st-forbidden operators all flip the nuclear parity, so don't contribute at all to the allowed transitions between states of same parity.

Density of final states

We have to make sure that momentum and energy are conserved properly among the 3-body final state.

We start by writing the ν density as a statistical mechanical result (and integrate over all angles for the time being):

$$dn_\nu = \frac{V}{2\pi^2\hbar^3} p_\nu^2 dp_\nu$$

$E_\nu^2 = m_\nu^2 + p_\nu^2$ but $m_\nu < 3 \text{ eV} \approx 0$ so $E_\nu = p_\nu$.

We can ignore the recoil energy for kinematics

(though keeping it produces corrections to correlations, 'recoil order terms' ~ 0.01) which gives the relation:

$$E_\nu = Q - K_e$$

where Q is the total kinetic energy released in the decay, and K_e is the kinetic energy of the electron. (This kinetic energy is sometimes written ' E ' in the literature)

I'll also make use of maximum total e energy $E_0 = Q + m_e$ and $E_\nu = E_0 - E_e$

Density of charged lepton final states: Fermi function

The density of charged-lepton states gets perturbed in the presence of the nuclear Coulomb field, so (also integrating over all angles)

$$dn_e = \frac{V}{2\pi^2\hbar^3} F(Z, K_e) p_e^2 dp_e; \quad \text{where } F(Z, K_e) \text{ is the "Fermi function":}$$

Lepton nonrelativistic,

pointlike nucleus,

$|\psi(\mathbf{r} = \mathbf{0})|^2$ from the nonrelativistic

Coulomb wavefunction gives:

$$F(Z, K_e) = \left| \frac{x}{1 - e^{-x}} \right|$$

$x = -1 \times \pm 2\pi\alpha Zc/v$ for β^\pm decay,
 $\alpha \approx 1/137$.

Increases the total decay rate by 2

between $Z=0$ and $Z=26$ (for $Q=7$ MeV),

Better formulations include e^- screening of the atom, exchange between outgoing and atomic e^- ...

See Sir Denys Wilkinson's 5-part series in Nuclear Inst. and Meth.

For a decay rate with better than $\sim 10\%$ accuracy:

Dirac eq $|\psi(\mathbf{r})|^2 \xrightarrow{\mathbf{r} \rightarrow \mathbf{0}} \infty$

so Fermi evaluated at the nuclear surface

deShalit and Feshbach eq. IX.2.15; Fermi Zeit. Physik 88 (1934) 161)

$$F(Z, K_e) = 2(2kr)^{2(s-1)} \frac{1+s}{s^2 + \eta^2} \left| \frac{e^{\pi\eta/2} \Gamma(s+1+i\eta)}{\Gamma(2s+1)} \right|^2$$

using the Γ function, $\eta = x/(2\pi)$, and $s = \sqrt{1 - (\alpha Z)^2}$

Good to a few percent

β energy spectrum for allowed decay

Integrating $p_e^2 dp_e p_\nu^2 dp_\nu \delta(Q - K_e - E_\nu)$ over p_ν , ignoring recoil-order terms and forbidden decay (so the nuclear matrix elements have no spatial/momentum dependence),

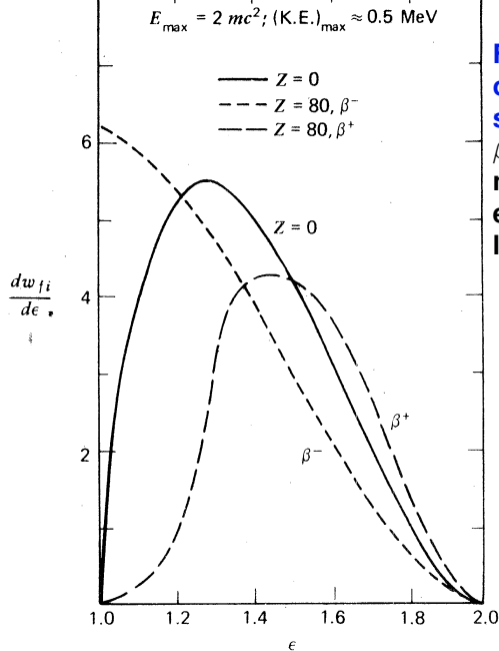
$$W(p_e) dp_e = \frac{1}{2\pi^3 \hbar^3 c^3} \sum_{\mu m_f} |\langle J_f m_f r | O_{\lambda\mu}(\beta) | J_i m_i r' \rangle|^2 F(Z, K_e) p_e^2 (Q - K_e) \sqrt{(Q - K_e)^2 - m_\nu^2} dp_e$$

with $O_{\lambda\mu} = \sum_{j=1}^A (\mathbf{G}_V \tau_\pm(j) + \mathbf{G}_A \vec{\sigma}(j) \tau_\pm(j))$. Differentiating $E^2 = p^2 + m^2 \Rightarrow pdp = EdE$,

$$W(E_e) dE_e \propto F(Z, E_e) E_e p_e (E_0 - E_e) \sqrt{(E_0 - E_e)^2 - m_\nu^2} dE_e$$

- The decay rate $\sim Q^5$, a large dependence. This is just from three powers of momentum for each lepton, minus one for energy conservation.
- The spectrum gets distorted at the very endpoint (large K_e , near Q), by the ν mass, which has upper limit (from ${}^3\text{H}$ decay KATRIN) 1.1 eV at 90% confidence.

(Most forbidden decay operators produce large changes in this energy spectrum)



Fermi function effect on β energy spectrum

β^- is 'pulled into' nucleus... a big effect for high Z and low E_β

ft value for allowed decay

After doing the phase space integration, we can write down the answer:

$$ft = \frac{K}{|M_F|^2 + g_A^2 |M_{GT}|^2}$$

$$K = \frac{2\pi^3 \hbar^7 \ln 2}{m_\beta^5 c^4 G_V^2} = 6142 \pm 3.2\text{s}$$

If you include isospin mixing and ‘radiative’ corrections, you can define the quantity Ft that is actually constant for the Fermi transitions:

$$Ft = ft(1 - \delta_C)(1 + \delta_R) = \frac{K}{G_F^2 |V_{ud}|^2 |M_{fi}|^2 (1 + \Delta_R)}$$

where $|M_{fi}|^2 = T(T+1) - T_3(T_3+1)$ as below

Isospin-breaking corrections δ_C are parameterized by two sources:

- 1) isospin mixing with other 0^+ configurations
- 2) the spatial wavefunctions are slightly different because the protons repel each other ‘radial mismatch’.

Recent correction for f Chien-Yeah Seng, arXiv:2212.02681 (accepted by PRL)

JB mentioned out loud the 2-3 σ difference from CKM unitarity in the corrected Ft values from this theorist and colleagues, from two sets of radiative corrections of both nucleon and nucleus.

The new work considers a correction to the phase space integral f (!)

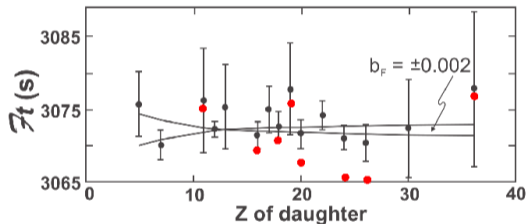
There is a recoil-order correction

$$\propto q^2 R^2_{\text{ChargedWeak}} \neq q^2 R^2_{\text{Charge}}$$

(This is a standard expansion of a pointlike nucleus to include its spatial distribution, related by a Fourier transform to the momentum transfer q)

Holstein RMP: One can get $R^2_{\text{ChargedWeak}}$ by comparing isobaric triplets of measured R^2_{Charge} , but no one has done this correctly before.

The Ft values move about this much (JB reading Table II of Seng and shifting naively the centroids) for the ones that have been measured.



Seng reports that this could account for the whole CKM unitarity discrepancy, but it's work in progress.

Seng also comments that the CVC test would not work out as well.

Superaligned Ft values

Consider Fermi beta decay in many 0^+ to 0^+ cases. We can sum over the nucleons

$$\sum_{k=1}^A \tau_{\pm}(k) = T_{\pm}$$

where T_{\pm} lowers or raises the 3rd component of SU(2) isospin for the whole nucleus, just like the lowering and raising operators for SU(2) spin. The Fermi operator's matrix element is

$$\begin{aligned} & \langle \mathbf{J}_f M_f T_f T_{0f} | \sum_{k=1}^A \tau_{\pm} | \mathbf{J}_i M_i T_i T_{0i} \rangle \\ & = \sqrt{T_i(T_i + 1) - T_{0i}(T_{0i} \pm 1)} \end{aligned}$$

if $\mathbf{J}_f = \mathbf{J}_i$, $M_f = M_i$, $T_f = T_i$, and $T_{0f} = T_{0i} \pm 1$; 0 otherwise.

For these cases, the ft value then given by just some constants, which are given by the weak interaction strength. (f=integral over phase space). I.e. they all should have the same intrinsic strength.

The vector operator is related to the electric charge operator. We know electric charge is conserved. The “conserved vector current” hypothesis of Feynman and Gell-Mann: by analogy they theorized that the vector part of the weak interaction is also conserved. This eventually leads to electroweak unification. This has many consequences. For example, for the vector part of the weak interaction we can go straight from the quark matrix element to the nucleon one to the nucleus one.

Effect of different Fermi functions on superallowed Ft's

Z	Q	no Fermi	Non-rel	Fermi's	Towner '05	error
5	0.88577	3540.7	3005.8	3030.2	3073.0	4.9
7	1.80851	3618.7	2985.0	3028.2	3071.9	2.6
12	3.21071	3957.0	2905.2	3015.4	3072.9	1.5
16	4.46971	4252.2	2832.3	3010.9	3071.7	1.9
18	5.02234	4400.4	2786.2	3002.9	3072.2	2.1
20	5.40358	4548.1	2732.9	2991.7	3075.6	2.5
22	6.02863	4696.5	2679.4	2979.5	3078.5	2.4
24	6.61039	4846.2	2622.4	2966.1	3071.1	2.7
26	7.22056	5004.4	2566.7	2956.2	3071.2	2.8

Towner's include isospin mixing corrections.

Note Fermi's 1934 function isn't really good enough for this, while "Towner" includes 1% corrections from isospin mixing, or rather the difference in isospin mixing between the parent and daughter. These are parameterized by:

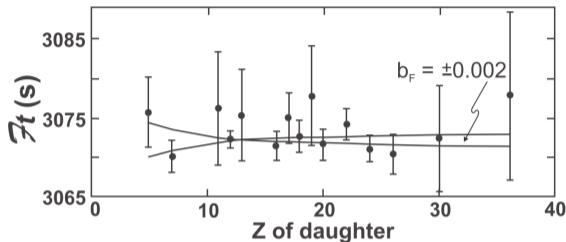
- 1) different isospin configurations mixed;
- 2) different wavefunctions because the nuclei have different radii.

IMME is fit mass-by-mass, adjusting an effect Coulomb interaction in a shell model. (A technical check of neutron occupancies is used in the 2020 versions.)

Charge-dependent nuclear interaction to fit the c IMME coefficient Towner'08, Ormand Brown'85

Consistency of Ft 's tests CVC hypothesis

J.C. Hardy, I.S.Towner, PRC 102 044501 (2020)



Notice a constant goes through all— 5/15 should miss at 1σ .

There is a common systematic uncertainty from a radiative correction which is folded into each of these points.

This test is used to gain confidence in the isospin breaking calculations.

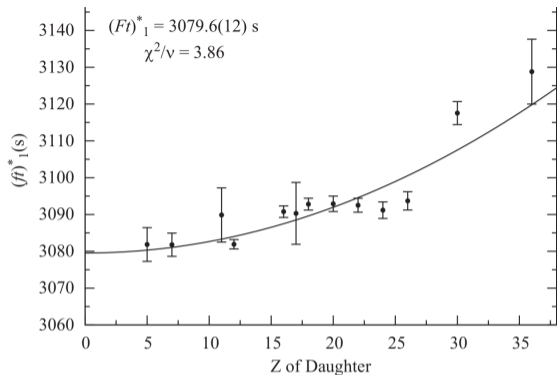


Fig. 2. Plot of the $(ft)_1^*$ data points that do not include theoretical corrections for isospin symmetry breaking and the resulting quadratic fit giving the global trend

Grinyer, Svensson, Brown NIMA 622 236 (2010)

Using 'Wilkinson Method 2'

Correcting fluctuations in Ft in each shell, yet allowing magnitude of isospin breaking to vary phenomenologically with Z^2 .

V_{ud} changed by 0.2σ . σ_{Vud} increased by 1.3.

Quark eigenstates in the weak interaction: Cabibbo angle

To explain some weak decays, in particular ratios of semileptonic baryon decays with and without strangeness,

the weak interaction mixes the d and s quarks, so you can think of the u changing to d in β decay as:

$$|u\rangle \rightarrow |d\rangle + \epsilon|s\rangle \quad \text{i.e.} \quad |u\rangle \rightarrow \cos(\theta_C)|d\rangle + \sin(\theta_C)|s\rangle$$

θ_C , the Cabibbo angle, is a parameter whose value (13.04°) is unexplained so far from underlying physics. (Like any mixing ‘angle’, the angle is in an abstract space, and it’s just a simple way to normalize wavefunctions)

For 3 families of particles, this generalizes to

→ 3x3 unitary “CKM” matrix between $|d\rangle$, $|s\rangle$, $|b\rangle$

From superallowed Ft values we get a vital physics constant: V_{ud}

The quark eigenstates of the weak interaction are not the same as the mass eigenstates. They are related by a unitary transformation.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

As for any unitary matrix, top row has the property:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

The superallowed Ft value, compared to muon decay (the strength of the leptonic weak interaction), gives you V_{ud} . (V_{ub} is very small and does not matter.)

There's been a long struggle over V_{us} , which comes from kaon decays or hyperon β decay, with useful checks from theory with more than one possible solution.

CKM unitarity test is off by 2-3 σ at 0.1% from most recent reevaluations of radiative corrections (see Towner Hardy review 2020).

Again, each Ft value has an isospin mixing calculation done phenomenologically, because initial and final wavefunctions are not identical. The uncertainty and centroids of these calculations are still an open question.

$\log(ft)$ for β decay

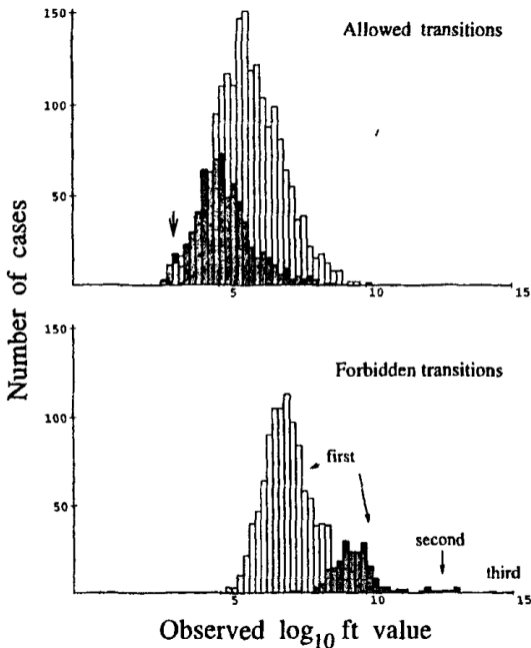
Wong Figure 5.8

As we said above, G-T transitions preserve nuclear π , while 1st-forbidden transitions flip nuclear π .

If the ft values are different enough, that can distinguish the transition and be used to determine π .

However, the ft values for G-T and 1st forbidden overlap.

Sometimes the nuclear matrix element for G-T decay is accidentally small.
(E.g. ^{14}C GT decay has $\log(ft)$ of 9.0, five orders slower rate than the fastest GT's)



Selection rules

Fermi

G-T

γ_5 dominates $0^- \rightarrow 0^+$

$\sigma \cdot r$ suppressed by r/λ

'1st forb. unique' $2^\pm \leftrightarrow 0^\mp$

One operator \Rightarrow calculable correlations from spin,

A_β large

TABLE I. Allowed and first-forbidden nuclear matrix elements and their selection rules (K designates the rank of the transition operator, when regarded as a tensor).

Matrix element	K	ΔJ	$\Delta\pi$
Allowed $C_V f 1$	0 0		+1
$C_A f \hat{\mathbf{d}}$	1 0, ± 1 (no $0 \rightarrow 0$)		+1
First for- bidden $C_A f \gamma_5$	0 0		-1
$C_A f (\hat{\mathbf{d}} \cdot \mathbf{r}/i)$			
$C_V f \mathbf{r} i$	1 0, ± 1 (no $0 \rightarrow 0$)		-1
$C_V f \boldsymbol{\alpha}$			
$C_A f (\hat{\mathbf{d}} \times \mathbf{r})$			
$C_A f i B_{ij}$	2 0, $\pm 1, \pm 2$ (no $0 \rightarrow 0$, no $1 \rightarrow 0$, no $0 \rightarrow 1$)		-1

Weidenmüller Rev Mod Phys 33 574 (1961)

Some correlations in 1st-forbidden β decay are simple

If there's one operator, correlation is given by angular momentum coupling, no nuclear structure dependence

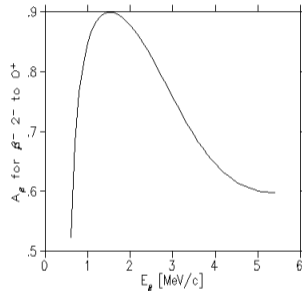
- $2^\pm \leftrightarrow 0^\mp$ “1st forbidden unique” has one operator. (one must flip nuclear spin and have leptons carry off $L=1$ to change J by 2 and flip nuclear parity)

Behrens and Buhning Chs 7, 14; AbetaFirstForbiddenUniqueJB.pdf

Coefficient of $\cos\theta_\beta$ wrt spin:

$$A_\beta \frac{v}{c} = \frac{p}{E} \frac{p_\nu^2 + \frac{3}{5} p_\beta^2}{p_\nu^2 + p_\beta^2}$$

So it's not just Gamow-Teller and Fermi that have large predictable β asymmetry



- $0^- \rightarrow 0^+$ decay: 2 operators, but one is suppressed wrt other by $R_{\text{nucleus}}/E_\beta$

\sim few % in fission products Hayen PRC 100 054323 (2019)

$\Rightarrow E_\beta$ spectrum is \sim allowed; $a_{\beta\nu} \approx 1$

- Warburton PRC 26 1186 (1982) has E_β spectrum and $a_{\beta\nu}$ for 1st forbidden for light nuclei
- Glick-Magid and Gazit, J. Phys. G 49 105105 (2022) forbidden β expansion in 5 small quantities+ Coulomb corrections

Clarification of g_V and V_{ud}

I said $g_V=1.00$ was experimentally shown, which was pretty sloppy. Better to say $g_V=1$ is a prediction of CVC:

- The $0^+ \rightarrow 0^+$ Ft values are experimentally constant, testing whether g_V is a constant for all transitions, but not necessarily $g_V=1.000\dots$

- Backing up, G_V is determined by

$$\mathcal{F}t \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^V)}, \quad (1)$$

where $K/(\hbar c)^6 = 2\pi^3 \hbar \ln 2 / (m_e c^2)^5 = 8120.27648(26) \times 10^{-10} \text{ GeV}^{-4}$ s, G_V is the vector coupling constant for semileptonic weak interactions, δ_C is the isospin-symmetry-breaking correction, and Δ_R^V is the transition-independent part of the radiative correction. The terms δ'_R and δ_{NS} comprise the transition-dependent part of the radiative correction, the

and then V_{ud} is determined by

$$V_{ud} = G_V/G_F, \quad (2)$$

where G_F is the well-known weak-interaction constant for muon decay. Once the value of V_{ud} is established it can be

So the present deficit in V_{ud} could also be a change for g_V from its value of 1 from electroweak unification.

E.g., e and μ weak couplings could be different.

Crivellin and Hoferichter PRL 125 111801 (2020) consider keeping CKM unitarity while considering constraints from

$$(\pi \rightarrow e\nu)/(\pi \rightarrow \mu\nu)$$

Axial vector is not conserved. Is g_A the same in nuclei? G-T ("Ikeda") sum rule:

define the sum rule strength

$$S_{\pm} = G_A^{-2} \sum_f |\langle f | \mathbf{O}_{GT}(\beta^{\pm}) | i \rangle|^2$$

$$S_{\pm} = G_A^{-2} \sum_f \langle f | \mathbf{O}_{GT}(\beta^{\pm}) | i \rangle^* \langle f | \mathbf{O}_{GT}(\beta^{\pm}) | i \rangle$$

$$= G_A^{-2} \sum_f \langle i | \mathbf{O}_{GT}^{\dagger}(\beta^{\pm}) | f \rangle \langle f | \mathbf{O}_{GT}(\beta^{\pm}) | i \rangle$$

$$= G_A^{-2} \langle i | \mathbf{O}_{GT}^{\dagger}(\beta^{\pm}) \mathbf{O}_{GT}(\beta^{\pm}) | i \rangle$$

operators involved here have the following p

$$\sigma_{\mu}^{\dagger} = (-1)^{\mu} \sigma_{-\mu} \qquad \tau_{\mp}^{\dagger} = \tau_{\pm}$$

$$S_{+} = \langle i | \sum_{k=1}^A \sum_{\mu} (-1)^{\mu} \sigma_{-\mu}(k) \tau_{+}(k) \sigma_{\mu}(k) \tau_{-}(k) | i \rangle$$

$$= \langle i | \sum_{k=1}^A \sigma^2(k) \tau_{+}(k) \tau_{-}(k) | i \rangle$$

In a spherical basis, the scalar product is

$$\mathbf{J} \cdot \mathbf{V} = \sum_q (-1)^q J_{1q} V_{1,-q}$$

$$\tau_{+} \tau_{-} | p \rangle = | p \rangle \qquad \tau_{+} \tau_{-} | n \rangle = 0$$

expectation value of σ^2 is 3.

$$S_{+} = \langle i | \sum_{k=1}^Z \sigma^2(k) | i \rangle = 3Z$$

Similarly, $S_{-} = 3N$, and $S_{+} - S_{-} = 3(Z-N)$

[Formally similar sum rule arguments, applied to the nucleon, express g_A in terms of π -nucleon cross-sections,

calculating 1.16 Weisberger PRL 14 1047 (1965)

and 1.24 Adler PRL 14 1051 (1965).

Experiment then in $n \beta$ decay was 1.18 ± 0.02 , now 1.26.]

Recent progress on Gamow-Teller strength

Decades of detailed studies with high- Q β decay (and (p,n) and (n,p) reactions at 100-200 MeV) consistently found $\approx 75\%$ of the GT strength in many nuclei.

Comparison is not to the sum rule: not all of the strength can be measured. Bertsch Esbensen 1987 Rep. Prog. Phys. 50

607

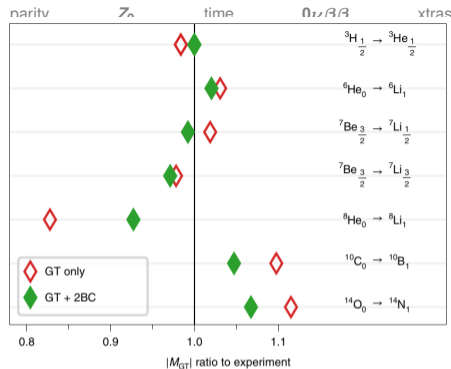
- mixing with the Δ moves strength to high excitation?
- Nuclear structure calculations lack configurations?
- g_A changing in nuclei?
- Maybe π becomes effectively massless in nuclei (chiral symmetry taken to its extreme) and π degrees of freedom go away completely $\rightarrow g_A=1$? Rho Ann Rev Nucl Part 34 531 (1984)

Recent calculations Gysberg Nat Phys 15 428 2019 reproduce GT strength with about 5-10% accuracy, combining chiral EFT's with accurate many-body techniques, considering 2-body currents.

2-body currents are the chiral EFT equivalent of meson exchange currents, and are treated consistently with 3-nucleon forces in chiral EFT in Gysberg et al.

So both more configurations and 2-body currents are important.

The need for 2-body currents \iff the axial current strength is changing in nuclei.



Jackson, Treiman, Wyld 1957 wrote down 4-Fermi vertex interaction H for nucleon beta decay. (These could be written in a more natural helicity basis...)

You construct Lorentz-invariant quantities, i.e. a Lorentz scalar, from the possible objects which Lorentz transform like vectors, axial vectors, scalars, tensors, pseudoscalars (it turns out all combinations of more Dirac matrices reduce to these).

Assuming pointlike high-mass bosons only, one could now call this an EFT Lee did consider gradients:

Quark-lepton interactions have been found experimentally to be V,A only so far. V is assumed conserved (like electric charge), so $C_V=1$ is often assumed. QCD still can change A, and 'induce' all the other terms for hadron-lepton interactions, changing all these constants but C_V . We've seen how this creates interesting ways to test QCD's influence on weak interactions, and we've already seen $|C_A| = 1.26...$

I.e. this looks a lot like the S.M. quark-lepton Lagrangian but of course we have to be careful about the C_x 's

$$\begin{aligned}
 H_{\text{int}} = & (\bar{\psi}_p \psi_n) (C_S \bar{\psi}_e \psi_\nu + C_{S'} \bar{\psi}_e \gamma_5 \psi_\nu) \\
 & + (\bar{\psi}_p \gamma_\mu \psi_n) (C_V \bar{\psi}_e \gamma_\mu \psi_\nu + C_{V'} \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu) \\
 & + \frac{1}{2} (\bar{\psi}_p \sigma_{\lambda\mu} \psi_n) (C_T \bar{\psi}_e \sigma_{\lambda\mu} \psi_\nu + C_{T'} \bar{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_\nu) \\
 & - (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n) (C_A \bar{\psi}_e \gamma_\mu \psi_\nu + C_{A'} \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu) \\
 & + (\bar{\psi}_p \gamma_5 \psi_n) (C_P \bar{\psi}_e \psi_\nu + C_{P'} \bar{\psi}_e \gamma_5 \psi_\nu) \\
 & + \text{Hermitian co}
 \end{aligned}$$

Jackson, Treiman, Wyld 1957 wrote down observables before angular integration, and the answers

$$\begin{aligned} \omega(\langle J \rangle | E_e, \Omega_e, \Omega_\nu) dE_e d\Omega_e d\Omega_\nu \\ = \frac{1}{(2\pi)^5} p_e E_e (E^0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu \xi \left\{ 1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} + b \frac{m}{E_e} \right. \\ \left. + c \left[\frac{1}{3} \frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} - \frac{(\mathbf{p}_e \cdot \mathbf{j})(\mathbf{p}_\nu \cdot \mathbf{j})}{E_e E_\nu} \right] \left[\frac{J(J+1) - 3\langle (\mathbf{J} \cdot \mathbf{j})^2 \rangle}{J(2J-1)} \right] \right. \\ \left. + \frac{\langle J \rangle}{J} \left[A \frac{\mathbf{p}_e}{E_e} + B \frac{\mathbf{p}_\nu}{E_\nu} + D \frac{\mathbf{p}_e \times \mathbf{p}_\nu}{E_e E_\nu} \right] \right\}. \end{aligned}$$

Rather than use JTW's answers here,

Re: the 'Fierz interference' term $b \frac{m_e}{E_e}$,

product of a SM term with normal helicity and a *SM* term with non-normal helicity:

$$\sqrt{1 + \frac{p_e}{E_e}} \times \sqrt{1 - \frac{p_e}{E_e}} = \sqrt{1 - \frac{p_e^2}{E_e^2}} = \frac{m_e}{E_e} \text{ take care with particle physics 'chirality' vs. 'helicity'}$$

Reference for non-Dirac treatment: R. Hong, M. Sternberg, A. Garcia, "Helicity and nuclear β decay correlations," American Journal of Physics 85 p 45 (2017).

we'll assume the functional form of the correlations. In limiting cases, assumptions about S.M. lepton helicity will then let us deduce the S.M. predictions soon.

The S.M. weak interaction makes left-handed leptons and right-handed antileptons in decays, Helicity $\hat{s} \cdot \hat{p}$

Note $\frac{p}{E}$ is, of course, $\frac{v}{c}$. One can always boost to a frame moving faster than a massive particle—reversing \hat{p} but preserving \hat{s} . That's intuitively why there's a factor of $\frac{v}{c}$ multiplying the helicities.

Measure ν helicity $\epsilon = \hat{s}_\nu \cdot \hat{k}_\nu$ directly: transfer \hat{s}_ν to γ circular polarization; boost \vec{k}_γ by $\pm \vec{k}_\nu$

Goldhaber, Grodzins, Sunyar
Phys Rev 109 1015 (Dec 1957)



• ν with $\hat{s} = -1$ populates
 $\langle J_z \rangle = 0, +1$ **not -1**

• So γ is circularly polarized—
transmission through magnet
depends on iron polarization:

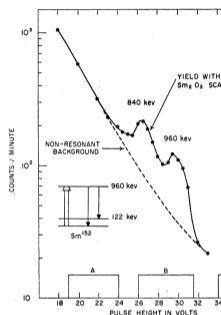
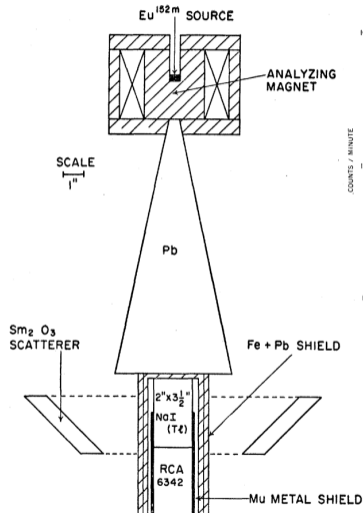
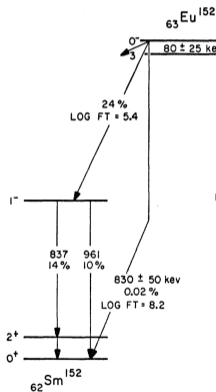
$$\frac{N_+ - N_-}{N_+ + N_-} = 0.017 \pm 0.003$$

• Upward ν boosts γ
momentum so it can be
absorbed on-resonance

$\Rightarrow \nu$ helicity $-1 \pm 10\%$

(• $\bar{\nu}$ helicity $\sim +1$)

Palathingal PRL 524 24 '69)



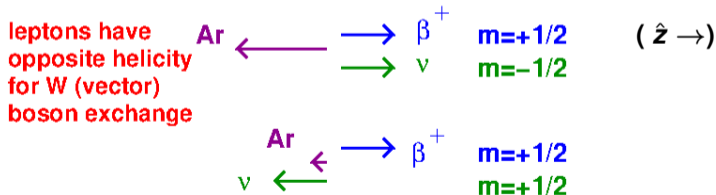
Surprisingly enough, this is the best **direct** measurement of ν helicity $= \hat{s}_\nu \cdot \hat{k}_\nu$

The β - ν angular distribution in the SM

$$W[\theta_{\beta\nu}] = 1 + a \frac{v_\beta}{c} \cos \theta_{\beta\nu}$$

For ^{38m}K , $0^+ \rightarrow 0^+$ decay:

$a = +1$ 'Proof':



The β - ν angular distribution in the SM

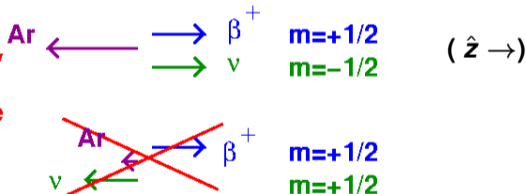
$$W[\theta_{\beta\nu}] = 1 + a \frac{v_\beta}{c} \cos \theta_{\beta\nu}$$

For ^{38m}K , $0^+ \rightarrow 0^+$ decay:

$a = +1$ 'Proof':



leptons have
opposite helicity
for W (vector)
boson exchange



For scalar exchange, lepton helicities are same: $a = -1$

No nuclear structure corrections until 10^{-6} accuracy (Isospin breaking only mixes in 0^+ configurations)

Note $a_{\beta\nu}$ depends on the relative helicity of β and ν , but not the absolute sign. The observable is parity-even \rightarrow is not actually sensitive to \mathcal{P}

$a_{\beta\nu} = -1/3$ Gamow-Teller decay

If you were to work the matrix element (trace of Dirac matrices...) you would see this is a consequence of the $\gamma(1+\gamma_5)$ Lorentz structure from the W^+ .

I.e. a Gamow-Teller decay, just like Fermi decay, still makes left-handed ν and right-handed β^+ (or right-handed $\bar{\nu}$ and left-handed β^-)

But the angular distribution result is different because of the nonzero nuclear spin involved.

This is simplest to see in $0^+ \text{ } ^6\text{He} \rightarrow 1^+ \text{ } ^6\text{Li} + \beta^- + \bar{\nu}$

The final ^6Li can have 3 different spin projections. Orient \hat{z} up:

$m(^6\text{Li})$	m_{β^-}	m_{ν}	β, ν relative direction	a contribution
+1	+1/2	+1/2	opposite	-1
-1	-1/2	-1/2	opposite	-1
0	m_{β}	$-m_{\beta}$	same	+1 (like Fermi $0^+ \rightarrow 0^+$)
				ave -1/3

For any Gamow-Teller transition, if the weak interaction produces opposite-handed leptons and antileptons, $a_{\beta\nu} = -1/3$.

Scalar and tensor Lorentz currents produce same-handed leptons and antileptons, and for Gamow-Teller $a_{\beta\nu} = +1/3$

$a_{\beta\nu}$ experiments

Feynman&Gell-Mann paper PR 109 193 (1957)

proposing CVC and $V\pm A$:

"These theoretical arguments seem to the authors to be strong enough to suggest that the disagreement with the He^6 recoil experiment... indicates that these experiments are wrong.

The $\pi \rightarrow e + \bar{\nu}$ problem may have a more subtle solution."

successes listed:

μ decay rate to 2%

asymmetry in direction of

$\pi \rightarrow \mu \rightarrow e$ chain

$a_{\beta\nu}$ in ^{35}Ar

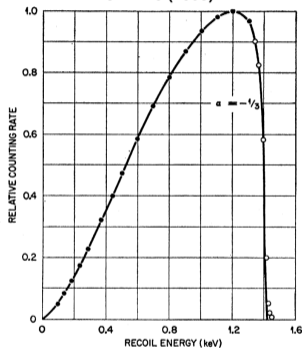
Undistorted E_β spectra

e^- polarization from β decay

^6He Gamow-Teller decay

Johnson, Pleasonton, Carlson PhysRev

132 1149 (1963)



$a_{\beta\nu} = -0.3308 \pm 0.0030$
 agreed much better with
 V, A ($-1/3$)
 than
 S, T ($+1/3$).

Fermi $0^+ \rightarrow 0^+$ decays:

Adelberger PRL 83 1299 (1999) (err. 83 3101)

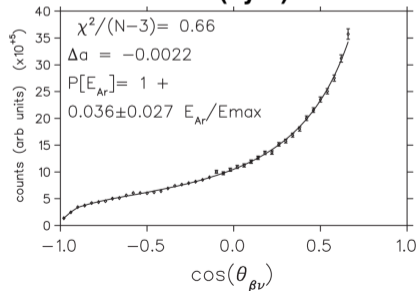
^{32}Ar

$$\tilde{a} = 0.9989 \pm 0.0052(\text{stat}) \pm 0.0039(\text{syst})$$

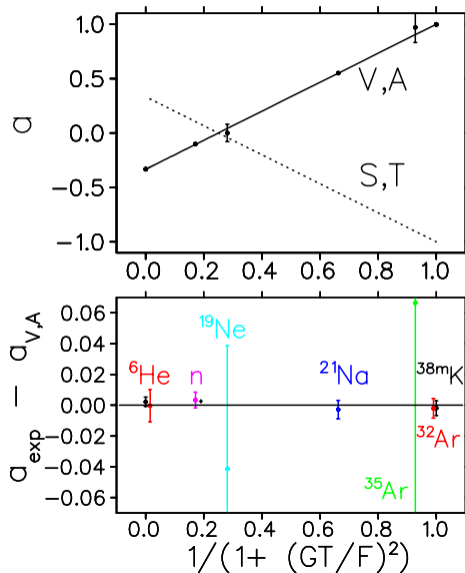
Gorelov PRL 94 142501 (2005)

^{38m}K

$$\tilde{a} = 0.9981 \pm 0.0030(\text{stat}) \pm 0.0037(\text{syst})$$



Summarizing info on Lorentz structure from $\beta\nu$ correlation



Interaction is mostly vector and axial vector, i.e. V and A

[Except aSPECT has difference in a for neutron $(2.57 \pm 0.84) \times 10^{-3}$

Beck PRC 101 055506 (2020)

Explainable by a finite Lorentz tensor allowed by other nuclear β decay

Falkowski JHEP04 (2021) 126

but recent ${}^8\text{Li}$ $\beta\alpha\alpha$ correlation agrees with ${}^6\text{He}$ $a_{\beta\nu}$ with higher precision

Burkey PRL 128 202502 (2022); Sargsyan PRL 128 202503 (2022)]

For the sign between them, we need to consider parity violation \rightarrow

Symmetries: Continuous, Discrete

● Noether's theorem (1915):

Continuous symmetry	→	Conserved quantity
Time-translational invariance	→	Energy
Space-translational invariance	→	Momentum
Rotational invariance	→	Angular momentum
(Laplace-Runge-Lenz vector)	→	name?

THE LATE EMMY NOETHER.

Professor Einstein Writes in Appreciation of a Fellow-Mathematician.

To the Editor of The New York Times:

Discrete symmetries in quantum mechanics are quite different, but we'll appeal to classical intuition concerning observables for \mathcal{P}, \mathcal{T} .

gan. In the realm of algebra, in which the most gifted mathematicians have been busy for centuries, she discovered methods which have proved of enormous importance in the development of the present-day younger generation of mathematicians. Pure mathematics is, in its way, the poetry of logical ideas. One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulae are discovered necessary for the deeper penetration into the laws of nature.

Emmy Noether's
**WONDERFUL
THEOREM**

Noether's Theorem:
If under the infinitesimal transformation

$$t' = t + \epsilon \tau + \dots$$

$$q'^\mu = q^\mu + \epsilon \zeta^\mu + \dots$$

the functional

$$\Gamma = \int_{t_0}^{t_1} L(t, q^\mu, \dot{q}^\mu) dt$$

is both invariant and extremal, then the following conservation law holds:

$$p_{\mu} \zeta^\mu - H \tau = \text{const.}$$

Revised and Updated Edition
DWIGHT E. NEUENSCHWANDER

Historical Ideas about P , T breaking

- Wigner considered implications of P , T symmetry conservation in atomic spectra 1926-28. Showed $\langle T\psi_i, T\psi_f \rangle = \langle \psi_f, \psi_i \rangle^*$

“In quantum theory, invariance principles permit even further reaching conclusions than in classical mechanics.” (D. Gross, Physics Today 48 46 (1995))

- Weyl 1931 considered C , P , T and CPT in “Maxwell-Dirac theory”: $C \Rightarrow$ Dirac eq. negative energy states had to have same mass as the e^- plato.stanford.edu

- From “CP Violation Without Strangeness” Khriplovich and Lamoreaux: 1949 Dirac “I do not believe there is any need for physical laws to be invariant under reflections in space and time although the exact laws of nature so far known do have this invariance.”

- 1956 Lee and Yang proposed \not{P} in weak decays to fix the θ - τ puzzle

- Feynman gives Ramsey 50:1 odds \not{P} would not be observable
Ramsey experiment starting at ORNL gets derailed by fission experiments...
it's OK, Ramsey won 1989 Nobel for his fringes

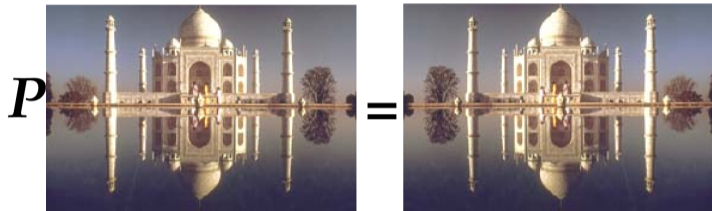
- 1957 3 simultaneous experimental measurements of $\not{P} \rightarrow$

Parity (From A. Zee “Fearful Symmetry”)

As of 1956, we thought
all interactions
respected parity

Parity operator

$$P \psi(\vec{r}) \rightarrow \pm \psi(-\vec{r})$$

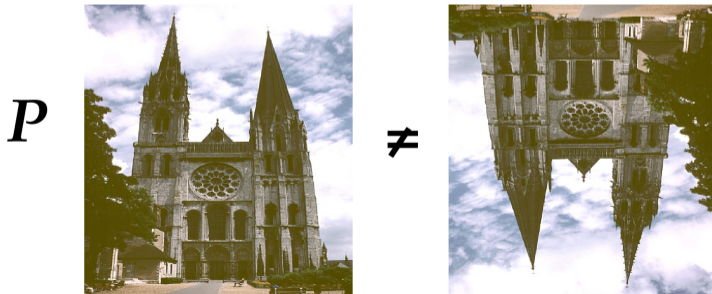


1957:

$\tau - \theta$ Puzzle

+ μ decay

+ ^{60}Co decay \Rightarrow



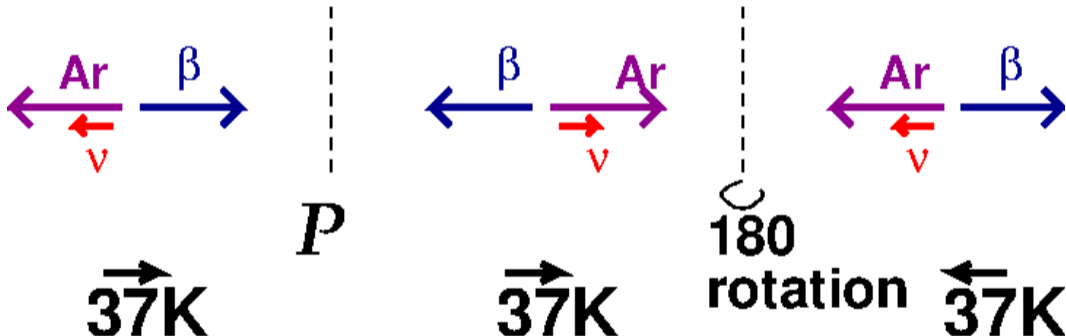
Decays: Parity Operation can be simulated by Spin Flip

Under Parity operation P :

$$\vec{r} \rightarrow -\vec{r}$$

$$\vec{p} \sim \frac{d\vec{r}}{dt} \rightarrow -\vec{p}$$

$$\vec{J} = \vec{r} \times \vec{p} \rightarrow +\vec{J}$$



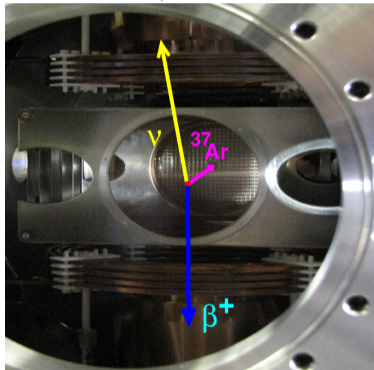
\Rightarrow A spin flip corresponds exactly to P reversal

Decays don't exactly test T -reversal symmetry \rightarrow

\mathcal{T} correlation of 3 of 4 momenta

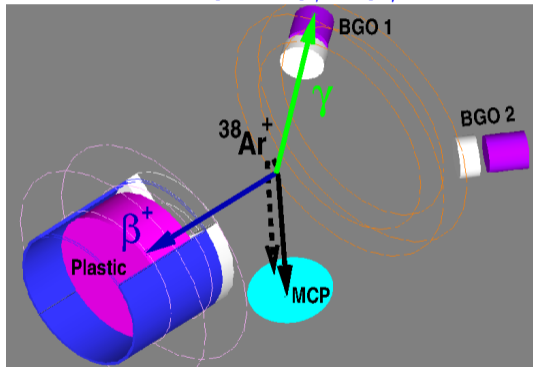
$$\mathbf{t} \rightarrow -\mathbf{t} \Rightarrow \vec{\mathbf{p}} \propto \frac{d\vec{\mathbf{r}}}{dt} \rightarrow -\vec{\mathbf{p}}$$

$$\text{but } \vec{\mathbf{p}}_{\text{recoil}} \cdot \vec{\mathbf{p}}_{\beta} \times \vec{\mathbf{p}}_{\nu} \equiv 0 \text{ 😞}$$



$$\vec{\mathbf{p}}_{\nu} \cdot \vec{\mathbf{p}}_{\beta} \times \vec{\mathbf{p}}_{\gamma} = -\vec{\mathbf{p}}_{\text{recoil}} \cdot \vec{\mathbf{p}}_{\beta} \times \vec{\mathbf{p}}_{\gamma}$$

$$\xrightarrow{\mathbf{t} \rightarrow -\mathbf{t}} \vec{\mathbf{p}}_{\text{recoil}} \cdot \vec{\mathbf{p}}_{\beta} \times \vec{\mathbf{p}}_{\gamma}$$



- We can test symmetry of apparatus with coincident pairs 😊
- Not exact. Outgoing particles interact → fake \mathcal{T}

One experimental discovery of parity violation

Wu, Ambler, Hayward, Hopper, Hobson, PR 105 1413; (Garwin Lederman Weinrich PR 105 1415 Feb '57; Hargittai Physics World 13 Sep 2012)
 Abashian BNL PR 105 1927 1957 (Lee: gradient!); emulsion Friedman Telegdi PR 106 1290

Dilution Refrigerator to spin-polarize with nuclear polarization

$$P = \langle \frac{J_z}{J} \rangle$$



$$W[\theta] = 1 + PA\hat{J} \cdot \frac{\vec{p}_\beta}{E_\beta}$$

$$= 1 + AP\frac{v}{c}\cos[\theta]$$

$$A_{\beta^-} = -1.0$$

Note: $5^+ \rightarrow 4^+$ Null

if left-handed $\hat{v}_{\beta^-} = \hat{J}_i$

Let $\hat{J}_i = +\hat{z}$

enforce $m_i = m_f$

$$m_i^j = +5$$

$$m_\beta = -1/2, m_f^j = +4$$

can't happen \Rightarrow

$$A_{\beta^-} = -1 \text{ QED}$$

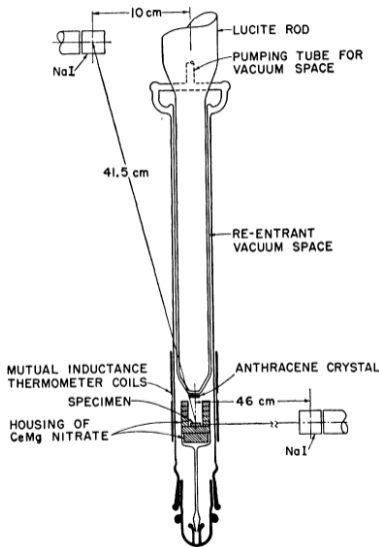
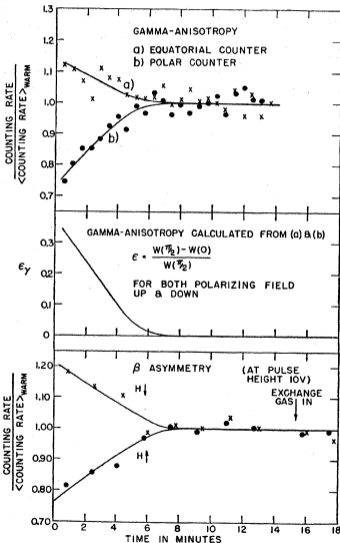


Fig. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

Lookup for allowed β -decay correlations

Jackson Treiman Wyld NuclPhysA 4 206 (1957)

± 1 , no nuclear parity change, $J = 0 \rightarrow J' = 0$ forbidden. J and J' are the angular momenta of the original and final nuclei. $\delta_{JJ'}$ is the Kronecker delta symbol and

$$\lambda_{JJ'} = \begin{cases} 1 & J \rightarrow J' = J-1 \\ \frac{1}{J+1} & J \rightarrow J' = J \\ -\frac{J}{J+1} & J \rightarrow J' = J+1 \end{cases} \quad (\text{A.1})$$

$$A_{JJ'} = \begin{cases} 1 & J \rightarrow J' = J-1 \\ -\frac{(2J-1)}{J+1} & J \rightarrow J' = J \\ \frac{J(2J-1)}{(J+1)(2J+3)} & J \rightarrow J' = J+1 \end{cases} \quad (\text{A.2})$$

Z is the atomic number of the final nucleus, α is the fine structure constant, and $\gamma = (1 - \alpha^2 Z^2)^{\frac{1}{2}}$.

For pure G-T: $A_{\beta\pm} = \pm \lambda_{J',J}$

Textbooks with calculations:

a, the ' $\beta - \nu$ correlation':

Halzen&Martin "Quarks&Leptons,"

my notes [ph505jbVIII.2005.aBetaNu.WithDirac.pdf](#)

Melconian's notes include Fierz term!

A, the ' β asymmetry wrt spin':

Greiner and Müller "Gauge Theory of Weak Interactions"

Towner's notes within mine

upper sign for β^- , lower sign for β^+

$$\xi = |M_F|^2 (|C_S|^2 + |C_V|^2 + |C'_S|^2 + |C'_V|^2) + |M_{GT}|^2 (|C_T|^2 + |C_A|^2 + |C'_T|^2 + |C'_A|^2) \quad (\text{A.3})$$

$$a\xi = |M_F|^2 \left\{ [-|C_S|^2 + |C_V|^2 - |C'_S|^2 + |C'_V|^2] \mp \frac{\alpha Z m}{p_e} 2 \operatorname{Im} (C_S C_V^* + C'_S C'_V^*) \right\}$$

$$+ \frac{|M_{GT}|^2}{3} \left\{ [|C_T|^2 - |C_A|^2 + |C'_T|^2 - |C'_A|^2] \pm \frac{\alpha Z m}{p_e} 2 \operatorname{Im} (C_T C_A^* + C'_T C'_A^*) \right\} \quad (\text{A.4})$$

$$b\xi = \pm 2\gamma \operatorname{Re} [|M_F|^2 (C_S C_V^* + C'_S C'_V^*) + |M_{GT}|^2 (C_T C_A^* + C'_T C'_A^*)] \quad (\text{A.5})$$

$$c\xi = |M_{GT}|^2 A_{JJ'} \left[|C_T|^2 - |C_A|^2 + |C'_T|^2 - |C'_A|^2 \pm \frac{\alpha Z m}{p_e} 2 \operatorname{Im} (C_T C_A^* + C'_T C'_A^*) \right] \quad (\text{A.6})$$

$$A\xi = |M_{GT}|^2 \lambda_{JJ'} \left[\pm 2 \operatorname{Re} (C_T C'_T^* - C_A C'_A^*) + \frac{\alpha Z m}{p_e} 2 \operatorname{Im} (C_T C'_A^* + C'_T C_A^*) \right]$$

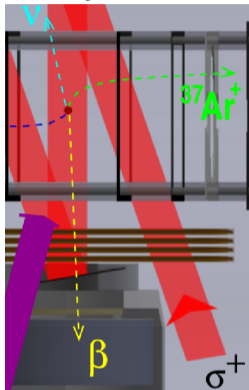
$$+ \delta_{JJ'} M_F M_{GT} \sqrt{\frac{J}{J+1}} \left[2 \operatorname{Re} (C_S C'_T^* + C'_S C_T^* - C_V C'_A^* - C'_V C_A^*) \right]$$

$$\pm \frac{\alpha Z m}{p_e} 2 \operatorname{Im} (C_S C'_A^* + C'_S C_A^* - C_V C'_T^* - C'_V C_T^*) \quad (\text{A.7})$$

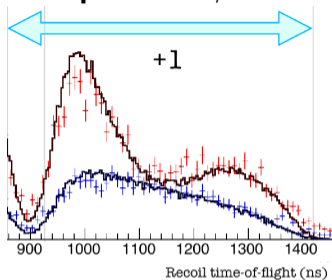
+ $B_\nu, \mathcal{T}D, \dots$

A different isospin mirror-decay spin-polarized observable

Isobaric mirror decay has helicity-driven null



2014 polarized β-recoil



$v_{\text{TOFaxis}} = 0$ suppressed.
Dip would be deeper with ion MCP position cut or $\cos(\theta_{\beta-\nu})$ determination

$$W(\theta, P) \approx 1 + a_{\text{pol}} \cos(\theta_{\beta\nu})$$

with $a_{\text{pol}} = (A_{\beta} - B_{\nu})P - a_{\beta\nu} + 2c/3 = 1$ or 0 , independent of $\frac{M_{\text{GT}}}{M_{\text{F}}}$

The neutron community checks this combination of observables for consistency
Mostovoi+Frank Pis'ma Zh. Eksp. Teor. Fiz. 24 45 (1976)

a_{pol} is likely to be statistics-limited.
We consider this the most direct measurement of the ν helicity since Goldhaber et al., though the $I^{\pi} = 3^{+} \rightarrow 2^{+}$ decay of ^{38}gK would require both leptons to have non-SM helicity and be more complete

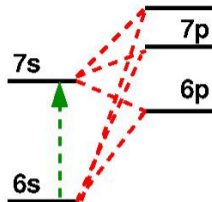
Weak Neutral Current

Existence of Z^0 boson, spin-1 partner of W^\pm and the photon, was a S.M. prediction.

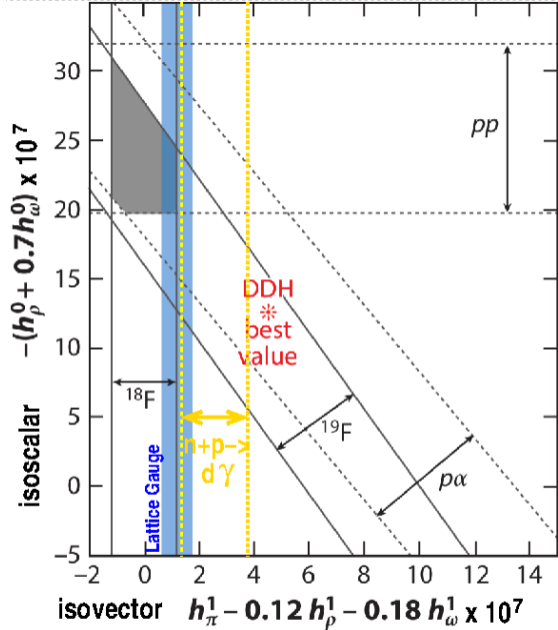


Searched for in :

- ν scattering (winner: Gargamelle) (not parity-violating!)
- Atomic \mathcal{P} by mixing atomic states of opposite parity (1st answers came in small, creating concern for the S.M. prediction; now using cesium the best low-energy measurement of e^- - q weak neutral coupling) $\propto Z^2 N$



- **parity violating nucleon-nucleon interaction, via γ asymmetries from decay of nuclear states.** \mathcal{P} can also come from the known charged current (W^{+-}). It was noticed that isovector \mathcal{P} could only come from the neutral current, so that search was emphasized early on. It turns out the isovector \mathcal{P} was suppressed compared to isoscalar and isotensor for reasons only understood more recently, and the isovector \mathcal{P} has only been measured very recently to be nonzero. (Otherwise Queens and Cal State L.A. would have measured weak neutral current in ^{18}F and shared Nobel with Gargamelle.)
- SNO used neutral current breakup of d , independent of ν flavor
- \mathcal{P} electron scattering on the proton at SLAC and JLAB; for neutron skin of ^{48}Ca and ^{208}Pb .
- COHERENT scattering of ν from nuclei is agreeing so far with SM cross-section (not \mathcal{P} !)



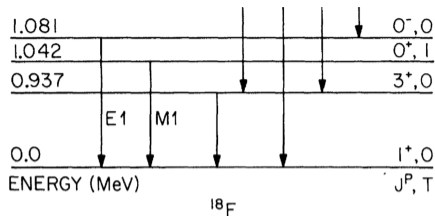
Weak interaction between nucleons, \mathcal{P}
 W^\pm, Z^0 ($m=80.4, 91.2$ GeV) are very short-ranged compared to mesons.

- Parameterized by meson exchange (emitted weakly, absorbed strongly...)
- The isovector piece was long expected to be dominated by the weak neutral current, but the $1/N_c$ expansion suppresses isovector/isoscalar by $\sin^2(\theta_W)/N_c \approx 1/12$ (Phillips et al. PRL 114 062301 (2015)).
- A formal EFT produces similar results.
 - Isovector and isoscalar parts now considered measured.
- An isotensor part is interesting and inspiring proposals like $\vec{\gamma} + d \rightarrow n + p$

E.g. measuring N-N ρ by mixing of ^{18}F 0^- and 0^+ states: much nuclear physics

• Observable is the circular polarization of the 1.081 MeV γ -ray, caused by E1 interference with the parity-violating M1, $-0.7 \pm 2.0 \times 10^{-3}$

Sensitivity is **enhanced** by:



Barnes et al. PRL 40 840 (1977)

- The J^+ ; $T = 0^-$; 0 and 0^+ ; 1 states lie **close together in energy**, admixture $\propto \frac{\langle 0^- | \mathcal{O}_{\text{WeakNN}} | 0^+ \rangle}{\Delta E}$
- The **E1 operator is isovector** (except for a tiny correction from the long-wavelength approximation), so is **suppressed** by $\sim 10^{-4}$ between the $T=0$ states, so the parity-violating M1 competes better so the circular polarization is larger 😊

• A hard-to-calculate nuclear matrix element is needed to extract the weak N-N physics. (We noticed the 0^- state involves excitations of the p shell, which is quite complicated.) The same effective operator contributes, with known β -decay constants of proportionality, to the forbidden β decay of the isobaric analog 0^+ ; $T = 1$ state in ^{18}Ne .

Summarized in Haxton PRL 46 698 (1981) and the experimental paper before it Adelberger, Hoyle, Swanson, Lintig 695

- The experimental asymmetry measured was $\sim 10^{-5}$, while in $n(p,d)\gamma$ was 3×10^{-8}



TRIUMF Physics and time reversal

When $t \rightarrow -t$, does anything change?

- Wave Equ. is 2nd-order in t : $\nabla^2 u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$ **symmetric in t**

- Heat Equ. is 1st-order in t : $\nabla^2 u = -\frac{\partial u}{\partial t}$ **$t \rightarrow -t$, boom?**

‘Dissipation’, like friction... The arrow of time remains a research problem in stat mech, but it’s not from (known) particle physics

- Schrodinger Equation is 1st order: $i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$

‘Take the complex conjugate’ (as Wigner did above)

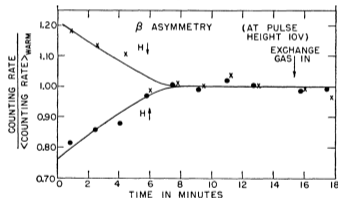
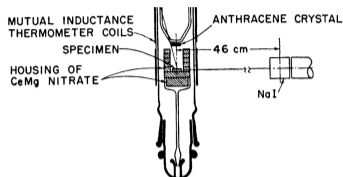
(but see Dressel et al. PRL 119 220507 (2017))

“Arrow of Time for Continuous Quantum Measurements”)

Microscopic physics was thought to be symmetric in t



Parity broken, why not Time?



Immediately after \mathcal{P} arity was seen to be totally broken in β decay (' ν left-handed')

**Wu, Ambler, Hayward, Hopper, Hobson,
PR 105 (1957) 1413**

Many T-odd observables were proposed:

PHYSICAL REVIEW

VOLUME 106, NUMBER 3

Possible Tests of Time Reversal Invariance in Beta Decay

J. D. JACKSON,* S. B. TREIMAN, AND H. W. WYLD, JR.

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received January 28, 1957)

Need scalar triple products of 3 vectors:
observables involving spin

$$D \hat{J} \cdot \frac{\vec{p}_\beta}{E_\beta} \times \frac{\vec{p}_\nu}{E_\beta} \quad R \vec{\sigma}_\beta \cdot \hat{J} \times \frac{\vec{p}_\beta}{E_\beta}$$

are consistent with $\mathcal{T} < 0.001$

but some has been found \rightarrow

Possible Tests of Time Reversal Invariance in Beta Decay

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\mathcal{CP} discovered in $K\bar{K}$ meson decays in 1963,
though not much (Cronin and Fitch Nobel prize 1980)

Quark eigenstates in the weak interaction:

To explain some weak decays we saw,

$$|u\rangle \rightarrow |d\rangle + \epsilon|s\rangle \quad \text{i.e.} \quad |u\rangle \rightarrow \cos(\theta_C)|d\rangle + \sin(\theta_C)|s\rangle$$

Maybe one reason for 3 families of particles,

→ 3x3 unitary “CKM” matrix between $|d\rangle$, $|s\rangle$, $|b\rangle$

There is one complex phase, which leads to this type of \mathcal{CP}

Any 2x2 unitary matrix, one can define away the phase as trivial

A reason for 3 generations of particles?

That one phase is consistent with CP in $K\bar{K}$ and $B\bar{B}$ systems

There have been hints in $K\bar{K}$ and $B\bar{B}$ of more CP than in the standard model,

$p\bar{p} \rightarrow \mu^+\mu^+$ or $\mu^-\mu^-$ CP at 3.6σ
Abazov PRD 2014 Fermilab;

so this 2001 cartoon was a little premature \rightarrow



J. Fabergé. CERN Courier, 6, No. 10, 193 (October 1966). [Courtesy of Madame Fabergé.]

T2K ν_μ oscillations different from $\bar{\nu}_\mu$ at 2 to 3 σ Nature 580 339 (2020)

CP could have some utility for cosmology \rightarrow

The excess of matter over antimatter can come from \mathcal{CP}

Sakharov JETP Lett 5 24 (1967) used \mathcal{CP} to generate the universe's excess of matter over antimatter:

- \mathcal{CP} ,
- baryon nonconservation, and
- nonequilibrium.

But known \mathcal{CP} is too small by 10^{10} , so 'we' need more to exist. Caveats:

- You could use \mathcal{CPT} though there are no complete models [Dolgov Phys Rep 222 (1992) 309]
- We need \mathcal{CP} in the early universe, not necessarily now

A concrete demonstrative example from Ramsey-Musolf at INT 2020

\cancel{CP} explaining T2K's ν vs. $\bar{\nu}$ result lets
heavy N decay this way in some
models

Other mechanisms have much more
abstract \cancel{CP}

Dine-Affleck models require non-SM
physics, but not explicit particles, and
don't need high-Temp early cosmology

I. Affleck and M. Dine, Nucl. Phys. B 249, 361 (1985)

So we look for more \cancel{CP} . How is this
related to \cancel{T} ?

www.int.washington.edu/Talks/WorkShops/int_20_2b/People/Ramsey-Musolf_M/Ramsey-Musolf.pdf

Neutrinos and the Origin of Matter

- Heavy neutrinos decay out of equilibrium in early universe
- Majorana neutrinos can decay to particles and antiparticles
- Rates can be slightly different (CP violation)

$$\Gamma(N \rightarrow \ell H) \neq \Gamma(N \rightarrow \bar{\ell} H^*)$$

- Resulting excess of leptons over anti-leptons partially converted into excess of quarks over anti-quarks by Standard Model sphalerons

\mathcal{T} is related to \mathcal{CP} by the “CPT Theorem”

“All local Lorentz invariant QFT’s are invariant under CPT”

Schwinger Phys Rev 82 914 (1951)

Lüders, Pauli, Bell 1954

- Gravity \rightarrow not flat:

K meson experiments Adler

PhysLettB 364 (1995) 239 test \mathcal{CPT} to

within 1000x expected from quantum gravity

direct tests include e^+e^- decay

asymmetry $< 10^{-4}$ Moskal Nat Comm 2021

Proofs still pursued \rightarrow

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On the CPT theorem

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ABSTRACT

We provide a careful development and rigorous proof of the CPT theorem within the framework of mainstream (Lagrangian) quantum field theory. This is in contrast to the usual rigorous proofs in purely axiomatic frameworks, and non-rigorous proof-sketches in the mainstream approach. We construct the CPT transformation for a general field directly, without appealing to the enumerative classification of representations, and in a manner that is clearly related to the requirements of our proof. Our approach applies equally in Minkowski spacetimes of any dimension at least three, and is in principle neutral between classical and quantum field theories: the quantum CPT theorem has a natural classical analogue. The key mathematical tool is that of complexification; this tool is central to the existing axiomatic proofs, but plays no overt role in the usual mainstream approaches to CPT.

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Assuming CPT, $\mathcal{CP} \Leftrightarrow \mathcal{T}$ in most physics theories

The matter excess then motivates \mathcal{T} searches

EDM in a fundamental particle breaks T : this is exact

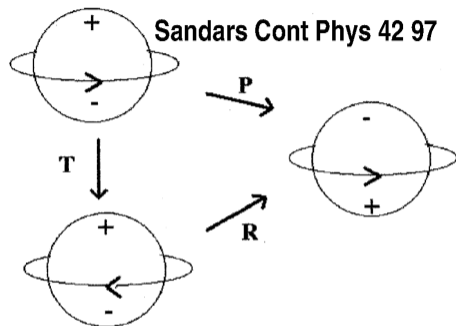
Landau, Nucl. Phys. 3 (1957) p. 127

Electric Dipole moment $\vec{d} = \sum q_i \vec{r}_i$

Since the angular momentum is the only vector in the problem, $\vec{d} = a\vec{J}$

Under T , $\vec{J} \xrightarrow{T} -\vec{J}$ $\vec{d} \xrightarrow{T} +\vec{d}$

If the physics is invariant under T , this is a contradiction, $\Rightarrow a = 0$



[• The other logical possibility: there are 2 states, with opposite sign of the EDM, and T just formally changes one state to the other.

For most fundamental particles, we know there aren't 2 states

Why do we know the electron doesn't have 2 states?

E.g. some polar molecules have a dipole moment listed in tables, which produces degenerate states and does not break T ...]

Schiff's Theorem: does a nuclear EDM make an atomic EDM?

Schiff's Theorem PR 132 2194 (1963): The nuclear electric dipole moment $d_{\text{nuclear}} = \sum q_i r_i \hat{r}_i$ causes the atomic e^- 's to rearrange themselves so they develop an opposite dipole moment. In the limit of nonrelativistic e^- 's and a point nucleus, the e^- 's dipole moment exactly cancels the nuclear moment, so that the net atomic dipole moment vanishes.

(For the e^- 's EDM, there is 'antiscreening,' and $d_{\text{atom}} \overset{Z \gg 1}{\gg} d_{e^-}$ Sandars Phys Lett 14 194 (1965))

The Schiff moment S involves $\sum q_i r_i^2 \hat{r}_i$ does not get screened completely:

$$\langle S \rangle = \sum q_i (r_i^2 - \frac{5}{3} \langle R_{\text{ch}}^2 \rangle) \approx R_{\text{nucleus}}^2 d_{\text{nucleus}}, \text{ so } d_{\text{atom}} / d_{\text{nucleus}} \sim R_{\text{nucleus}}^2 / R_{\text{atom}}^2 \sim 10^{-8}$$

Combination of Large Z and relativistic wf's offset by $10 Z^2 \approx 10^5$, with overall suppression of $d_{\text{atom}} \sim 10^{-3} d_{\text{nucleus}}$

Best measurements in diamagnetic (atomic total angular momentum 0) ^{199}Hg constrain strong interaction \mathcal{T} competitive with neutron EDM.

A nuclear magnetic quadrupole moment is also \mathcal{T} . This also produces an observable atomic EDM, yet with no screening Haxton+Henley PRL 51 1937 (1983), so it's more accurate to interpret experiments. (The total atomic angular momentum must be nonzero, so stray Larmor precession of 1000x greater μ makes experiments challenging.)

\mathcal{T} in QCD and nucleon-nucleon interactions

$$\mathcal{L}_{\mathcal{CP}} = \theta_{QCD} \frac{g^2}{32\pi^2} \mathbf{F}_\alpha^{\mu\nu} \mathbf{F}_{\alpha\mu\nu}^*$$

Neutron EDM bounds $\Rightarrow \theta_{QCD} \lesssim 0.5 \times 10^{-10}$

Peccei-Quinn mechanism drives θ_{QCD} small by a global U(1):

breaking the U(1) produces a 0^- axion with mass \propto (symmetry-breaking scale)/(coupling).

Null experiments drive that scale high.

Other \mathcal{T} physics in the N-N potential is parameterized by isoscalar, isovector, and isotensor terms, with a separate set for whether or not they break P .

(Terms can be related by chiral EFT deVries fphy.2020.00218 and the $1/N_c$ expansion Samart

PRC 94 024001 (2016))

θ_{QCD} is isoscalar

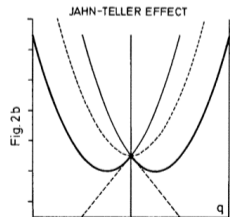
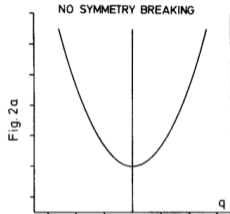
The QCD and effective nucleon-nucleon \mathcal{T} physics produces:

- \mathcal{T} nuclear Schiff and magnetic quadrupole moments,
- \mathcal{T} asymmetries in polarized beam experiments (Simonius PRL 78 4161 (1997))
- \mathcal{T} asymmetries in polarized neutron experiments on polarized targets ($\lesssim 10^{-5}$ Huffman et al. PRC 55 2284 (1997), with plans to improve these at next-generation neutron sources enough to complement n and ^{199}Hg EDM experiments.)

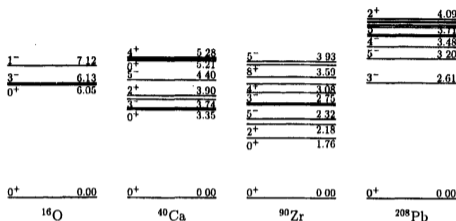
If one sees these asymmetries, they are from \mathcal{T} : unlike decays, they are free of 'final-state interaction' false effects.

Octupole physics

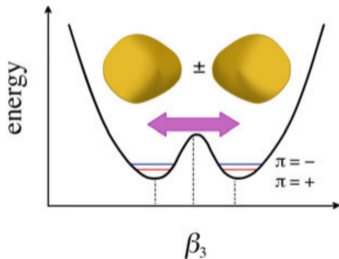
A good e.g. of the Jahn-Teller effect



Octupole $\Delta J^\pi=3^-$ vibrations can be excited in many nuclei:



Wong Fig 6.2



Obertelli and Sagawa Fig. 7.33

• Pear-shaped nucleus is P . Must construct good- P wf's:

$$\psi_{\pm} = (|a\rangle \pm |b\rangle) / \sqrt{2}$$

A parity doublet of otherwise nearly identical states, each with a band of same P .

• Relatively large E1's between the parity-doubled states are produced by the induced static electric dipole moment:

Bohr Mottelson NP4 529 (1957)

$$D = -\frac{81}{8\pi} 35^{-3/2} \left(\frac{e^2}{R_0} \frac{A}{C_1}\right) Z\beta_2\beta_3 eR_0$$

which implies

$$B(E1) = \frac{3}{4\pi} \langle D^2 \rangle_{\text{av}} \langle J_i 100 | J_i 1 J_f 0 \rangle^2$$

The E1 rate works for both $\langle \beta_3 \rangle^2$ (static octupole deformation) and $\langle \beta_3^2 \rangle$ which is nonzero for octupole vibrations.

Oc Def Schiff moment enhancement

The low-lying parity doublet:

- enhances mixing of opposite-parity states
- enhances the resulting Schiff moment because of the octupole and quadrupole deformations.

Schematic model estimates typically:

$$S \propto \frac{J}{J+1} \beta_2 \beta_3^2 Z A^{\frac{2}{3}} \frac{1}{E^- - E^+} e \eta \quad \text{Spevak PRC 56 1357 (1997)}$$

Result is 100-1000 x enhancement over ^{199}Hg

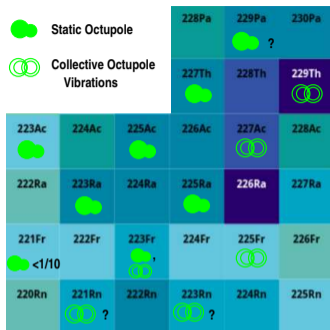
Graner PRL 116 161601 (2016), \sim restoring the full effect of the nuclear EDM, and in one case 10^4 or 10^5 enhancement going beyond (if a low-lying state is really the same J with opposite π).

- These models treat critical effective π NN interaction $\langle f | \sigma \cdot p | i \rangle$ macroscopically, an unresolved order-of-magnitude uncertainty in the best self-consistent mean field calculations Dobaczewski PRL 121 232501 (2018)
- Macroscopic models can't say what % of ψ is the octupole configuration. Spevak (Auerbach) estimate this effect, which greatly dilutes Schiff moment

enhancement in nuclei without octupole collectivity

- As with E1's, there are similar enhancements from $\langle \beta_3 \rangle^2$ and $\langle \beta_3^2 \rangle$, though the calculations are more complex for octupole vibrations

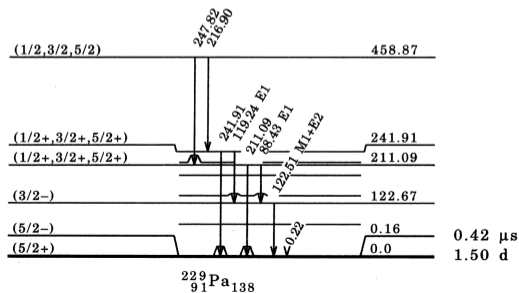
clear treatment: Engel Friar Hayes PRC61 035502 (2000)

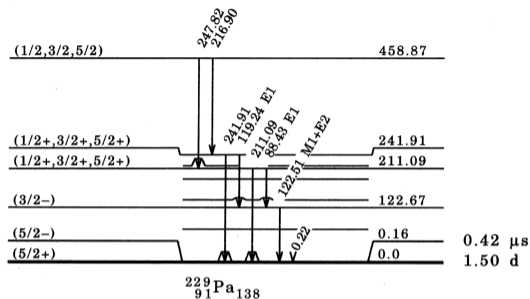


All demonstrated static octupole deformation is in radioactive nuclei Behr arXiv

2203.06758 resource

letter





Berman Fultz RMP 47 713 (1975):

The Thomas-Reiche-Kuhn (TRK) sum rule (see Levinger, 1960) is an expression giving the total integrated cross section for electric dipole photon absorption, in the absence of exchange forces, and is given by

$$\int_0^\infty \sigma(E) dE = \frac{2\pi^2 e^2 \hbar N Z}{M c A} = 60 \left(\frac{N Z}{A} \right) \text{MeV} \cdot \text{mb},$$

${}^{229}\text{Pa}$ could have an atomic EDM enhanced by its Schiff moment by $\sim 10^5$ times, because of its tiny parity doublet splitting and octupole phenomena

However, the $5/2^-$ state has not been identified. The splitting is known to be 60 ± 50 eV

Ahmad PRC 024313 (2015) “with this large uncertainty the existence of the parity doublet is not certain”

J. Singh from MSU proposes measuring that photon directly

Very little is known about atomic levels of Pa

If that is a parity doublet, what is known about the E1 strength in ${}^{229}\text{Pa}$ suggests the E1 between them would take up the entire Thomas-Reiche-Kuhn electric dipole sum rule

Haxton, MSU/FRIB EDM 2019 workshop.

Nuclear nearest-level spacing and \mathcal{T}

Bohr and Mottelson 2C-2:

Assume a Hamiltonian matrix with random values, the Gaussian Orthogonal Ensemble (GOE).

Diagonalizing the Hamiltonian produces a statistical distribution of level spacings ϵ in terms of average spacing D (the “Wigner distribution”)

$$P(\epsilon) = \frac{\pi}{2D^2} \epsilon e^{-\frac{\pi}{4} \frac{\epsilon^2}{D^2}}$$

$$P(\epsilon) \stackrel{\epsilon \rightarrow 0}{\propto} \epsilon$$

This was for time-reversal invariant interactions.

If you allow for \mathcal{T} ,

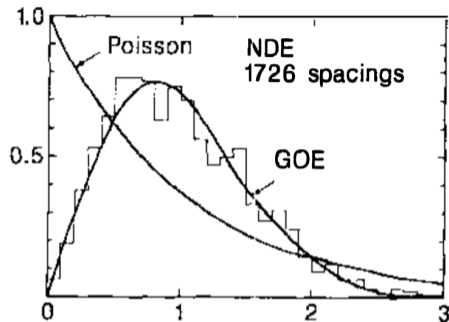
you have unitary matrices instead,

the Gaussian Unitary Ensemble (GUE) with twice

as many elements,

because they're complex. Then

$$P(\epsilon) \stackrel{\epsilon \rightarrow 0}{\propto} \epsilon^2$$



More sophisticated statistical measures extract an upper limit for the amount of \mathcal{T} in nuclear interactions $\alpha \lesssim 2 \times 10^{-3}$ (J.B.French Ann. Phys. 181 235 (1988)). It's treated as an upper bound, since nuclear level spacings are not necessarily random 😊

SM 2nd-order weak $\nu\nu\beta\beta$ vs SM $0\nu\beta\beta$ decay

We've already seen SM $\beta\beta\nu\nu$ decay. 1st measured geochemically, then directly in very-low-background experiments. In $0\nu\beta\beta$ all energy is captured, a distinctive signature.

Kayser Journal of Physics: Conference Series 173 (2009) 012013;
 Primakoff and Rosen 1959 Rep. Prog. Phys. 22 121

Particle physics for $0\nu\beta\beta$ to happen:

- Lepton number must not be conserved
- ν 's have mass ?

Cleanest statement: The non-SM physics that produces $0\nu\beta\beta$ generates a Majorana mass term.

A mass term for a Dirac ν_L needs a ν_R :

$$m\bar{\nu}\nu = m(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L)$$

(because $\bar{\nu}_L\nu_L = \bar{\nu}_R\nu_R = 0$ 'not its own antiparticle')

Majorana particles are their own antiparticles, making natural a mass term $\propto m\nu_L^c\nu_L$

(blithely ignoring much interesting physics formalism!)

Diagonalizing the Dirac+Majorana mass matrix then economically generates the light ν_L mass observed and very heavy ν_R : seesaw mechanism but is not the only possibility

Schechter and Valle qualify in text: some other physics could precisely cancel this diagram and keep the Majorana mass term 0.

All variations (e.g. photon exchange between u and \bar{d}) must be cancelled, "extremely unlikely."

Schechter, Valle PRD 25 2951 (1982)

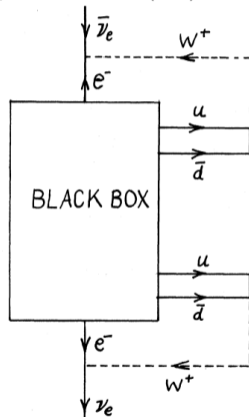


FIG. 2. Diagram showing how any neutrinoless double- β decay process induces a $\bar{\nu}_e$ -to- ν_e transition, that is, an effective Majorana mass term.

$0\nu\beta\beta$ rate, if due to exchange of light ν 's:

$\Gamma = G|M|^2(\sum_i U_{ei}^2 m_i)^2$ if due to exchange of light ν 's,

where G is the lepton phase space factor (trivial),

U_{ei} the ν mass mixing matrix,

M the nuclear matrix element (hard to calculate).

Suhonen Front. Phys. 5 art 55 p.1 (2017)

- 0^+ parent, progeny, $\nu\nu\beta\beta$ dominated by 1^+ intermediate states, GT transitions.

- $0\nu\beta\beta$ has contributions from forbidden operators and more spins, so $\nu\nu\beta\beta$ is not a complete benchmark for theory.

- Formally, two- γ emission from an excited state also sums over virtual states, QRPA developed for these experiments

Schirmer PRL 53 1897 (1984) but operators and states are different.

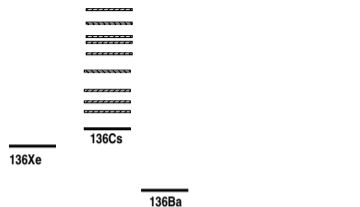
- A variety of approximate many-body answers vary by $2-4 \times$. ^{48}Ca calculable by complete many-body methods \rightarrow

(Boehm+Vogel "Physics of massive ν 's" crudely set non-rel

$F(Z, E) \sim \frac{E}{p} \frac{2\pi\alpha Z}{1-e^{-2\pi\alpha Z}}$ (sort of ok for spectrum, poor for rates)

to allow analytic phase space integrals $\sim E_0^{11}$ for $\nu\nu\beta\beta$ and

$\sim E_0^5$ for $0\nu\beta\beta$)



A contact term from chiral EFT changes the nuclear calculation

Cirigliano PRL 126 172002 (2021)

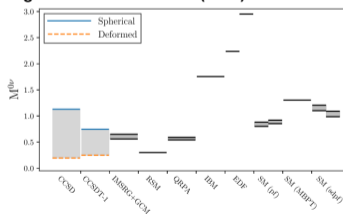


FIG. 1. Comparison of the NME for the $0\nu\beta\beta$ decay of ^{48}Ca .

Novario PRL 126 182502 (2021)

β - ν correlation from recoil momentum spectrum Kofoed-Hansen Dan. Mat. Fys. Medd. 28 nr9 (1954)

The recoil momentum spectrum is straightforward and analytic:

If we write angular distribution in terms of E (β total energy), θ (β - ν angle), p (β momentum), q (ν momentum) (it's understood we have to evaluate q to conserve energy-momentum; it's not a free parameter)

$$P(E, \theta) dE d\Omega_\theta =$$

$$F(Z, E) p E q^2 \left(1 + \frac{b}{E} + a \frac{p}{E} \cos \theta \right) dE d\Omega_\theta$$

Then if the recoil momentum is r , energy conservation $E+q=E_0$ ($E_0=Q+m_\beta$), then we just use law of cosines:

$$p^2 + q^2 + 2pq \cos \theta = r^2$$

differentiate θ with respect to r :

$$|\sin \theta d\theta| = 2d\Omega_\theta = \frac{r}{pq} dr$$

we immediately get the recoil momentum spectrum

$$P(E, r) =$$

$$\frac{1}{2} F(Z, E) \left(rEq + brq + r \frac{a}{2} (r^2 - p^2 - q^2) \right) dEdr$$

at fixed E , it's linear in recoil energy R

$$P(E, R) dEdR =$$

$$\frac{M}{2} F(Z, E) \left(Eq + bq + \frac{a}{2} (2MR - p^2 - q^2) \right) dEdR$$



β^+ asymmetry ^{37}K data



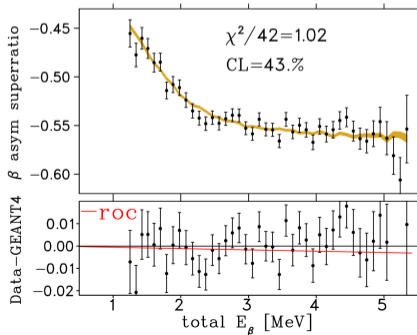
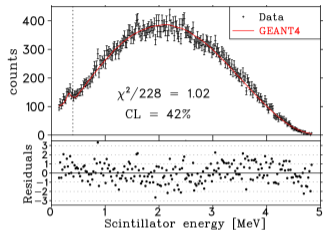
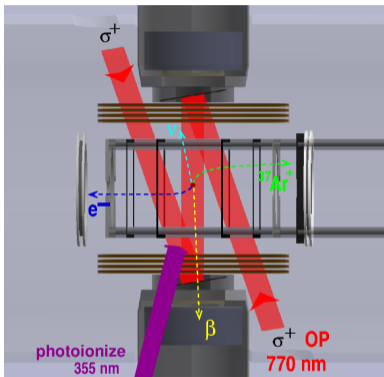
Fenker et al. Phys Rev Lett 120, 062502 (2018)

A_β [experiment] =
 -0.5707 ± 0.0019

A_β [theory] =
 -0.5706 ± 0.0007

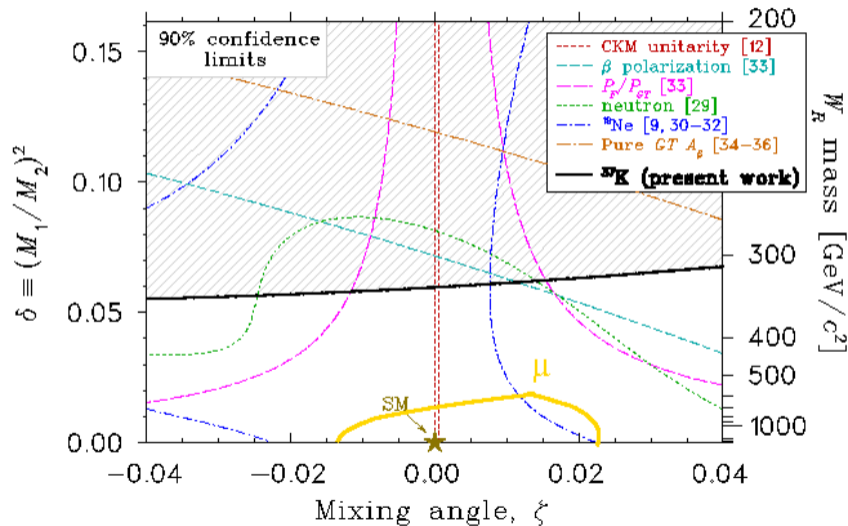
theory prediction needs
 GT/F ratio from $t_{1/2}$

The best fractional
 accuracy achieved in
 nuclear or neutron β
 decay





Still no wrong-handed ν 's



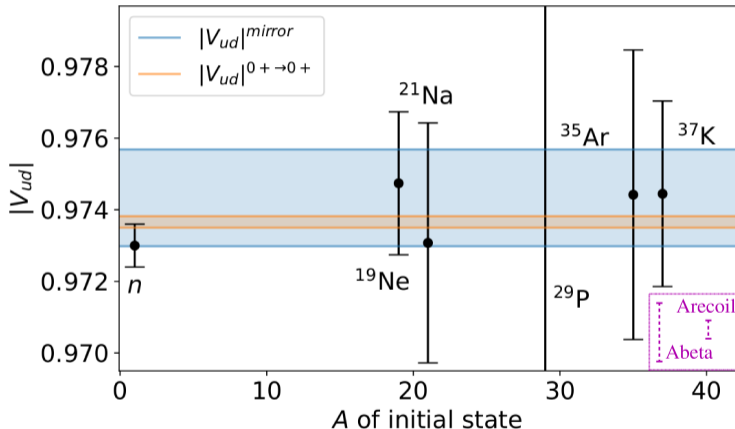
Extra W' with heavier mass, couples to wrong-handed ν_R

We can evade TWIST limits by assuming the muon ν_R is heavy

LHC $M'_W > 3.7$ TeV 90%



Weak interaction: same strength, all nuclei?



Deduced V_{ud}
from mirror decays

Are people overestimating
their uncertainties? We
aren't 😊

We project to reach 0.0005
accuracy, as good as any
 $0^+ \rightarrow 0^+$ except ^{26m}Al .

Assumes 5% isospin
breaking calculation.

Hayen and Severijns, arXiv:1906.09870 (June 2019)