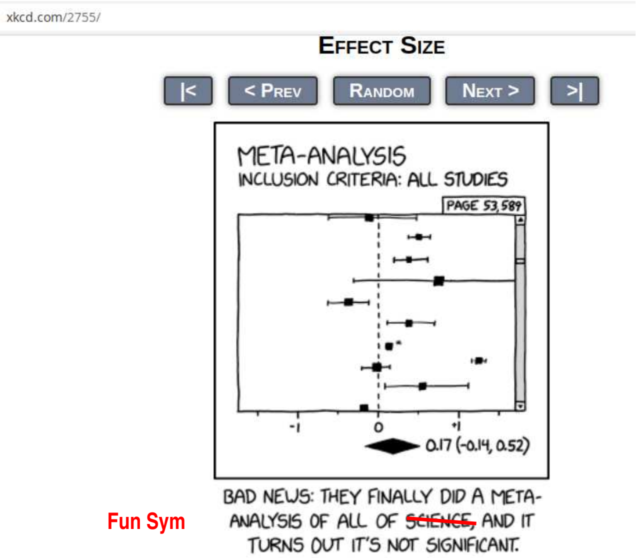
“Truth loves its limits,
for there it meets the beautiful”

Rabindrinath Tagore, “Fireflies”

“Good people are key. Be nice.”

Single-bullet slide, Jan Hall
Nobel Prize talk on frequency combs
APS DAMOP 2006

Fun Sym and Weak interactions I JB
Mar 21, 26, 2025
Nuclear Astro BD Mar 28
Fun Sym and Weak Interactions II JB
April 2,4, 2025
HW10 (#1,2 only) due Mon Apr 7;
HW11 available 11am, due Fri Apr 11
Final exam Wed April 16 9:30 am Pacific



E1 is isovector, M1 is mostly isovector

Weak Interactions and Nuclei

- **Why the weak interaction is weak (at low energies)**
- **Quark-lepton interaction+QCD induces nucleon-lepton interaction terms:**
  Conserved Vector Current,
  Partially conserved Axial Current
- **β decay observables**
  Lepton long-λ expansion, Fermi function
  Allowed,Forbidden Decay; Selection Rules
- **Weak quark eigenstates, CKM matrix unitarity**

Fun Symmetries:

- **$\not{P}$ (complete) : lepton helicity**
  Decay correlations
- **Weak neutral current examples:**
  Weak interaction between nucleons
- **$\not{T}$ (tiny)**
  CKM phase
  Atomic Electric Dipole Moments from:
  Nuclear Schiff, magnetic quadrupole, and from QCD Lagrangian
  Nuclear level spacing: Wigner distribution
- **$0\nu\beta\beta$ decay intro**

Refs.: Wong 5.5-5.6;
Commins and Bucksbaum "Weak Interactions of Leptons and Quarks" and
Commins "Weak Interactions (Physics) 1st edition."
Commins' Notes Ph 250 UCB 1996 (see Canvas "Lecture Notes")

v2 adds π W slide; v3 adds slide 43 on forbidden β decay

E1 transitions are isovector Bohr and Mottleson Vol I p. 44

The E1 radiative transition strength depends on the matrix element of the electric dipole operator:

$D = \sum_k e_k z_k = e \sum_k \frac{1}{2}(1 - \tau_z(k)) z_k$

$= \frac{1}{2} e \sum_k z_k - \frac{1}{2} e \sum_k \tau_z(k) z_k$

1st term depends only on position of c.o.m. of the whole nucleus, no transitions, Thompson scattering

2nd term: z component of a vector in isospin, so $|T_i - T_f| \leq 1 \leq |T_i + T_f|$

Consequence: for N=Z all transitions with $T_f = T_i$ are forbidden.

This selection rule has many phenomenological consequences.
An E1 multipole that would otherwise change $^{12}C + \alpha \rightarrow ^{16}O$ is greatly suppressed.

Isovector part of M1 is expected to be about one order larger than isoscalar

magnetic moment (in units of nuclear magnetons) can be expressed in the form

$$\boldsymbol{\mu} = \sum_k (g_s(k)\mathbf{s}_k + g_l(k)\mathbf{l}_k)$$
$$= \sum_k \{\tfrac{1}{2}(1 - \tau_z(k))(g_p\,\mathbf{s}_k + \mathbf{l}_k) + \tfrac{1}{2}(1 + \tau_z(k))g_n\,\mathbf{s}_k\}$$
$$= \tfrac{1}{2}\mathbf{I} + 0.38\sum_k \mathbf{s}_k - \sum_k \tau_z(k)(4.71\ \mathbf{s}_k + \tfrac{1}{2}\mathbf{l}_k) \qquad (1\text{-}65)$$

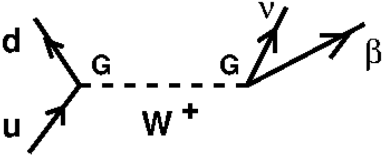
where we have inserted the values $g_p = 5.59$ and $g_n = -3.83$ for the spin g factors for proton and neutron.

The first term in Eq. (1-65) is proportional to the total angular momentum **I** and does not contribute to transitions between different states. The second term is a scalar in isospace, but has a coefficient that is an order of magnitude smaller than that for the last term (the isovector part). Thus we expect the isospin selection rules discussed above for $E1$ radiation to be also approximately valid for $M1$ radiation (Morpurgo, 1958). This is confirmed in the examples shown in Fig. 1-8,

The suppression of isoscalar M1's was used to help determine isospin mixing of the 12.71 MeV 1^+;T=0 level with the 15.11 1^+;T=1 **Adelberger PL62B 29 (1976)**

Why the Weak Interaction is weak at low energy

Consider the propagator in the Feynman diagram for $W^\pm$ exchange:



Propagator+vertices: T $\propto \frac{G_W(-g^{\mu\nu}+p^\mu p^\nu/M_W^2)G_W}{p^2-M_W^2} \xrightarrow{p<<M_W} \frac{G_W^2}{M_W^2} \Rightarrow$

Rates $\propto \frac{G_W^4}{M_W^4}$. Other physics with G_x, M_x have cross-terms $\propto \frac{G_W^2 G_x^2}{M_W^2 M_x^2}$

So the massive W^+ makes the interaction strength small for β decay with $p \sim MeV$

At high $p \sim M_W$, the interaction has the same coupling strength as E&M

For nucleons, G can and is different from the quark-lepton couplings

β decay is purely weak $\Rightarrow$ physics at scale M_W = 80GeV/c^2? Sure, if you want to assume an electroweak coupling

Conserved Vector Current hypothesis with Dirac formalism

CVC was developed in the late 50's. It was realized you could treat at least part of the weak interaction like electromagnetism.
CVC is sometimes considered more for its consequences than for the physics behind it, so I'm going through the physics assumptions.

- **Construct the E&M current for pointlike particles and show its derivative is zero, simply because of conservation of electric charge.**
- **Consider what happens if the particles are composite, like nucleons. One gets some relations for 'form factors' describing the nucleons, relations necessary to keep this current conserved.**
- **Hypothesize that the vector part of the weak current should be similarly conserved, and show what that implies for weak interaction physics.**

I'll use Dirac formalism, because the currents are all relativistic:
I'll cite the limited formalism I need as I go along.

The S.M. interaction has W exchange, which at momenta $<< M_W$ produces this quark-lepton current-current Langrangian density that is purely 'V-A' (using the opposite-signed convention) (γ_μ reduces to Fermi, $\gamma_\mu\gamma_5$ to Gamow-Teller):

$$L = \frac{G}{\sqrt{2}} J^\mu \bar{J}^\dagger_\mu + h.c. \text{ with } J_\mu = J^{(lep)}_\mu + J^{(had)}_\mu \text{ and } J^{(lep)}_\mu = \bar{\psi}_e \gamma_\mu (1+\gamma_5)\psi_{\text{neutrino}}$$

We would really like to just deal with quarks, so that we could write something like:

$$J^{(had)}_\mu = J^{quark}_\mu = \bar{\psi}_d \gamma_\mu (1+\gamma_5)\psi_u$$

because then everything would be automatically V-A, just like purely leptonic weak interactions (like μ decay).
However, we're stuck with nucleons, composite particles made of quarks. So QCD can 'induce' other terms as it combines quarks into the nucleon wf's.
So we have to go back and construct a general Lorentz vector for the hadrons (to make a bilinear covariant with the Lorentz vector of the leptons), along with an a Lorentz axial vector for the hadrons (to make a bilinear covariant with the axial current of the leptons).
First we'll back up and do this for the E&M current, which is purely vector. The fact that this vector current is conserved (electric charge is conserved) puts constraints on the composite terms:

First we consider the E&M current, take its divergence, and use Dirac equation $(\gamma_\mu\partial_\mu + m)\psi = (\not{p} + m)\psi = 0$

$J_\mu = -e\bar{\psi}\gamma_\mu\psi$ (more properly, matrix element $\langle p'|J_\mu(E\&M)|p\rangle$) for particle with momentum $p \to p'$)

using plane-wave solution to Dirac eq. $\psi = u(p)e^{ip\cdot x}$

$$
\begin{aligned}
\partial_\mu J_\mu/(-e) &= \\
\partial_\mu \left[\bar{u}(p_2)\gamma_\mu u(p_1)e^{i(p_1-p_2)\cdot x}\right] \\
&= \left[\bar{u}(p_2)(p_1 - p_2)_\mu\gamma_\mu u(p_1)\right] e^{i(p_1-p_2)\cdot x} \\
&= \left[\bar{u}(p_2)\ \not{p}_1 u(p_1) - \bar{u}(p_2)\ \not{p}_2 u(p_1)\right] e^{i(p_1-p_2)\cdot x} \\
&= i(m_1 - m_2)\bar{u}(p_2)u(p_1)e^{i(p_1-p_2)\cdot x} = 0
\end{aligned}
$$

because $m_1 = m_2$ in E+M interactions
So this E+M current is conserved, so charge is conserved, QED ☺

Electromagnetic current for composite particles

Before we go back to the weak interaction, it is instructive to write down a general electromagnetic current for a composite particle, take its divergence, and set that to zero. We will get a direct prediction about a corresponding term in β decay from CVC.
For composite particles like nucleons, we have to again write the most general Lorentz vector that can be constructed from γ_μ's and momenta, subject to:
a) momentum conservation means there are only two independent momenta, the difference $k_\mu = (p_2)_\mu - (p_1)_\mu$ and the average $K_\mu = 1/2(p_2 + p_1)_\mu$ of the individual momenta
b) not more than two γ matrices, because three γ's can be written as one γ with γ_5: I, γ_μ, and $\sigma_{\mu\nu} = \frac{1}{2i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$
c) use Dirac eq., i.e. replace $\not p_1$ with im_1 when adjacent to spinors.
Then the most general forms of Lorentz vectors are: γ_μ, $\sigma_{\mu\nu}k_\nu$, k_ν, $\sigma_{\mu\nu}K_\nu$, K_ν.
It turns out that the matrix elements of the last two can be rewritten in terms of the other 3. (Perhaps just reflecting that in the CM frame the total momentum is zero.)

So we can write a general form for the E&M current of a composite particle:

$$J_\mu/e = \bar{\psi}\left[F_1\gamma_\mu - \frac{F_2}{2m}\sigma_{\mu\nu}k_\nu + iF_3k_\mu\right]\psi$$

where F_1, F_2, F_3 are form factors, scalar functions of k^2. Is this conserved?

$$-\partial_\mu J_\mu/e = \bar{u(p_2)}\left[F_1 \not{k} - \frac{F_2}{2m}\sigma_{\mu\nu}k_\nu k_\mu + iF_3k^2\right]u(p_1)e^{ik\cdot x}$$

The 1st term vanishes as above
The 2nd term is zero independent of F_2, because $\sigma_{\mu\nu}$ is completely antisymmetric.

$$\sum_{\mu\nu}\sigma_{\mu\nu}k_\nu k_\mu \equiv \sum_{\mu<\nu}\left[\sigma_{\mu\nu}k_\nu k_\mu + \sigma_{\nu\mu}k_\mu k_\nu\right] = \sum_{\mu<\nu}\left[\sigma_{\mu\nu} + \sigma_{\nu\mu}\right]k_\mu k_\nu = 0$$

The third term is <u>not</u> zero, so for CVC to hold, $F_3(k^2)$=0.
The 2nd term can be related to the magnetic moments, in particular the non-Dirac 'anomalous' magnetic moments, so:
For the proton, $F_1^p(0)$=1, $F_2(p)$=μ_p-1 = 1.793
For the neutron, $F_1^n(0)$=0, $F_2(n)$=μ_n =-1.913

Formally setting up isospin-changing operators for a weak 'current':

Recall results from angular momentum algebra: define isospin raising/lowering operators

$T_{\pm} = T_1 \pm iT_2$

$T_{\pm}|T, T_z\rangle = \sqrt{T(T+1) - T_z(T_z \pm 1)}|T, T_z \pm 1\rangle$

For spin-1/2, $T_z|1/2, \pm 1/2\rangle = \pm 1/2|1/2, \pm 1/2\rangle$

$T_+|1/2, -1/2\rangle = |1/2, 1/2\rangle$

Now write the E&M vertex function in terms of isoscalar and isovector parts:

$$e\left(\frac{1}{2}\left[F_1^S\gamma_\mu - \frac{F_2^S}{2m}\sigma_{\mu\nu}k_\nu\right] + \left[F_1^V\gamma_\mu - \frac{F_2^V}{2m}\sigma_{\mu\nu}k_\nu\right]T_z\right)$$

$$F_1^S = F_1^{(p)} + F_1^{(n)} = 1 + 0 = 1$$

$$F_1^V = F_1^{(p)} - F_1^{(n)} = 1 + 0 = 1$$

$$F_2^S = F_2^{(p)} + F_2^{(n)} = -0.120$$

$$F_2^V = F_2^{(p)} - F_2^{(n)} = +3.706$$

Now we can finally write the weak vertex function for the hadron part:

$$\frac{g}{2\sqrt{2}} V_{ud} \left[\left(g_V \gamma_\mu - \frac{g_M}{2m} \sigma_{\mu\nu} k_\nu + i g_S k_\mu \right) + \left(g_A \gamma_\mu - \frac{g_T}{2m} \sigma_{\mu\nu} k_\nu + i g_P k_\mu \right) \gamma_5 \right] T_\pm$$

where we have also included the similar axial vector terms, to form the covariant piece with the lepton axial vector current.

The CVC hypothesis includes some bold assertions:

a) Vector portion of weak current is conserved, analogous to E&M current

b) The two vector weak currents– the β^+ and β^- decay, given by the terms with T$_\pm$ isospin raising/lowering operators– and the isovector part of the electromagnetic current are members of an isotriplet of current operators

This implies:

i) $g_V = F_1^V$=1.00. Presence of strong interactions has left this term completely untouched $\Rightarrow$ unrenormalized. This has many physics consequences.

ii) $g_M = F_2^V = \mu_p - \mu_n - 1 = 3.70$

This term in the weak current of the nucleon is related to the anomalous magnetic moments of the nucleons, called "Weak magnetism"

iii) g_S = 0! The "induced scalar" term must be zero for CVC to hold

So now our full lepton-nucleon interaction density is (Morita Hyp. Int. 21 143 (1985)):

$$\sqrt{2}L = [V_\lambda + A_\lambda]\left[\bar{\psi}_e\gamma_\lambda(1+\gamma_5)\psi_\nu\right] + \left[V'_\lambda + A'_\lambda\right]\left[\bar{\psi}_\nu\gamma_\lambda(1+\gamma_5)\psi_e\right]$$

with explicitly different forms for $\beta^\pm$ decay:

$$V_\lambda = \bar{\psi}_p\left(g_V\gamma_\lambda + \frac{g_M}{2m}\sigma_{\lambda\rho}k_\rho + ig_Sk_\lambda\right)\psi_n \quad A_\lambda = \bar{\psi}_p\gamma_5\left(g_A\gamma_\lambda + \frac{g_T}{2m}\sigma_{\lambda\rho}k_\rho + ig_Pk_\lambda\right)\psi_n$$

$$V'_\lambda = \bar{\psi}_n\left(g_V^*\gamma_\lambda + \frac{g_M^*}{2m}\sigma_{\lambda\rho}k'_\rho - ig_S^*k'_\lambda\right)\psi_p \quad A'_\lambda = \bar{\psi}_n\gamma_5\left(g_A^*\gamma_\lambda - \frac{g_T^*}{2m}\sigma_{\lambda\rho}k'_\rho + ig_Pk'_\lambda\right)\psi_n$$

$$k = k_p - k_n = -k'$$

Yes, the hadron part, because of the QCD-driven "dressing" within the nucleon, is more complicated than the lepton part.
g_S and g_T terms change sign from electron to positron decay. These are therefore odd under charge symmetry. So they vanish in isobaric analog decays to the extent that charge symmetry is good. These are called "2nd-class currents" $\rightarrow$

There are at least 2 ways to make 2nd-class currents in a quark model:

• Remembering Standard Model has $\bar{u}\gamma_\mu d$ and $\bar{u}\gamma_5 d$ terms only, add derivative terms like $\partial_\mu \bar{u} d$ and $\partial^\nu \bar{u}\sigma_{\mu\nu}\gamma_5 d$ Chiral EFT has these

These are not renormalizable, one large reason they were excluded from the Standard Model (Weinberg Phys. Rev. 112 1375 (1958)).

[One perspective is that the Standard Model itself may be an Effective Field Theory good up to some very high energy. Naively, maybe that means renormalizability is not an exact logical requirement. However, deliberately introducing a manifestly unrenormalizable term would still be a very complicated move for the main part of one's basic theory.]

• Introduce a new quantum number in addition to color and flavor! (Feynman famously called this q.n. 'smell'?). You can also interpret this as a second set of quarks (Holstein Treiman PRD 13 3059 (1976)) carrying this quantum number.

A related scenario: recently people consider extra sectors of particles not interacting much with us, but interacting strongly among themselves. QCD-like symmetries turn out to be a feasible way to generate dark matter. There are tight constraints from experiment on such scenarios.

• The best experimental limits on 2nd-class currents, from dedicated β decay measurements, allow 2nd-class current effects about an order of magnitude larger than the known ones from charge-symmetry breaking in QCD.

Formal extension from nucleons to nuclei The hadron current we have written is for the spin-1/2 nucleon, where the μ is the only non-Dirac electromagnetic moment.

If you are describing nuclei (or hadrons) with spin $>$ 1/2, then higher-rank electromagnetic moments also, by CVC, contribute to the weak vector current.
E.g., the electric quadrupole moment produces a component in the weak vector current.
Similarly, additional nuclear-structure dependent form factors appear for $J > 1$ in the axial vector current.

Holstein generalizes from nucleons to nuclei and writes decay correlations: Rev. Mod. Phys. 46 789 (1974) erratum 48 673; or “Weak Interactions in Nuclei”.
Nuclei are treated as “elementary particles” and form factors are introduced to include moments and effects from their nonpointlike size.
In isobaric analog decays, the vector current part is given by the measured electromagnetic moments. The g_T term in isobaric analog decays is zero, but in pure Gamow-Teller decay it is not zero, producing a part that depends on a nuclear structure calculation whose accuracy can limit the sensitivity to new physics.
Holstein’s approach considers ‘recoil-order’ terms $\sim (E_\beta/M)^N$ for N=1,2,3. Convergence is not guaranteed of such a series.

Behrens&Bühring “Electron Wavefunctions and Nuclear β Decay” has forbidden β decay

Finite nuclear S.M. expressions gain complexity with those corrections (Holstein)

$$l^\mu\langle\beta|V_\mu|\alpha\rangle = \delta_{JJ'}\delta_{MM'}\left(a(q^2)\frac{P\cdot l}{2M} + e(q^2)\frac{q\cdot l}{2M}\right) + ib(q^2)\frac{1}{2M}C^{M'k;M}_{J'1;J}(\mathbf{q}\times\mathbf{l})_k$$
$$+ C^{M'k;M}_{J'2;J}\left[\frac{1}{2M}f(q^2)C^{nn';k}_{11;2}l_n q_{n'} + \frac{1}{(2M)^3}g(q^2)P\cdot l\sqrt{\frac{4\pi}{5}}\,Y_2^k(\mathbf{q})\right]$$

$$a = g_V \qquad c = \sqrt{3}g_A$$
$$b - a = \sqrt{3}g_M \qquad d = \sqrt{3}g_T$$
$$e = g_S \qquad h = \sqrt{3}g_P.$$

$$l^\mu\langle\beta|A_\mu|\alpha\rangle = C^{M'k;M}_{J'1;J}\varepsilon_{ijk}\varepsilon_{ij\lambda\eta}\frac{1}{4M}\left[c(q^2)l^\lambda P^\eta - d(q^2)l^\lambda q^\eta\right.$$
$$\left. + \frac{1}{(2M)^2}h(q^2)q^\lambda P^\eta q\cdot l\right]$$
$$+ C^{M'k;M}_{J'2;J}C^{nn';k}_{12;2}l_n\sqrt{\frac{4\pi}{5}}\,Y_2^{n'}(\mathbf{q})\frac{1}{(2M)^2}j_2(q^2)$$
$$+ C^{M'k;M}_{J'3;J}C^{nn';k}_{12;3}l_n\sqrt{\frac{4\pi}{5}}\,Y_2^{n'}(\mathbf{q})\frac{1}{(2M)^2}j_3(q^2),$$

for decay between isobaric analogs:

$$\langle I, I_z \pm 1|V_\mu{}^W|I, I_z\rangle,$$

$$a(0) = [(I \mp I_z)(I \pm I_z + 1)]^{1/2}$$
$$b(0) = a(0)\sqrt{\frac{J+1}{J}}(\mu_\beta - \mu_\alpha)$$
$$e(0) = f(0) = 0$$
$$g(0) = -a(0)\left(\frac{(J+1)(2J+3)}{J(2J-1)}\right)^{1/2}\frac{2M^2}{3}(Q_\beta - Q_\alpha),$$

where $x(q^2) = x_0 + x_1 q^2\ldots$ "form factors" including finite size

Valence nucleon shell-model expressions for G-T, weak mag

This unpaired nucleon expression is incomplete for G-T transitions: (de-Shalit+Talmi Table 9.1)

THE VALUES OF $\langle\sigma\rangle^2$ FOR SINGLE NUCLEON TRANSITIONS

j_f \ j_i	$l+\frac{1}{2}$	$l-\frac{1}{2}$
$l+\frac{1}{2}$	$\frac{2l+3}{2l+1}=\frac{j_f+1}{j_f}$	$4\frac{l+1}{2l+1}$
$l-\frac{1}{2}$	$\frac{4l}{2l+1}$	$\frac{2l-1}{2l+1}=\frac{j_f}{j_f+1}$

but it can lend qualitative understanding for why the G-T/Fermi ratio is so different in *n*, ^{19}Ne, ^{37}K... e.g. both μ and G-T transitions are smaller for $d_{3/2}$ proton because the orbital term partly cancels the intrinsic spin term.

Weak magnetism in G-T transitions (Wang+Hayes PRC 95 064313 (2017):

$$\frac{d\omega}{dE_e} = \frac{G_F^2\cos^2\theta_C}{2\pi^3}\, p_e\, E_e(E_0-E_e)^2 F(E_e,Z) g_A^2 |\langle\vec{\Sigma}\rangle|^2 \times\left(1+\frac{4}{3}\left[\frac{\mu_v+\frac{\langle J_f||\vec{\Lambda}||J_i\rangle}{\langle J_f||\vec{\Sigma}||J_i\rangle}}{2\,M_N\,g_a}\right](2E_e-m_e^2/E_e-E_0)\right)$$

$$\delta_{LS}^{j_f j_i} = \frac{\langle nlj_f||\vec{\Lambda}||nlj_i\rangle}{\langle nlj_f||\vec{\Sigma}||nlj_i\rangle}$$

with $j_i = l\mp 1/2$ and $j_f = l\pm 1/2$

μ_v = 4.7 (isovector nucleon moment $\mu_p - \mu_n$)

$$\delta_{LS}^{--} = -(l+1),\quad \delta_{LS}^{-+} = -1/2,$$
$$\delta_{LS}^{+-} = -1/2,\quad \delta_{LS}^{++} = +l.$$

For reactor ν production, some simple estimates assumed the nucleon contribution $\pm$ 100%.

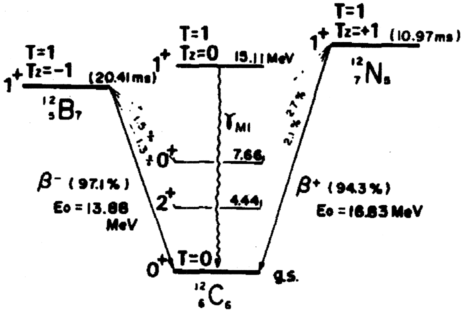
Weak Magnetism tests

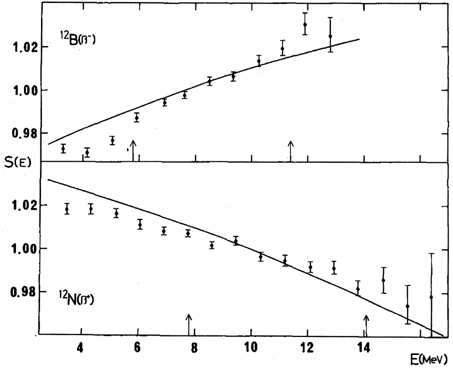
- For isobaric analog decays, the 'weak magnetism' $\frac{g_M}{2m}\sigma_{\mu\nu}k_\nu$ term is directly predicted by CVC by the 'anomalous' magnetic moment difference of the parent and daughter.
- For 'isospin mirror' Gamow-Teller decays, it is related to the isovector M1 γ-decay strength in the T_z=0 nucleus (Gell-Mann PhysRev 111 362 (1958)).

The k-dependence makes 20% distortions in the energy spectrum.

The axial vector $\frac{g_T}{2m}\sigma_{\mu\nu}k_\nu\gamma_5$ term, cancels in the difference unless there is a 2nd-class g_T.

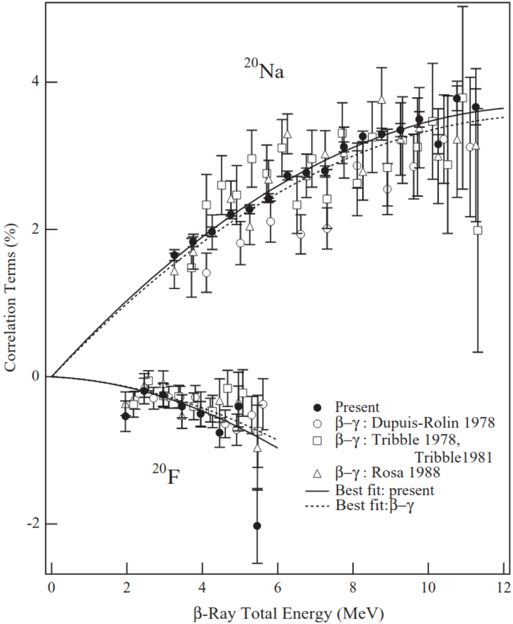
The results are consistent with the CVC prediction to $\sim$10% of the g_M term

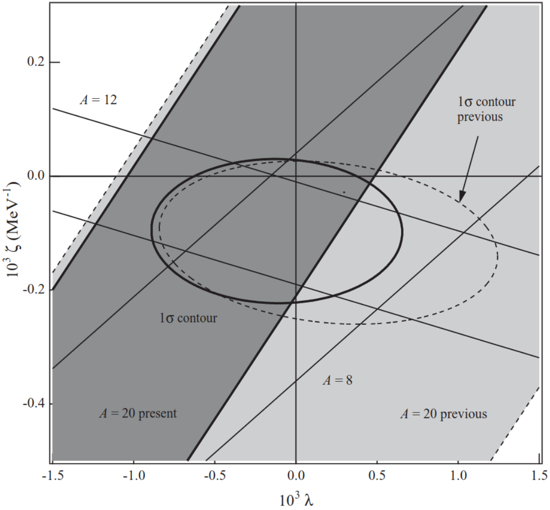




Further weak magnetism test:

The angular distribution of β's is isotropic wrt alignment $\langle M_J^2 \rangle$ for Gamow-Teller decay.
Results agree with CVC to $\sim$5%



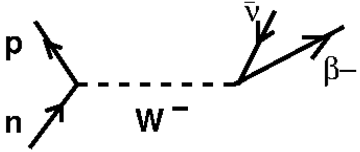


Minamisono PRC84 055501 (2011)

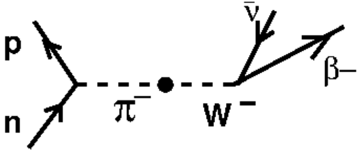
2nd class constraints →

Axes are contact interaction vs meson-ranged

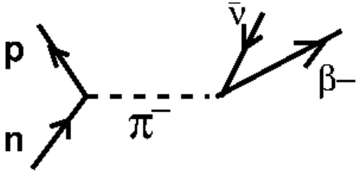
Sketch of lowest-order calculation of g_P Compare these diagrams:



A



B



C

Because W is short-range, C is same as B

For A (in a pure Gamow-Teller case), transition rate is:

$$T_{fi} = \frac{g}{2\sqrt{2}} V_{ud} \bar{u}_p \left(g_A \gamma_\mu \gamma_5 + i g_P k_\mu \gamma_5 \right) u_n \frac{1}{m_W^2} \frac{g}{2\sqrt{2}} \bar{u}_e \gamma_\mu (1 + \gamma_5) v_{\nu e}$$

For C:

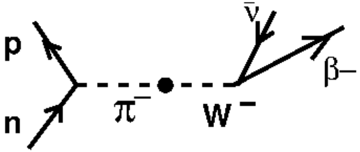
$$T_{fi} = g_{\pi NN} \sqrt{2} \bar{u}_p \gamma_5 u_n \frac{1}{k^2 + m_\pi^2} \frac{G}{\sqrt{2}} i f_\pi k_\mu \bar{u}_e \gamma_\mu (1 + \gamma_5) v_{\nu e} V_{ud}$$

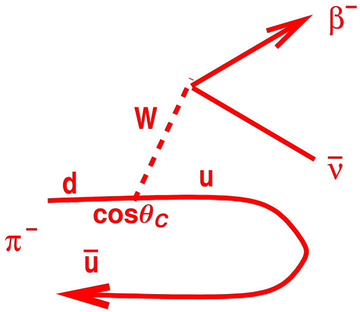
So C is like the g_P part of A; if we declare C responsible for all of it:

$$g_P(k^2) = \frac{g_{\pi NN} \sqrt{2} f_\pi}{k^2 + m_\pi^2}$$

In β decay this is small, but in μ capture it is a large contribution: (in computing the decay lifetime, g_P becomes multiplied by the lepton mass.)

the right-hand half of diagram B is related to semileptonic decay of the pion





$d\bar{u} \rightarrow u\bar{u}$ + W and $u\bar{u}$ vanishes into the vacuum (Fig. 4.10 Commins and Bucksbaum)

Continuing g_P, weak and strong interactions together:

see Gorringe and Fearing Rev.Mod.Phys. 76 (2004) 1 for chiral perturbation theory
Further arguments (including PCAC below) give the 'Goldberger-Treiman' expression for the QCD-induced 'pseudoscalar' coupling:

$$g_P(q^2) = \frac{2m_\mu m_N}{m_\pi^2 - q^2} g_A(0)$$

This is now understood as the first term in an expansion using "chiral perturbation theory"; modern calculations $g_p(-0.88m_\mu^2) = 8.23$.
"Chiral perturbation theory" is a systematic expansion in $m_\pi / m_{\text{nucleon}}$, guided by similar concepts to chiral EFT's for NN interaction (small m_{quark}, π's as Goldstone bosons from the underlying broken chiral symmetry...). But a calculation, not with free parameters.
History: Experiments in radiative capture on hydrogen: 12.4$\pm$0.9$\pm$0.4
Experiments as of 2004 in ordinary μ capture on hydrogen: 10.5$\pm$1.8
As of 2004, not good enough to help yet: PSI was working on it
This rigorous prediction of low-energy QCD's effects on weak interaction was not working in 2004. A more accurate PSI experiment resolved the discrepancy with theory: Andreev Phys Rev Lett 110 012504 (2013) $g_p(-0.88\mu^2)$=8.06$\pm$0.55.

Conserved Vector Current and 'Partially Conserved Axial Current': qualitative

One consequence of the conserved 'V' vector current is that the equivalent g_V=1. I.e. the interaction between quarks goes directly over to the interaction between nucleons and nuclei because the 'vector current' is conserved.
People looked pretty hard to find a way to find an axial 'A' current that was also conserved (look at Feynman and Gell-Mann PR '57).

Wong writes Eq. 5-52:

$$\sum_{\mu=1}^{4} \frac{\partial V_\mu}{\partial x_\mu} = 0$$

and then by analogy

$$\sum_{\mu=1}^{4} \frac{\partial A_\mu}{\partial x_\mu} = \mathrm{constant}\,\phi_\pi$$

where ϕ_π represents the pion field.

This predicts a relation between the weak coupling constants G_A and G_V, given by :

$$g_A \equiv \frac{G_A}{G_V} = \frac{f_\pi g_{\pi N}}{M_N c^2}$$

where f_π scales π decay and $g_{\pi N}$ can be deduced from π-nucleon scattering. This 'Goldberger-Treiman relation' predicts $|g_A| = 1.31$; experimental value is g_A=-1.259$\pm$0.004
This either 'confirms PCAC' or enforces that 'PCAC is a bad name for a poor approximation'.
Lattice QCD is at 1% accuracy for g_A
Note that this is all at momentum transfer $q^2\sim0$: the constants are really 'form factors,' functions of momentum.

PCAC in more detail

Axial (hadronic) Current:

$$A_\mu = -i\frac{g}{2\sqrt{2}}\bar{u}(p_2)(g_A\gamma_\mu\gamma_5 + ig_P k_\mu\gamma_5)u(p_1)e^{i(p_1-p_2)\cdot x}$$

PCAC hypothesis: the non-conservation of this current is due entirely to pions, and A_μ becomes conserved as m_π goes to 0:

$$\partial_\mu A_\mu \overset{m_\pi\to 0}{\longrightarrow} 0$$

Zee Ch IV.2

So evaluate the divergence of this current:

$$\partial_\mu A_\mu =$$

$$\frac{-ig}{2\sqrt{2}}\bar{u}(p_2)\left(g_A i\,\not{p}_1\gamma_5 - g_A i\,\not{p}_2\gamma_5 + g_P k^2\gamma_5\right)u(p_1)e^{i(p_1-p_2)\cdot x} =$$

using Dirac eq.

$$\frac{-ig}{2\sqrt{2}}\bar{u}(p_2)\left(2mg_A + g_P k^2)\gamma_5\right)u(p_1)e^{i(p_1-p_2)\cdot x}$$

By PCAC this vanishes as $m_\pi\to 0$, so:

$$g_A \overset{m_\pi\to 0}{\longrightarrow} \frac{g_P k^2}{2m} =$$

$$\frac{g_{\pi NN}\sqrt{2}f_\pi}{k^2+m_\pi^2}\frac{k^2}{2m} =$$

$$-\frac{g_{\pi NN}\sqrt{2}f_\pi}{2m}$$

the Goldberger-Treiman relation

Summary of hadronic weak current form factors in S.M.

- **Exact Predictions of CVC for vector current:**

1) g_V=1...: Experimental $0+\rightarrow 0^+$ Ft values same to $\approx$ 0.001. CKM unitarity has a 0.001 deficit at 2 to 3 σ. ($\pi^+\rightarrow\pi_0 + \nu + \beta^+$ agrees to 0.005 (PIBETA)

2) g_M=3.70: Weak magnetism measured to $\approx$ 5% of its value

3) g_S=0: Ft, and relative helicity of leptons from β-ν correlation and $\pi\rightarrow e\nu$, show no evidence for scalar term at C_S<0.05 level.

- **Estimates from PCAC (Goldberger-Treiman) and similar:**

1) $g_A = \frac{g_{\pi NN}\sqrt{2}f_\pi}{2m} = -1.32$; Decay of neutron $\Rightarrow$ -1.26 LGT gets this to $\approx$ 0.01

2) $m_\mu g_P = \frac{g_{\pi NN}\sqrt{2}f_\pi m_\mu}{m_\pi^2} = 9.2$ Including chiral perturbation theory more like 8.0, PSI's μCAP experiment μ capture on hydrogen agrees well.

Charge Symmetry (G-parity): No 2nd-class currents: $f_3\approx 0$, $g_2\approx 0$

(The best tests of this SU(2) symmetry are still in β decay: similar tests in hadronic decays of τ)

- **V and A are the dominant known couplings for nuclear β decay. (The most precise a_β measurement in the neutron disagrees badly, suggesting a small Lorentz tensor interaction.) Interesting that a couple of simple surmises determined 6 couplings so well– reasonable to call it an "effective field theory" for the lepton-nucleon weak interaction. See Ando PhysLettB595 250 (2004) for an EFT of neutron β decay including radiative**

β decay: Energy release, other basics, orbital angular momentum

$Q_{\beta-}$ = M(Z,N) - M(Z+1,N-1) using atomic masses
(this is in some sense accidental: the β is created in the nucleus and leaves; if nothing else happens, this would create a negative atomic ion...)

Q_{EC} = M(Z,N) - M(Z-1,N-1) - $|B.E.(\text{electron})|$

$Q_{\beta+}$ = M(Z,N)-M(Z+1,N-1) + 2 m_e

Sometimes EC is allowed energetically when β^+ is not.
Atomic electron overlap with nucleus is greater as one goes heavier; EC $\sim$ 1% at Z$\sim$40 isotopes where β^+ is allowed, but can be 10's of % at Z=82
Ratio is given well by atomic wavefunctions, and has some sensitivity to the weak interaction nature (Brysk and Rose, Rev Mod Phys 30 (1958) 1169)

- **Q can vary from 18 keV (t to ^{3}He) to $>$ 10 MeV**
(m_β= 0.511 MeV, so β's can be relativistic or non-relativistic.)

- **electron DeBroglie wavelength: λ= h/p = 2π(197 MeV fm)/ $\sqrt{E^2 - m_e^2}$**
For kinetic energy 1 MeV, this is 870 fm, much larger than the nucleus.
So the long-wavelength expansion we're about to make is a good one.
Similarly, ℓ=rxp$\sim$ 0.005 $\hbar$ is typically small

Fermi's Golden Rule, applied to β Decay to get rates

For now write the transition probability

$$W = \frac{2\pi}{\hbar}|\langle\phi_f(\vec{r})|H|\phi_i(\vec{r})\rangle|^2\rho(E_f)$$

The initial state is simply the parent nucleus at rest:

$$|\phi_i(\vec{r})\rangle = |J_i m_i \vec{r}\rangle$$

The final state consists of 3 particles. Ignoring for now Coulomb effect between the β and final nucleus, this is a product of 3 parts, with plane waves for the leptons:

$$|\phi_k(\vec{r})\rangle = \frac{1}{\sqrt{V}}e^{i\vec{k}_e\cdot\vec{r}}\frac{1}{\sqrt{V}}e^{i\vec{k}_\nu\cdot\vec{r}}|J_f m_f r'\rangle$$

The V's normalize the plane waves. Expand the plane waves in terms of spherical harmonics (we could do this for γ-rays, too: we're about to do a 'long-wavelength expansion'):

$$e^{i\vec{k}\cdot\vec{r}} = \sum_0^\infty \sqrt{4\pi(2\lambda+1)}\,i^\lambda j_\lambda(kr)\,Y_{\lambda 0}(\theta,0)$$

where $\vec{k} = \vec{k}_e + \vec{k}_\nu$ and θ is the angle between $\vec{k}$ and $\vec{r}$.

Now we make the long-wavelength expansion:

$$j_\lambda(kr) \stackrel{kr<<1}{\rightarrow} \frac{(kr)^\lambda}{(2\lambda+1)!!}$$

so that the final state wavefunction becomes:

$$|\phi_k(\vec{r})\rangle = \frac{1}{V}\left(1 + i\sqrt{\frac{4\pi}{3}}(kr)Y_{10}(\theta,0) + O(k^2r^2)\right)|J_f m_f r'\rangle$$

Even without the formal weak interaction theory, we can now surmise the form of *H*, the nuclear part of the β-decay operator.
Neutrons are transformed into protons $\Rightarrow$ the nuclear operator:
1) must be one-body, i.e.only one nucleon is involved at a time;
2) must involve single particle isospin raising/lowering operator $\tau_\pm$ (this comes from the 'vector' V 'Fermi' part of 'V-A')

• **The axial vector 'A' 'Gamow-Teller' part produces a product of σ and $\tau_\pm$**
Then we can write the matrix element $\langle\phi_f(\vec{r})|H|\phi_i(\vec{r})\rangle =$

$$\frac{1}{V}\langle J_f m_f r|\sum_{j=1}^{A}(G_V\tau_\pm(j)+G_A\vec{\sigma}(j)\tau_\pm(j))\left(1-i\sqrt{\frac{4\pi}{3}}(kr)Y_{10}(\theta,0)+O(k^2r^2)\right)|J_i m_i r'\rangle$$

• **The Fermi operator does nothing to space/spin. So it only links isobaric analog states, or pieces of isobaric analog states, i.e. states with same wavefuntion except proton/neutron interchange.**

• **This form shows both the allowed terms and some '1st forbidden' terms: these are from the same nuclear operators σ and τ, but including the next order of the lepton long wavelength expansion and thus suppressed. However, the nuclear matrix elements also vary, so some 1st forbidden rates are faster than some G-T.**

The 1st-forbidden operators all flip the nuclear parity, so don't contribute at all to the allowed transitions between states of same parity. **more p. 42 →**

Density of final states

We have to make sure that momentum and energy are conserved properly among the 3-body final state.
We start by writing the ν density as a statistical mechanical result (and integrate over all angles for the time being):

$$dn_\nu = \frac{V}{2\pi^2\hbar^3} p_\nu^2 dp_\nu$$

$E_\nu^2 = m_\nu^2 + p_\nu^2$ **but** $m_\nu < 3$ **eV** ≈ 0 **so** $E_\nu = p_\nu$.
We can ignore the recoil energy for kinematics
(though keeping it produces corrections to correlations, 'recoil order terms' $\sim$ 0.01)
which gives the relation:
$E_\nu = Q - K_e$
where Q is the total kinetic energy released in the decay, and K_e is the kinetic energy of the electron. (This kinetic energy is sometimes written '*E*' in the literature)
I'll also make use of maximum total *e* energy $E_0 = Q + m_e$ and $E_\nu = E_0 - E_e$

Density of charged lepton final states: Fermi function

The density of charged-lepton states gets perturbed in the presence of the nuclear Coulomb field, so (also integrating over all angles)

$dn_e = \frac{V}{2\pi^2\hbar^3} F(Z,K_e) p_e^2 dp_e$; **where $F(Z,K_e)$ is the "Fermi function":**

Lepton nonrelativistic, pointlike nucleus, $|\psi(r=0)|^2$ from the nonrelativistic Coulomb wavefuction gives:

$$F(Z,K_e) = \left|\frac{x}{1-e^{-x}}\right|$$

$x = -1 \times \pm 2\pi\alpha Zc/v$ for $\beta^{\pm}$ decay, $\alpha \approx 1/137$.
Increases the total decay rate by 2 between Z=0 and Z=26 (for Q=7 MeV),

For a decay rate with better than $\sim$10% accuracy:
Dirac eq $|\psi(r)|^2 \overset{r\to 0}{\to} \infty$
so Fermi evaluated at the nuclear surface

deShalit and Feshbach eq. IX.2.15; Fermi Zeit. Physik 88 (1934) 161)

$$F(Z,K_e) = 2(2kr)^{2(s-1)} \frac{1+s}{s^2+\eta^2} \left|\frac{e^{\pi\eta/2}\Gamma(s+1+i\eta)}{\Gamma(2s+1)}\right|^2$$

using the Γ function, η=x/(2π), and $s = \sqrt{1-(\alpha Z)^2}$
Good to a few percent

Better formulations include e^- screening of the atom, exchange between outgoing and atomic e^-...

See Sir Denys Wilkinson's 5-part series in Nuclear Inst. and Meth.

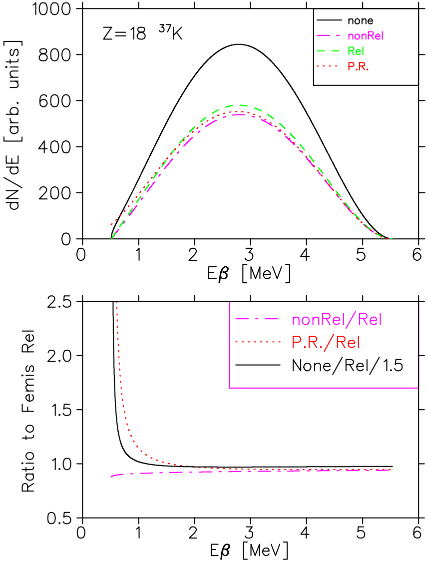
Approximate Fermi function:

$$2\pi Z\alpha \frac{E}{P} \frac{1}{1-e^{\pm 2\pi Z\alpha}}$$

Primakoff and Rosen 1959 Rep. Prog. Phys. 22 121

preserves the main 1/v dependence– lower v_β is distorted more

Use only for ft estimates, and only then for certain Z- the P&R low-energy E_β spectrum for some Z is worse than no Fermi function at all.



Several papers recently recalculating from scratch the distortion of the outgoing β from the Coulomb field

β energy spectrum for allowed decay

Integrating $p_e^2 dp_e p_\nu^2 dp_\nu \delta(Q - K_e - E_\nu)$ over p_ν, ignoring recoil-order terms and forbidden decay (so the nuclear matrix elements have no spatial/momentum dependence),

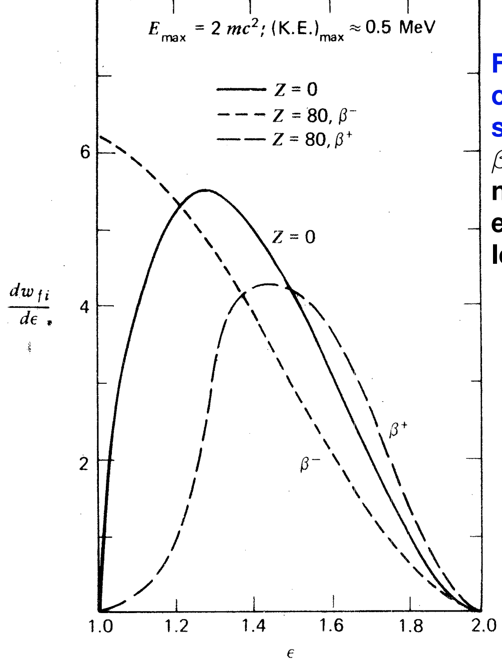
$$W(p_e)dp_e = \frac{1}{2\pi^3\hbar^3 c^3} \sum_{\mu\ m_f} |\langle J_f m_f r | O_{\lambda\mu}(\beta) | J_i m_i r' \rangle|^2 F(Z, K_e) p_e^2 (Q - K_e) \sqrt{(Q - K_e)^2 - m_\nu^2}\, dp_e$$

with $O_{\lambda\mu} = \sum_{j=1}^{A} (G_V \tau_\pm(j) + G_A \vec{\sigma}(j)\tau_\pm(j))$. Differentiating $E^2 = p^2 + m^2 \Rightarrow pdp = EdE$,

$$W(E_e)dE_e \propto F(Z, E_e) E_e p_e (E_0 - E_e) \sqrt{(E_0 - E_e)^2 - m_\nu^2}\, dE_e$$

- **The decay rate $\sim Q^5$, a large dependence. This is just from three powers of momentum for each lepton, minus one for energy conservation.**
- **The spectrum gets distorted at the very endpoint (large K_e, near Q), by the ν mass, which has upper limit (from ^{3}H decay KATRIN) 1.1 eV at 90% confidence.**

(Most forbidden decay operators produce large changes in this energy spectrum)



Fermi function effect on β energy spectrum

β^- is 'pulled into' nucleus... a big effect for high Z and low E_β

ft value for allowed decay

After doing the phase space integration, we can write down the answer:

$$ft = \frac{K}{|M_F|^2 + g_A^2|M_{GT}|^2}$$

$$K = \frac{2\pi^3\hbar^7 \ln 2}{m_\beta^5 c^4 G_V^2} = 6142 \pm 3.2\mathrm{s}$$

If you include isospin mixing and 'radiative' corrections, you can define the quantity *Ft* that is actually constant for the Fermi transitions:

$$Ft = ft(1-\delta_C)(1+\delta_R) = \frac{K}{G_F^2|V_{ud}|^2|M_{fi}|^2(1+\Delta_R)}$$

where $|M_{fi}|^2$ = T(T+1)-T_3(T_3+1) as below
Isospin-breaking corrections δ_C are parameterized by two sources:
1) isospin mixing with other 0^+ configurations
2) the spatial wavefunctions are slightly different because the protons repel each other 'radial mismatch'.

Recent correction for f Chien-Yeah Seng, arXiv:2212.02681 (accepted by PRL)

JB mentioned out loud the 2-3 σ difference from CKM unitarity in the corrected *Ft* values from this theorist and colleagues, from two sets of radiative corrections of both nucleon and nucleus.

The new work considers a correction to the phase space integral *f* (!)

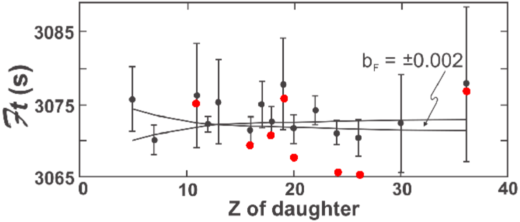
There is a recoil-order correction

$\propto q^2 R^2_{\mathrm{ChargedWeak}} \neq q^2 R^2_{\mathrm{Charge}}$

(This is a standard expansion of a pointlike nucleus to include its spatial distribution, related by a Fourier transform to the momentum transfer *q*)

Holstein RMP: **One can get $R^2_{\mathrm{ChargedWeak}}$ by comparing isobaric triplets of measured R^2_{Charge}, but no one has done this correctly before.**

The *Ft* values move about this much (JB reading Table II of Seng and shifting naively the centroids) for the ones that have been measured.



Seng reports that this could account for the whole CKM unitarity discrepancy, but it's work in progress.
Seng also comments that the CVC test would not work out as well.

more recent work relates the Coulomb part of isospin breaking to charge radii by assuming Coulomb is r^2 inside nucleus, but ignores the strong interaction isospin breaking.

Superallowed Ft values

Consider Fermi beta decay in many 0^+ to 0^+ cases. We can sum over the nucleons

$$\sum_{k=1}^{A} \tau_\pm(k) = T_\pm$$

where $T_\pm$ lowers or raises the 3rd component of SU(2) isospin for the whole nucleus, just like the lowering and raising operators for SU(2) spin. The Fermi operator's matrix element is

$$\langle J_f M_f T_f T_{0f} | \sum_{k=1}^{A} \tau_\pm | J_i M_i T_i T_{0i} \rangle = \sqrt{T_i(T_i+1) - T_{0i}(T_{0i} \pm 1)}$$

if $J_f = J_i,\ M_f = M_i,\ T_f = T_i$, and $T_{0f} = T_{0i} \pm 1$; 0 otherwise.

For these cases, the ft value then given by just some constants, which are given by the weak interaction strength. (f=integral over phase space). I.e. they all should have the same intrinsic strength.
The vector operator is related to the electric charge operator. We know electric charge is conserved. The "conserved vector current" hypothesis of Feynman and Gell-Mann: by analogy they theorized that the vector part of the weak interaction is also conserved. This eventually leads to electroweak unification. This has many consequences. For example, for the vector part of the weak interaction we can go straight from the quark matrix element to the nucleon one to the nucleus one.

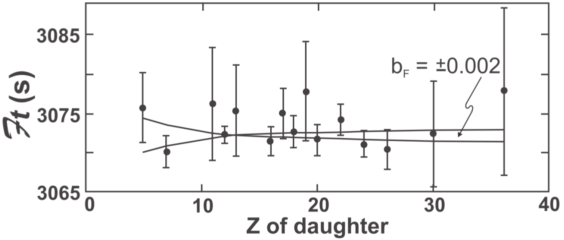
Effect of different Fermi functions on superallowed Ft's

Z	Q	no Fermi	E/p	Non-rel	Fermi's	Towner '05	error
5	0.88577	3540.7	4422	3005.8	3030.2	3073.0	4.9
7	1.80851	3618.7	3586	2985.0	3028.2	3071.9	2.6
12	3.21071	3955.5	3184	2905.2	3015.4	3072.9	1.5
16	4.46971	4252.2	3006	2832.3	3010.9	3071.7	1.9
18	5.02234	4400.4	2933	2786.2	3002.9	3072.2	2.1
20	5.40358	4548.1	2865	2732.9	2991.7	3075.6	2.5
22	6.02863	4696.5	2790	2679.4	2979.5	3078.5	2.4
24	6.61039	4846.2	2719	2622.4	2966.1	3071.1	2.7
26	7.22056	5004.4	2651	2566.7	2956.2	3071.2	2.8

Towner's include isospin mixing corrections.
Note Fermi's 1934 function isn't really good enough for this, while "Towner" includes 1% corrections from isospin mixing, or rather the difference in isospin mixing between the parent and daughter. These are parameterized by:
1) different isospin configurations mixed;
2) different wavefunctions because the nuclei have different radii.
IMME is fit mass-by-mass, adjusting an effect Coulomb interaction in a shell model.
(A technical check of neutron occupancies is used in the 2020 versions.)
Charge-dependent nuclear interaction to fit the *c* IMME coefficient Towner'08, Ormand Brown'85

Consistency of *Ft*'s tests CVC hypothesis

J.C. Hardy, I.S.Towner, PRC 102 044501 (2020)



Notice a constant goes through all– 5/15 should miss at 1 σ.
There is a common systematic uncertainty from a radiative correction which is folded into each of these points.
This test is used to gain confidence in the isospin breaking calculations.

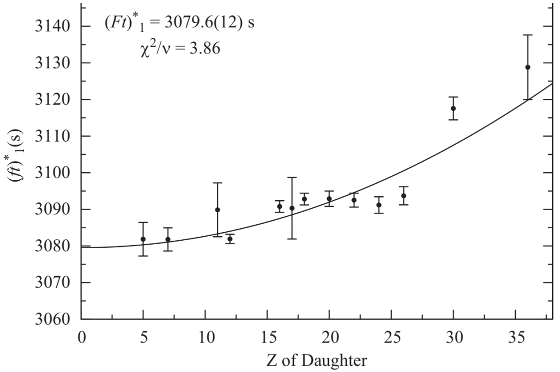


Fig. 2. Plot of the $(ft)^*_1$ data points that do not include theoretical corrections for isospin symmetry breaking and the resulting quadratic fit giving the global trend

Grinyer, Svensson, Brown NIMA 622 236 (2010)

Using 'Wilkinson Method 2'
Correcting fluctuations in *Ft* in each shell, yet allowing magnitude of isospin breaking to vary phenomenologically with Z^2.
V_{ud} changed by 0.2σ. σ_{Vud} increased by 1.3.

Quark eigenstates in the weak interaction: Cabibbo angle

To explain some weak decays, in particular ratios of semileptonic baryon decays with and without strangeness,

the weak interaction mixes the *d* and *s* quarks, so you can think of the *u* changing to *d* in β decay as:

$|u\rangle \rightarrow |d\rangle + \epsilon|s\rangle$ **i.e.** $|u\rangle \rightarrow \cos(\theta_C)|d\rangle + \sin(\theta_C)|s\rangle$

θ_C, the Cabibbo angle, is a parameter whose value (13.04^o) is unexplained so far from underlying physics. (Like any mixing 'angle', the angle is in an abstract space, and it's just a simple way to normalize wavefunctions)

For 3 families of particles, this generalizes to

$\rightarrow$ 3x3 unitary "CKM" matrix between $|d\rangle, |s\rangle, |b\rangle$

From superallowed ft values we get a vital physics constant: V_{ud}

The quark eigenstates of the weak interaction are not the same as the mass eigenstates. They are related by a unitary transformation.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

As for any unitary matrix, top row has the property:
$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

The superallowed *Ft* value, compared to muon decay (the strength of the leptonic weak interaction), gives you V_{ud}. (V_{ub} is very small and does not matter.)

There's been a long struggle over V_{us}, which comes from kaon decays or hyperon β decay, with useful checks from theory with more than one possible solution.

CKM unitarity test is off by 2-3 σ at 0.1% from most recent reevaluations of radiative corrections (see Towner Hardy review 2020).

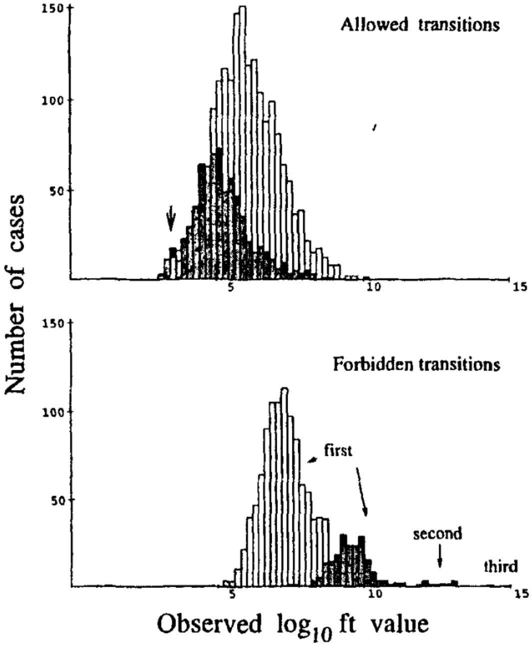
Again, each *Ft* value has an isospin mixing calculation done phenomenologically, because initial and final wavefunctions are not identical. The uncertainty and centroids of these calculations are still an open question.

log(ft) for β decay

Wong Figure 5.8

As we said above, G-T transitions preserve nuclear π, while 1st-forbidden transitions flip nuclear π.

If the *ft* values are different enough, that can distinguish the transition and be used to determine π.
However, the *ft* values for G-T and 1st forbidden overlap.
Sometimes the nuclear matrix element for G-T decay is accidentally small.
(E.g. ^{14}C GT decay has log(ft) of 9.0, five orders slower rate than the fastest GT's)



Selection rules

Fermi

G-T

γ_5 dominates $0^- \rightarrow 0^+$

$\sigma \cdot r$ suppressed by r/λ

'1st forb. unique' $2^\pm \leftrightarrow 0^\mp$
One operator $\Rightarrow$ calculable correlations from spin,
A_β large

TABLE I. Allowed and first-forbidden nuclear matrix elements and their selection rules (K designates the rank of the transition operator, when regarded as a tensor).

	Matrix element	K	ΔJ	$\Delta\pi$
Allowed	$C_V\int 1$	0	0	+1
	$C_A\int \boldsymbol{\sigma}$	1	0, ±1 (no 0→0)	+1
First forbidden	$C_A\int \gamma_5$, $C_A\int (\boldsymbol{\sigma}\cdot\mathbf{r}/i)$	0	0	−1
	$C_V\int \mathbf{r}i$, $C_V\int \boldsymbol{\alpha}$, $C_A\int (\boldsymbol{\sigma}\times\mathbf{r})$	1	0, ±1 (no 0→0)	−1
	$C_A\int iB_{ij}$	2	0, ±1, ±2 (no 0→0, no 1→0, no 0→1)	−1

Weidenmüller Rev Mod Phys 33 574 (1961)

Some correlations in 1st-forbidden β decay are simple
If there's one operator, correlation is given by angular momentum coupling, no nuclear structure dependence

- **$2^{\pm} \leftrightarrow 0^{\mp}$ "1st forbidden unique" has one operator.**

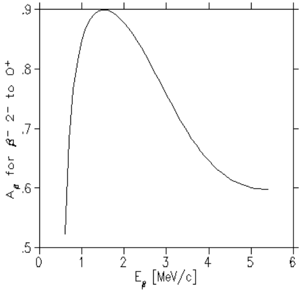
(one must flip nuclear spin and have leptons carry off L=1 to change *J* by 2 and flip nuclear parity)

Behrens and Buhring Chs 7, 14; AbetaFirstForbiddenUniqueJB.pdf

Coefficient of $\cos\theta_\beta$ wrt spin:

$$A_\beta \frac{v}{c} = \frac{p}{E} \frac{p_\nu^2 + \frac{3}{5} p_\beta^2}{p_\nu^2 + p_\beta^2}$$

So it's not just Gamow-Teller and Fermi that have large predictable β asymmetry



- **$0^- \rightarrow 0^+$ decay: 2 operators, but one is suppressed wrt other by $R_{\text{nucleus}}/E_\beta$ $\sim$ few % in fission products** Hayen PRC 100 054323 (2019)

$\Rightarrow E_\beta$ spectrum is $\sim$ allowed; $a_{\beta\nu} \approx 1$

- **Warburton PRC 26 1186 (1982) has E_β spectrum and $a_{\beta\nu}$ for 1st forbidden for light nuclei**
- **Glick-Magid and Gazit, J. Phys. G 49 105105 (2022) forbidden β expanision in 5 small quantities+ Coulomb corrections**

Clarification of g_V and V_{ud}

I said g_V=1.00 was experimentally shown, which was pretty sloppy. Better to say g_V=1 is a prediction of CVC:

- **The $0^+ \rightarrow 0^+$ *Ft* values are experimentally constant, testing whether g_V is a constant for all transitions, but not necessarily g_V=1.000...**

- **Backing up, G_V is determined by**

$$\mathcal{F}t \equiv ft(1+\delta_R')(1+\delta_{NS}-\delta_C) = \frac{K}{2G_V^2\left(1+\Delta_R^V\right)}, \qquad (1)$$

where $K/(\hbar c)^6 = 2\pi^3\hbar \ln 2/(m_e c^2)^5 = 8120.27648(26) \times 10^{-10}$ GeV^{-4}s, G_V is the vector coupling constant for semileptonic weak interactions, δ_C is the isospin-symmetry-breaking correction, and Δ_R^V is the transition-independent part of the radiative correction. The terms δ_R' and δ_{NS} comprise the transition-dependent part of the radiative correction, the

and then V_{ud} is determined by

$$V_{ud} = G_V/G_F, \qquad (2)$$

where G_F is the well-known weak-interaction constant for muon decay. Once the value of V_{ud} is established it can be

So the present deficit in V_{ud} could also be a change for g_V from its value of 1 from electroweak unification.

E.g., *e* and μ weak couplings could be different.
Crivellin and Hoferichter PRL 125 111801 (2020) consider keeping CKM unitarity while considering constraints from $(\pi \rightarrow e\nu)/(\pi \rightarrow \mu\nu)$

Axial vector is not conserved. Is g_A the same in nuclei? G-T ("Ikeda") sum rule:

define the sum rule strength

$$S_\pm = G_A^{-2} \sum_f |\langle f|O_{\rm GT}(\beta^\pm)|i\rangle|^2$$

$$\begin{aligned} S_\pm &= G_A^{-2} \sum_f \langle f|O_{\rm GT}(\beta^\pm)|i\rangle^* \langle f|O_{\rm GT}(\beta^\pm)|i\rangle \\ &= G_A^{-2} \sum_f \langle i|O^\dagger_{\rm GT}(\beta^\pm)|f\rangle \langle f|O_{\rm GT}(\beta^\pm)|i\rangle \\ &= G_A^{-2} \langle i|O^\dagger_{\rm GT}(\beta^\pm) O_{\rm GT}(\beta^\pm)|i\rangle \end{aligned}$$

operators involved here have the following p

$$\sigma^\dagger_\mu = (-1)^\mu \sigma_{-\mu} \qquad \tau^\dagger_\mp = \tau_\pm$$

$$\begin{aligned} S_+ &= \langle i|\sum_{k=1}^{A} \sum_\mu (-1)^\mu \sigma_{-\mu}(k) \tau_+(k) \sigma_\mu(k) \tau_-(k)|i\rangle \\ &= \langle i|\sum_{k=1}^{A} \sigma^2(k) \tau_+(k) \tau_-(k)|i\rangle \end{aligned}$$

In a spherical basis, the scalar product m

$$J \cdot V = \sum_q (-1)^q J_{1q} V_{1,-q}$$

$$\tau_+\tau_-|p\rangle = |p\rangle \qquad \tau_+\tau_-|n\rangle = 0$$

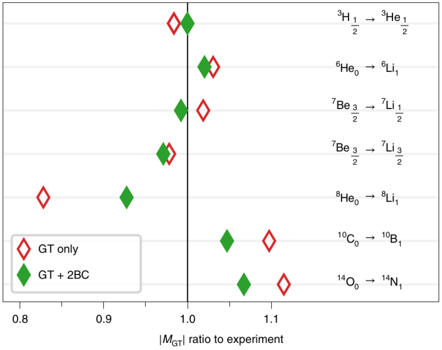
expectation value of σ^2 is 3.

$$S_+ = \langle i|\sum_{k=1}^{Z} \sigma^2(k)|i\rangle = 3Z$$

Similarly, S_- = 3N , and $S_+ - S_-$=3(Z-N)

[Formally similar sum rule arguments, applied to the nucleon, express g_A in terms of π-nucleon cross-sections, calculating 1.16 Weisberger PRL 14 1047 (1965) **and 1.24** Adler PRL 14 1051 (1965)**.**
Experiment then in n β decay was 1.18$\pm$0.02, now 1.26.]

Recent progress on Gamow-Teller strength



Decades of detailed studies with high-Q β decay (and (p,n) and (n,p) reactions at 100-200 MeV)consistently found $\approx$ 75% of the GT strength in many nuclei. Comparison is not to the sum rule: not all of the strength can be measured. Bertsch Esbensen 1987 Rep. Prog. Phys. 50 607

- **mixing with the Δ moves strength to high excitation?**
- **Nuclear structure calculations lack configurations?**
- **g_A changing in nuclei?**
- **Maybe π becomes effectively massless in nuclei (chiral symmetry taken to its extreme) and π degrees of freedom go away completely $\rightarrow$ g_A=1?** Rho Ann Rev Nucl Part 34 531 (1984)

Recent calculations Gysberg Nat Phys 15 428 2019 **reproduce GT strength with about 5-10% accuracy, combining chiral EFT's with accurate many-body techniques, considering 2-body currents.**

2-body currents are the chiral EFT equivalent of meson exchange currents, and are treated consistently with 3-nucleon forces in chiral EFT in Gysberg et al.

So both more configurations and 2-body currents are important.
The need for 2-body currents $\Longleftrightarrow$ the axial current strength is changing in nuclei.

Jackson, Treiman, Wyld NP 4 206 (1957) **rewrote** Lee Yang 104 254 (1956) **4-Fermion interaction H for nucleon beta decay:**

You construct Lorentz-invariant quantities, i.e. a Lorentz scalar, from the possible objects which Lorentz transform like vectors, axial vectors, scalars, tensors, pseudoscalars (it turns out all combinations of more Dirac matrices reduce to these). Assuming pointlike high-mass bosons, one could now call this an EFT derivatives produce small corrections

$$
\begin{aligned}
H_{\text{int}} = & (\bar\psi_p\psi_n)(C_S\bar\psi_e\psi_\nu + C_S'\bar\psi_e\gamma_5\psi_\nu) \\
& + (\bar\psi_p\gamma_\mu\psi_n)(C_V\bar\psi_e\gamma_\mu\psi_\nu + C_V'\bar\psi_e\gamma_\mu\gamma_5\psi_\nu) \\
& + \tfrac{1}{2}(\bar\psi_p\sigma_{\lambda\mu}\psi_n)(C_T\bar\psi_e\sigma_{\lambda\mu}\psi_\nu + C_T'\bar\psi_e\sigma_{\lambda\mu}\gamma_5\psi_\nu) \\
& - (\bar\psi_p\gamma_\mu\gamma_5\psi_n)(C_A\bar\psi_e\gamma_\mu\gamma_5\psi_\nu + C_A'\bar\psi_e\gamma_\mu\psi_\nu) \\
& + (\bar\psi_p\gamma_5\psi_n)(C_P\bar\psi_e\gamma_5\psi_\nu + C_P'\bar\psi_e\psi_\nu) \\
& + \text{Hermitian co}
\end{aligned}
$$

Pauli wrote down C_X: Lee Yang added C_X' for $\not P$

Quark-lepton interactions have been found experimentally to be V,A only so far. V is assumed conserved (like electric charge), so C_V=1 is often assumed. QCD still can change A, and 'induce' all the other terms for hadron-lepton interactions, changing all these constants but C_V. We've seen how this creates interesting ways to test QCD's influence on weak interactions, and we've already seen $|C_A|$ = 1.26...

I.e. this looks a lot like the S.M. quark-lepton Lagrangian but of course we have to be careful about the C_x's

Jackson, Treiman, Wyld 1957 wrote down observables before angular integration, and the answers

$$\omega(\langle J\rangle | E_e,\Omega_e,\Omega_\nu)dE_e d\Omega_e d\Omega_\nu$$
$$= \frac{1}{(2\pi)^5} p_e E_e (E^0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu \xi \Big\{ 1 + a\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e}$$
$$+ c\left[\frac{1}{3}\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} - \frac{(\mathbf{p}_e\cdot\mathbf{j})(\mathbf{p}_\nu\cdot\mathbf{j})}{E_e E_\nu}\right]\left[\frac{J(J+1)-3\langle(\mathbf{J}\cdot\mathbf{j})^2\rangle}{J(2J-1)}\right]$$
$$+ \frac{\langle\mathbf{J}\rangle}{J}\cdot\left[A\frac{\mathbf{p}_e}{E_e} + B\frac{\mathbf{p}_\nu}{E_\nu} + D\frac{\mathbf{p}_e\times\mathbf{p}_\nu}{E_e E_\nu}\right]\Big\}.$$

Rather than use JTW's answers here,

we'll assume the functional form of the correlations. In limiting cases, assumptions about S.M. lepton helicity will then let us deduce the S.M. predictions soon.
The S.M. weak interaction makes left-handed leptons and right-handed antileptons in decays, Helicity $\hat{s}\cdot\hat{p}$
Note $\frac{p}{E}$ is, of course, $\frac{v}{c}$. One can always boost to a frame moving faster than a massive particle– reversing $\hat{p}$ but preserving $\hat{s}$. That's intuitively why there's a factor of $\frac{v}{c}$ multiplying the helicities.

Re: the 'Fierz interference' term $b\frac{m_e}{E_e}$, product of a SM term with normal helicity and a ~~SM~~ term with non-normal helicity:

$$\sqrt{1+\frac{p_e}{E_e}} \times \sqrt{1-\frac{p_e}{E_e}} = \sqrt{1-\frac{p_e^2}{E_e^2}} = \frac{m_e}{E_e}$$ **take care with particle physics 'chirality' vs. 'helicity'**

Reference for non-Dirac treatment: R. Hong, M. Sternberg, A. Garcia, "Helicity and nuclear β decay correlations," American Journal of Physics 85 p 45 (2017).

Measure ν helicity $\epsilon = \hat{s}_\nu \cdot \hat{k}_\nu$ directly: transfer $\hat{s}_\nu$ to γ circular polarization; boost $\vec{k}_\gamma$ by $\pm\vec{k}_\nu$

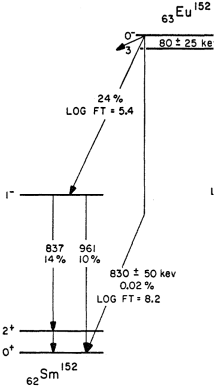
Goldhaber, Grodzins, Sunyar
Phys Rev 109 1015 (Dec 1957)

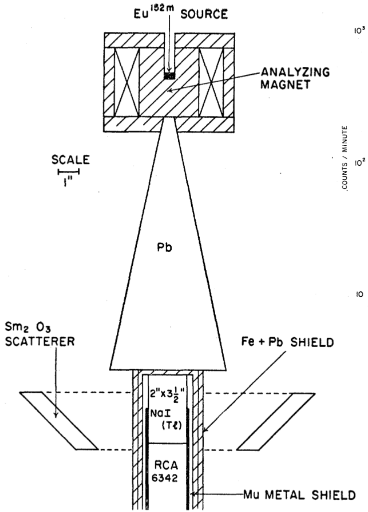
$$e^- + {}^{152m}\mathrm{Eu} \to \nu + {}^{152}\mathrm{Sm}$$

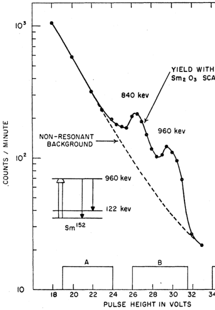
- ν with $\hat{s}$ = -1 populates $\langle J_z \rangle = 0, +1$ not -1
- So γ is circularly polarized– transmission through magnet depends on iron polarization: $\frac{N_+ - N_-}{N_+ + N_-} = 0.017 \pm 0.003$
- Upward ν boosts γ momentum so it can be absorbed on-resonance

$\Rightarrow$ ν helicity -1 $\pm$ 10%

(• $\bar{\nu}$ helicity $\sim$ +1
Palathingal PRL 524 24 '69)







Surprisingly enough, this is the best direct measurement of ν helicity = $\hat{s}_\nu \cdot \hat{k}_\nu$

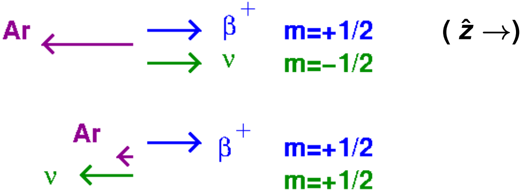
The β-ν angular distribution in the SM

$$W[\theta_{\beta\nu}] = 1 + a \frac{v_\beta}{c} \cos\theta_{\beta\nu}$$

For ^{38m}K, $0^+ \rightarrow 0^+$ decay:

***a*= +1 'Proof':**

leptons have opposite helicity for W (vector) boson exchange



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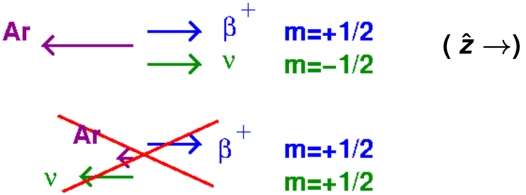


For scalar exchange, lepton helicities are same: a= -1

No nuclear structure corrections until 10^{-6} accuracy (Isospin breaking only mixes in 0^+ configurations)

Note $a_{\beta\nu}$ depends on the relative helicity of β and ν, but not the absolute sign. The observable is parity-even $\rightarrow$ is not actually sensitive to $\not{P}$

$a_{\beta\nu} = -1/3$ Gamow-Teller decay

If you were to work the matrix element (trace of Dirac matrices...) you would see this is a consequence of the $\gamma(1+\gamma_5)$ Lorentz structure from the W^+.
I.e. a Gamow-Teller decay, just like Fermi decay, still makes left-handed ν and right-handed β^+ (or right-handed $\bar{\nu}$ and left-handed β^-)
But the angular distribution result is different because of the nonzero nuclear spin involved.
This is simplest to see in 0^+ ^{6}He $\rightarrow$ 1^+ ^{6}Li + β^- + $\bar{\nu}$
The final ^{6}Li can have 3 different spin projections. Orient $\hat{z}$ up:

$m(^6Li)$	m_{β^-}	m_ν	β, ν relative direction	a contribution
+1	+1/2	+1/2	opposite	-1
-1	-1/2	-1/2	opposite	-1
0	m_β	$-m_\beta$	same	+1 (like Fermi $0^+ \rightarrow 0^+$)
				ave -1/3

For any Gamow-Teller transition, if the weak interaction produces opposite-handed leptons and antileptons, $a_{\beta\nu}$=-1/3.
Scalar and tensor Lorentz currents produce same-handed leptons and antileptons, and for Gamow-Teller $a_{\beta\nu}$=+1/3

$a_{\beta\nu}$ experiments

Feynman&Gell-Mann paper PR 109 193 (1957)

proposing CVC and V$\pm$A:

"These theoretical arguments seem to the authors to be strong enough to suggest that the disagreement with the He^6 recoil experiment... indicates that these experiments are wrong.

The $\pi \to e + \bar{\nu}$ problem may have a more subtle solution."

successes listed:

μ decay rate to 2%

asymmetry in direction of $\pi \to \mu \to e$ chain

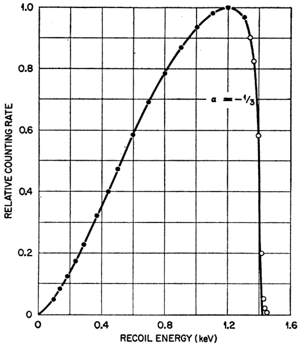
$a_{\beta\nu}$ in ^{35}Ar

Undistorted E_β spectra

e^- polarization from β decay

^{6}He Gamow-Teller decay

Johnson, Pleasonton, Carlson PhysRev 132 1149 (1963)



$a_{\beta\nu}$ = -0.3308 $\pm$ 0.0030 agreed much better with V,A (-1/3) than S,T (+1/3).

Fermi $0^+ \to 0^+$ decays:

Adelberger PRL 83 1299 (1999) (err. 83 3101)

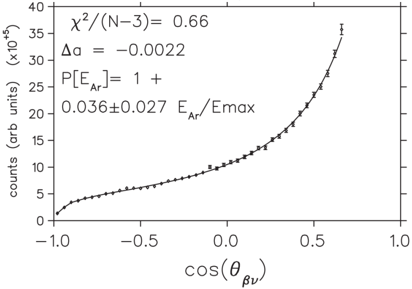
^{32}Ar

$\tilde{a}$=0.9989 $\pm$ 0.0052(stat) $\pm$ 0.0039(syst)

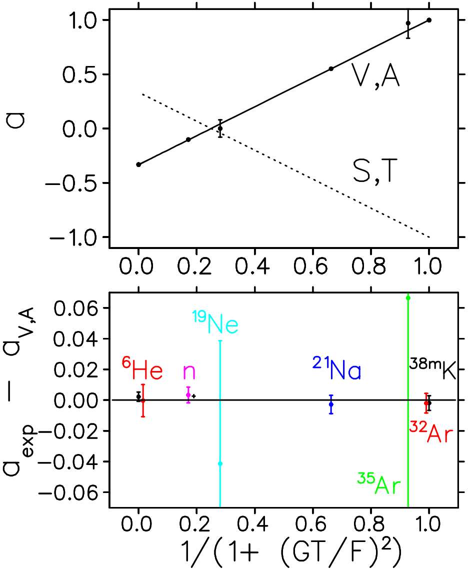
Gorelov PRL 94 142501 (2005)

^{38m}K

$\tilde{a}$=0.9981 $\pm$ 0.0030(stat) $\pm$ 0.0037(syst)



Summarizing info on Lorentz structure from β-ν correlation



Interaction is mostly vector and axial vector, i.e. V and A

[Except aSPECT has difference in *a* for neutron $(2.57 \pm 0.84)x10^{-3}$

Beck PRC 101 055506 (2020)

Explainable by a finite Lorentz tensor allowed by other nuclear β decay

Falkowski JHEP04 (2021) 126

but recent ^{8}Li $\beta\alpha\alpha$ correlation agrees with ^{6}He $a_{\beta\nu}$ with higher precision

Burkey PRL 128 202502 (2022); Sargsyan PRL 128 202503 (2022)]

For the sign between them, we need to consider parity violation →

Symmetries: Continuous, Discrete

- **Noether's theorem (1915):**

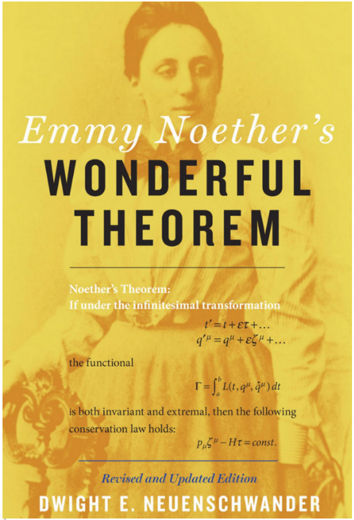
Continuous symmetry	$\rightarrow$	**Conserved quantity**
Time-translational invariance	$\rightarrow$	**Energy**
Space-translational invariance	$\rightarrow$	**Momentum**
Rotational invariance	$\rightarrow$	**Angular momentum**
(Laplace-Runge-Lenz vector)	$\rightarrow$	**name?**

THE LATE EMMY NOETHER.

Professor Einstein Writes in Appreciation of a Fellow-Mathematician.

To the Editor of The New York Times:

gan. In the realm of algebra, in which the most gifted mathematicians have been busy for centuries, she discovered methods which have proved of enormous importance in the development of the present-day younger generation of mathematicians. Pure mathematics is, in its way, the poetry of logical ideas. One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulae are discovered necessary for the deeper penetration into the laws of nature.



Discrete symmetries in quantum mechanics are quite different, but we'll appeal to classical intuition concerning observables for $\not{P}$, $\not{T}$.

Historical Ideas about *P*, *T* breaking

- **Wigner considered implications of P, T symmetry conservation in atomic spectra 1926-28. Showed $\langle T\psi_i, T\psi_f\rangle = \langle \psi_f, \psi_i\rangle^*$**
"In quantum theory, invariance principles permit even further reaching conclusions than in classical mechanics." (D. Gross, Physics Today 48 46 (1995))
- **Weyl 1931 considered *C*, *P*, *T* and *CPT* in "Maxwell-Dirac theory": C $\Rightarrow$ Dirac eq. negative energy states had to have same mass as the e^- plato.stanford.edu**
- **From "CP Violation Without Strangeness" Khriplovich and Lamoreaux:**
1949 Dirac "I do not believe there is any need for physical laws to be invariant under reflections in space and time although the exact laws of nature so far known do have this invariance."
- **1956 Lee and Yang proposed $\not{P}$ in weak decays to fix the θ-τ puzzle**
- **Feynman gives Ramsey 50:1 odds $\not{P}$ would not be observable**
Ramsey experiment starting at ORNL gets derailed by fission experiments...
it's OK, Ramsey won 1989 Nobel for his fringes
- **1957 3 simultaneous experimental measurements of $\not{P}$ $\rightarrow$**

Parity (From A. Zee "Fearful Symmetry")

As of 1956, we thought all interactions respected parity

Parity operator

$\boldsymbol{P}\,\psi(\vec{r}) \rightarrow \pm\,\psi(-\vec{r})$

P  $=$ 

1957:

$\tau - \theta$ Puzzle

+ μ decay

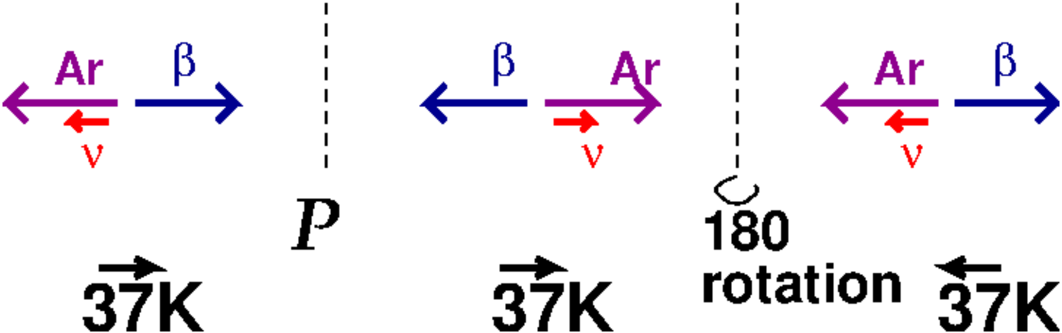
+ ^{60}Co decay $\Rightarrow$

P  $\neq$ 

Decays: Parity Operation can be simulated by Spin Flip

Under Parity operation *P*:

$\vec{r} \rightarrow -\vec{r}$ $\quad\quad \vec{p} \sim \frac{d\vec{r}}{dt} \rightarrow -\vec{p}$ $\quad\quad \vec{J} = \vec{r} \times \vec{p} \rightarrow +\vec{J}$



$\Rightarrow$ A spin flip corresponds exactly to *P* reversal

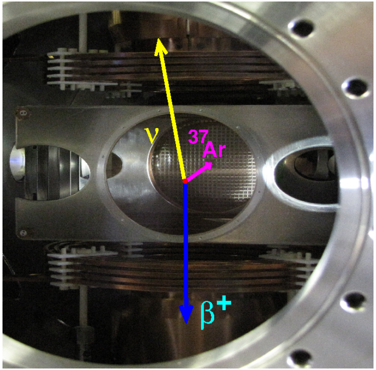
with one exception **Decays don't exactly test *T*-reversal symmetry $\rightarrow$**

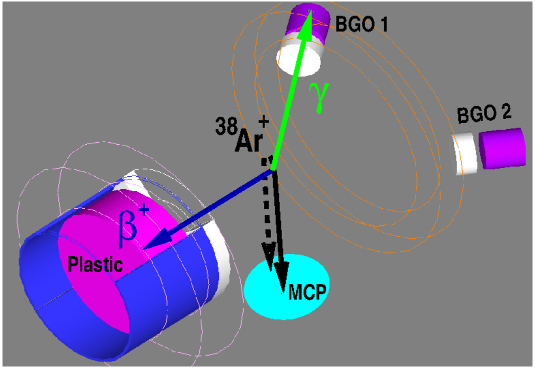
$\not T$ correlation of 3 of 4 momenta

$t \rightarrow -t \Rightarrow \vec{p} \propto \frac{d\vec{r}}{dt} \rightarrow -\vec{p}$

but $\vec{p}_{recoil} \cdot \vec{p}_\beta \times \vec{p}_\nu \equiv 0$ ☹

$$\vec{p}_\nu \cdot \vec{p}_\beta \times \vec{p}_\gamma = -\vec{p}_{recoil} \cdot \vec{p}_\beta \times \vec{p}_\gamma \xrightarrow{t \rightarrow -t} \vec{p}_{recoil} \cdot \vec{p}_\beta \times \vec{p}_\gamma$$



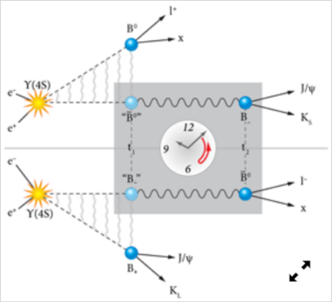


- We can test symmetry of apparatus with coincident pairs ☺
- Not exact. Outgoing particles interact → fake $\not T$

Entanglement in decays

There exists microscopic true $\not T$ in nature! independent of assumptions about QFT, CPT theorem, unitarity...

- **BABAR PRL 2012: Entanglement of B meson pairs enables $\psi_{\text{initial}} \leftrightarrow \psi_{\text{final}}$ also seen in K's KLOE-2 PLB 2023**



APS/Alan Stonebraker

Figure 1: Electron-positron collisions at SLAC produce a $\Upsilon(4s)$ resonance that results in an entangled pair of B mesons. When one meson decays at time t_1 , the identity of the other is "tagged" but not measured specifically. In the top panel, the tagged meson is a "$\overline{B}^0$". This surviving meson decays later at t_2 , encapsulating a time-ordered event, which in this case corresponds to "$\overline{B}^0$" $\to B_-$. To study time reversal, the BaBar collaboration compared the rates of decay in one set of events to the rates in the time-reversed pair. In the present case, these would be the "B_-" $\to \overline{B}^0$ events, shown in the bottom panel.

M. Zeller Physics 2012

One experimental discovery of parity violation

Wu, Ambler, Hayward, Hopper, Hobson, PR 105 1413; (Garwin Lederman Weinrich PR 105 1415 Feb '57; Hargittai Physics World 13 Sep 2012)
Abashian BNL PR 105 1927 1957 (Lee: gradient!); emulsion Friedman Telegdi PR 106 1290

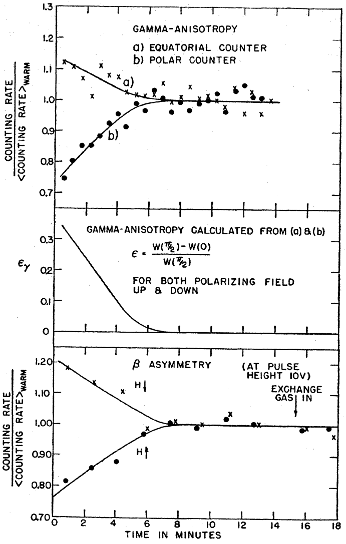
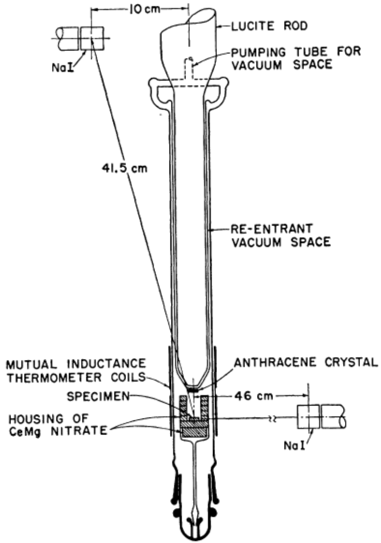


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.



Dilution Refrigerator to spin-polarize with nuclear polarization

$P = \langle \frac{J_z}{J} \rangle$

^{60}Co $\rightarrow$ ^{60}Ni + β^- + $\bar{\nu}$

$W[\theta] = 1 + PA\hat{J} \cdot \frac{\vec{p}_\beta}{E_\beta}$

$= 1 + AP\frac{v}{c}\cos[\theta]$

$A_{\beta-} = -1.0$

Note: $5^+ \rightarrow 4^+$ Null for left-handed $\hat{v}_{\beta-}$ = $\hat{J}_i$

Proof: Let $\hat{J}_i$= +$\hat{z}$

enforce $m_i = m_f$

If m_i=m_J=J=+5,

m_β=-1/2, $m_J^f \leq$ =+4

can't happen $\Rightarrow$

$A_{\beta-} = -1$ QED

Lookup for allowed β-decay correlations

one needs to take trace of Dirac matrices

Jackson Treiman Wyld NuclPhysA 4 206 (1957)

± 1, no nuclear parity change, $J=0\to J'=0$ forbidden. J and J' are the angular momenta of the original and final nuclei, $\delta_{J'J}$ is the Kronecker delta symbol and

$$\lambda_{J'J}=\begin{cases}1 & J\to J'=J-1\\ \dfrac{1}{J+1} & J\to J'=J\\ -\dfrac{J}{J+1} & J\to J'=J+1\end{cases}\qquad\text{(A.1)}$$

$$\Lambda_{J'J}=\begin{cases}1 & J\to J'=J-1\\ -\dfrac{(2J-1)}{J+1} & J\to J'=J\\ \dfrac{J(2J-1)}{(J+1)(2J+3)} & J\to J'=J+1.\end{cases}\qquad\text{(A.2)}$$

Z is the atomic number of the final nucleus, α is the fine structure constant, and $\gamma=(1-\alpha^2Z^2)^{\frac{1}{2}}$.

For pure G-T: $A_{\beta\pm} = \pm\,\lambda_{J',J}$

Textbooks with calculations:

***a*, the 'β — ν correlation':**
Halzen&Martin "Quarks&Leptons,"
my notes ph505jbVIII_2005_aBetaNu_WithDirac.pdf

Melconian's notes include Fierz term!

***A*, the 'β asymmetry wrt spin':**
Greiner and Müller "Gauge Theory of Weak Interactions"
Towner's notes within mine

upper sign for β^-, lower sign for β^+

$$\xi=|M_\mathrm{F}|^2(|C_\mathrm{S}|^2+|C_\mathrm{V}|^2+|C'_\mathrm{S}|^2+|C'_\mathrm{V}|^2)+|M_\mathrm{GT}|^2(|C_\mathrm{T}|^2+|C_\mathrm{A}|^2+|C'_\mathrm{T}|^2+|C'_\mathrm{A}|^2)\qquad\text{(A.3)}$$

$$a\xi=|M_\mathrm{F}|^2\left\{[-|C_\mathrm{S}|^2+|C_\mathrm{V}|^2-|C'_\mathrm{S}|^2+|C_\mathrm{V}|^2]\mp\frac{\alpha Zm}{p_e}\,2\,\mathrm{Im}\,(C_\mathrm{S}C_\mathrm{V}^*+C'_\mathrm{S}C'^*_\mathrm{V})\right\}+\frac{|M_\mathrm{GT}|^2}{3}\left\{[|C_\mathrm{T}|^2-|C_\mathrm{A}|^2+|C'_\mathrm{T}|^2-|C'_\mathrm{A}|^2]\pm\frac{\alpha Zm}{p_e}\,2\,\mathrm{Im}\,(C_\mathrm{T}C_\mathrm{A}^*+C'_\mathrm{T}C'^*_\mathrm{A})\right\}\qquad\text{(A.4)}$$

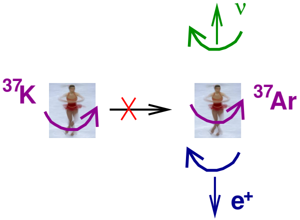
$$b\xi=\pm 2\gamma\,\mathrm{Re}[|M_\mathrm{F}|^2(C_\mathrm{S}C_\mathrm{V}^*+C'_\mathrm{S}C'^*_\mathrm{V})+|M_\mathrm{GT}|^2(C_\mathrm{T}C_\mathrm{A}^*+C'_\mathrm{T}C'^*_\mathrm{A})]\qquad\text{(A.5)}$$

$$c\xi=|M_\mathrm{GT}|^2\Lambda_{J'J}\left[|C_\mathrm{T}|^2-|C_\mathrm{A}|^2+|C'_\mathrm{T}|^2-|C'_\mathrm{A}|^2\pm\frac{\alpha Zm}{p_e}\,2\,\mathrm{Im}\,(C_\mathrm{T}C_\mathrm{A}^*+C'_\mathrm{T}C'^*_\mathrm{A})\right]\qquad\text{(A.6)}$$

$$A\xi=|M_\mathrm{GT}|^2\lambda_{J'J}\left[\pm 2\,\mathrm{Re}(C_\mathrm{T}C'^*_\mathrm{T}-C_\mathrm{A}C'^*_\mathrm{A})+\frac{\alpha Zm}{p_e}\,2\,\mathrm{Im}\,(C_\mathrm{T}C'^*_\mathrm{A}+C'_\mathrm{T}C_\mathrm{A}^*)\right]+\delta_{J'J}M_\mathrm{F}M_\mathrm{GT}\sqrt{\frac{J}{J+1}}\left[2\,\mathrm{Re}\,(C_\mathrm{S}C'^*_\mathrm{T}+C'_\mathrm{S}C_\mathrm{T}^*-C_\mathrm{V}C'^*_\mathrm{A}-C'_\mathrm{V}C_\mathrm{A}^*)\pm\frac{\alpha Zm}{p_e}\,2\,\mathrm{Im}\,(C_\mathrm{S}C'^*_\mathrm{A}+C'_\mathrm{S}C_\mathrm{A}^*-C_\mathrm{V}C'^*_\mathrm{T}-C'_\mathrm{V}C_\mathrm{T}^*)\right]\qquad\text{(A.7)}$$

+ B_ν, $\not{T}D$, ...

A spin-polarized angular distribution sensitive to ν helicity



If $I_z = I_{\rm initial}$ and $I_{\rm initial} = I_{\rm final}$, the leptons can't increase $I_{z\ \rm final}$

If β^+ down, the ν can't go up, unless either β or ν have wrong helicity

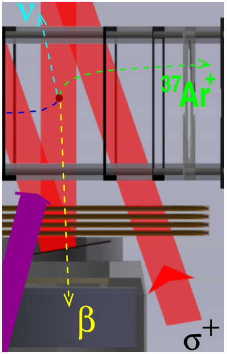
Any imperfect I_z/I mimics a wrong-handed ν

^{38}K G.T. $3^+ \to 2^+$ needs both ν and β^+ helicities wrong:

would be most direct ν helicity measurement

since Goldhaber 1957

Helicity-driven null



Fenker et al. PRL 2018

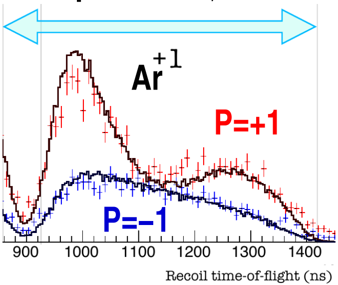
A_β= -0.5707$\pm$ 0.001913 in agreement with SM

achieved I_z/I = 0.991 $\pm$ 0.001

update James McNeil

VIR-L03 12:42 Wed

2014 polarized β-recoil



$v_{\rm TOFaxis} = 0$ suppressed. Dip would be deeper with ion MCP position cut or $\cos(\theta_{\beta-\nu})$ determination

$$W(\theta,P) \approx 1 + a_{\rm pol}\cos(\theta_{\beta\nu})$$

$$a_{\rm pol} = \frac{a_{\beta\nu} - 2c/3T + PB_\nu}{1 + PA_\beta + bm/E}$$

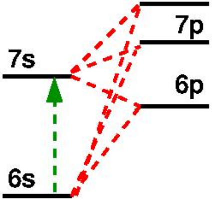
= 1 or 0, independent of $\frac{M_{GT}}{M_F}$

Weak Neutral Current

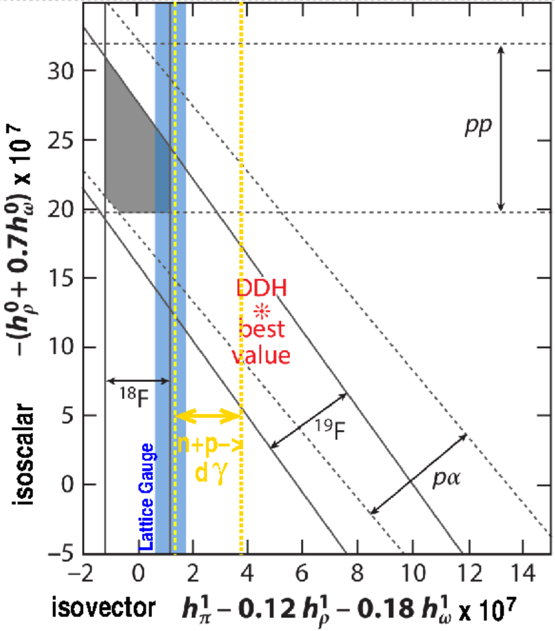
Existence of Z^0 boson, spin-1 partner of $W^{\pm}$ and the photon, was a S.M. prediction.

Searched for in :

- **ν scattering (winner: Gargamelle) (not parity-violating!)**
- **Atomic $\not{P}$ by mixing atomic states of opposite parity (1st answers came in small, creating concern for the S.M. prediction; now using cesium the best low-energy measurement of e$^-$-q weak neutral coupling) $\propto Z^2N$**



- **parity violating nucleon-nucleon interaction, via γ asymmetries from decay of nuclear states. $\not{P}$ can also come from the known charged current (W^{+-}). It was noticed that isovector $\not{P}$ could only come the neutral current, so that search was emphasized early on. It turns out the isovector $\not{P}$ was suppressed compared to isoscalar and isotensor for reasons only understood more recently, and the isovector $\not{P}$ has only been measured very recently to be nonzero. (Otherwise Queens and Cal State L.A. would have measured weak neutral current in ^{18}F and shared Nobel with Gargamelle.)**
- **SNO used neutral current breakup of d, independent of ν flavor**
- **$\not{P}$ electron scattering on the proton at SLAC and JLAB; for neutron skin of ^{48}Ca and ^{208}Pb.**
- **COHERENT scattering of ν from nuclei is agreeing so far with SM cross-section (not $\not{P}$!)**



Gardner Haxton Holstein ARNPS 2017+ Blythe PRL 121 242002 (2018)

Weak interaction between nucleons, $\not{P}$

$W^{\pm}$, Z^0 (m=80.4, 91.2 GeV) are very short-ranged compared to mesons.

- Parameterized by meson exchange (emitted weakly, absorbed strongly...)

The isovector piece was long expected to be dominated by the weak neutral current, but the 1/Nc expansion suppresses isovector/isoscalar by $\sin^2(\theta_W)/N_C \approx 1/12$ (Phillips et al. PRL 114 062301 (2015).

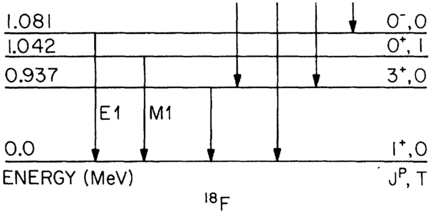
- A formal EFT produces similar results.
- Isovector and isoscalar parts now considered measured.
- $n + p \rightarrow d + \gamma$ isovector $\Rightarrow$ evidence for weak neutral current at 2σ

An isotensor part is interesting and inspiring proposals like $\vec{\gamma} + d \rightarrow n + p$

E.g. measuring N-N $\not{P}$ by mixing of ^{18}F 0^- and 0^+ states: much nuclear physics

- Observable is the circular polarization of the 1.081 MeV γ-ray, caused by E1 interference with the parity-violating M1, $-0.7 \pm 2.0 \times 10^{-3}$

Sensivity is enhanced by:



Barnes et al. PRL 40 840 (1977)

- The J^+; T= 0^-; 0 and 0^+; 1 states lie close together in energy, admixture $\propto \frac{\langle 0^- | \mathcal{O}_{\mathrm{WeakNN}} | 0^+ \rangle}{\Delta E}$
- The E1 operator is isovector (except for a tiny correction from the long-wavelength approximation), so is suppressed by $\sim 10^{-4}$ between the T=0 states, so the parity-violating M1 competes better so the circular polarization is larger ☺

- A hard-to-calculate nuclear matrix element is needed to extract the weak N-N physics. (We noticed the 0^- state involves excitations of the p shell, which is quite complicated.) The same effective operator contributes, with known β-decay constants of proportionality, to the forbidden β decay of the isobaric analog 0^+; $T = 1$ state in ^{18}Ne.

Summarized in Haxton PRL 46 698 (1981) and the experimental paper before it Adelberger, Hoyle, Swanson, Lintig 695

- The experimental asymmetry measured was $\sim 10^{-5}$, while in n(p,d)γ was 3x10^{-8}

TRIUMF

Physics and time reversal

When t $\rightarrow$ -t, does anything change?

- **Wave Equ. is 2nd-order in t:** $\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ **symmetric in t**
- **Heat Equ. is 1st-order in t:** $\nabla^2 u = -\frac{\partial u}{\partial t}$ **t$\rightarrow$ -t, boom?**

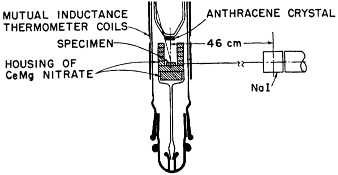
'Dissipation', like friction... The arrow of time remains a research problem in stat mech, but it's not from (known) particle physics

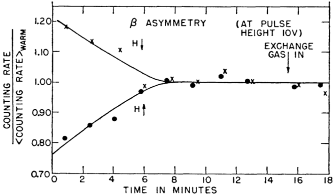
- **Schroedinger Equation is 1st order:** $i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$

'Take the complex conjugate' (as Wigner did above)
(but see Dressel et al. PRL 119 220507 (2017)
"Arrow of Time for Continuous Quantum Measurements")

Microscopic physics was thought to be symmetric in t

Parity broken, why not $\not{T}$ime?





Immediately after $\not{P}$arity was seen to be totally broken in β decay (‘ν left-handed’)

Wu, Ambler, Hayward, Hopper, Hobson,
PR 105 (1957) 1413

Many T-odd observables were proposed:

PHYSICAL REVIEW VOLUME 106, NUMBER 3

Possible Tests of Time Reversal Invariance in Beta Decay

J. D. Jackson,* S. B. Treiman, and H. W. Wyld, Jr.
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received January 28, 1957)

Need scalar triple products of 3 vectors: observables involving spin

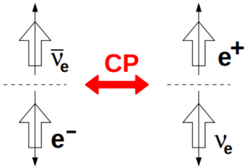
$$D\hat{J}\cdot\frac{\vec{p}_\beta}{E_\beta}\times\frac{\vec{p}_\nu}{E_\beta} \qquad R\vec{\sigma}_\beta\cdot\hat{\mathbf{J}}\times\frac{\vec{\mathbf{p}}_\beta}{E_\beta}$$

are consistent with $\not{T} < 0.001$

but some has been found $\rightarrow$

The Weak Interactions Can Also Violate CP

CP could be a good symmetry even if P and C were violated. Schematically



$$\Gamma(\pi^+ \to \mu^+ \nu_L) = \Gamma(\pi^- \to \mu^- \overline{\nu}_R) \quad ; \text{ CP invariance!}$$

Weak decays into hadrons, though, can violate *CP*.

There are "short-lived" and "long-lived" K states:

$$K_S \sim \frac{1}{\sqrt{2}}(K^0 - \overline{K}^0) \to \pi^+\pi^- \quad \text{(CP even)}$$

$$K_L \sim \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0) \to \pi^+\pi^-\pi^0 \quad \text{(CP odd)}$$

However, $K_L \to 2\pi$ as well! K_S and K_L do not have definite CP!

[Christenson, Cronin, Fitch, Turlay, PRL 13, 138 (1964).]

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Possible Tests of Time Reversal Invariance in Beta Decay

J. D. Jackson,* S. B. Treiman, and H. W. Wyld, Jr.
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$\not{CP}$ discovered in $K\bar{K}$ meson decays in 1963,
though not much (Cronin and Fitch Nobel prize 1980)

Quark eigenstates in the weak interaction:
To explain some weak decays we saw,

$|u\rangle \rightarrow |d\rangle + \epsilon|s\rangle$ **i.e.** $|u\rangle \rightarrow \cos(\theta_C)|d\rangle + \sin(\theta_C)|s\rangle$

Maybe one reason for 3 families of particles,
$\rightarrow$ 3x3 unitary "CKM" matrix between $|d\rangle, |s\rangle, |b\rangle$

There is one complex phase, which leads to this type of $\not{CP}$
Any 2x2 unitary matrix, one can define away the phase as trivial
A reason for 3 generations of particles?

That one phase is consistent with $\not{CP}$ in $K\bar{K}$ and $B\bar{B}$ systems

There have been hints in $K\bar{K}$ and $B\bar{B}$ of more $\not{CP}$ than in the standard model,

$p\bar{p} \to \mu^+\mu^+$ or $\mu^-\mu^-$ $\not{CP}$ at 3.6 σ Abazov PRD 2014 Fermilab;

so this 2001 cartoon was a little premature $\to$

J. Fabergé. *CERN Courier*. 6, No. 10, 193 (October 1966). [Courtesy of Madame Fabergé.]

T2K ν_μ oscillations different from $\bar{\nu_\mu}$ at 2 to 3 σ Nature 580 339 (2020)

$\not{CP}$ could have some utility for cosmology $\to$

The excess of matter over antimatter can come from $\not{CP}$

Sakharov JETP Lett 5 24 (1967) used $\not{CP}$ to generate the universe's excess of matter over antimatter:

- **$\not{CP}$,**
- **baryon nonconservation, and**
- **nonequilibrium.**

But known $\not{CP}$ is too small by 10^{10}, so 'we' need more to exist. **Caveats:**

- **You could use $\not{CPT}$ though there are no complete models [Dolgov Phys Rep 222 (1992) 309]**
- **We need $\not{CP}$ in the early universe, not necessarily now**

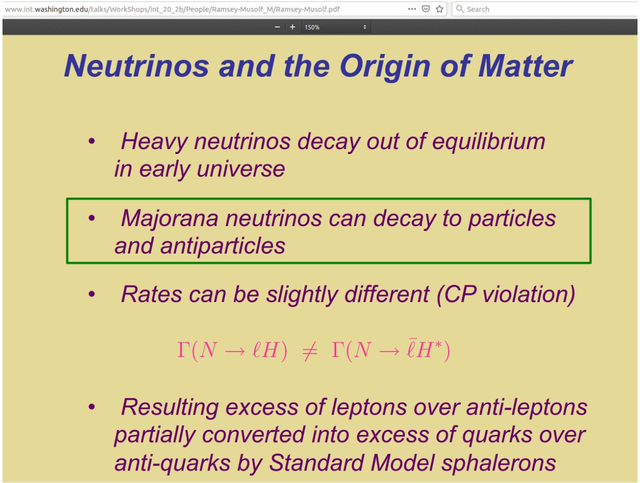
A concrete demonstrative example from Ramsey-Musolf at INT 2020

$\not{CP}$ explaining T2K's ν vs. $\bar{\nu}$ result lets heavy *N* decay this way in some models

Other mechanisms have much more abstract $\not{CP}$
Dine-Affleck models require non-SM physics, but not expicit particles, and don't need high-Temp early cosmology

I. Affleck and M. Dine, Nucl. Phys. B 249, 361 (1985)

So we look for more $\not{CP}$. How is this related to $\not{T}$?



$\not{T}$ is related to $\not{CP}$ by the "CPT Theorem"

"All local Lorentz invariant QFT's are invariant under CPT"
Schwinger Phys Rev 82 914 (1951)
Lüders, Pauli, Bell 1954

- **Gravity $\rightarrow$ not flat:**

K meson experiments Adler PhysLettB 364 (1995) 239 test $\not{CPT}$ to within 1000x expected from quantum gravity

direct tests include e^+e^- decay asymmetry $< 10^{-4}$ **Moskal Nat Comm 2021**

Proofs still pursued $\rightarrow$

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On the CPT theorem

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ABSTRACT

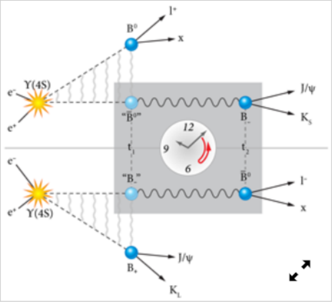
We provide a careful development and rigorous proof of the CPT theorem within the framework of mainstream (Lagrangian) quantum field theory. This is in contrast to the usual rigorous proofs in purely axiomatic frameworks, and non-rigorous proof-sketches in the mainstream approach. We construct the CPT transformation for a general field directly, without appealing to the enumerative classification of representations, and in a manner that is clearly related to the requirements of our proof. Our approach applies equally in Minkowski spacetimes of any dimension at least three, and is in principle neutral between classical and quantum field theories: the quantum CPT theorem has a natural classical analogue. The key mathematical tool is that of complexification; this tool is central to the existing axiomatic proofs, but plays no overt role in the usual mainstream approaches to CPT.

Assuming CPT, $\not{CP} \Leftrightarrow \not{T}$ in most physics theories
The matter excess then motivates $\not{T}$ searches

Entanglement in decays

There exists microscopic true $\not{T}$ in nature! independent of assumptions about QFT, CPT theorem, unitarity...

- **BABAR PRL 2012: Entanglement of B meson pairs enables $\psi_{\text{initial}} \leftrightarrow \psi_{\text{final}}$ also seen in K's KLOE-2 PLB 2023**



APS/Alan Stonebraker

Figure 1: Electron-positron collisions at SLAC produce a $\Upsilon(4s)$ resonance that results in an entangled pair of B mesons. When one meson decays at time t_1 , the identity of the other is "tagged" but not measured specifically. In the top panel, the tagged meson is a "$\overline{B}^0$". This surviving meson decays later at t_2 , encapsulating a time-ordered event, which in this case corresponds to "$\overline{B}^0$" $\rightarrow B_-$. To study time reversal, the BaBar collaboration compared the rates of decay in one set of events to the rates in the time-reversed pair. In the present case, these would be the "B_-" $\rightarrow \overline{B}^0$ events, shown in the bottom panel.

M. Zeller Physics 2012

EDM in a fundamental particle breaks *T*: this is exact

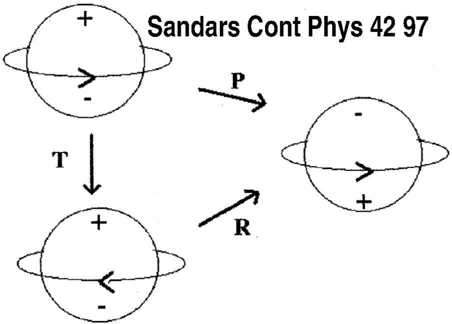
Landau, Nucl. Phys. 3 (1957) p. 127

Electric Dipole moment $\vec{d} = \sum q_i \vec{r}_i$

Since the angular momentum is the only vector in the problem, $\vec{d} = a\vec{J}$

Under *T*, $\vec{J} \overset{t \to -t}{\longrightarrow} -\vec{J} \quad \vec{d} \overset{t \to -t}{\longrightarrow} +\vec{d}$

If the physics is invariant under *T*, this is a contradiction, $\Rightarrow a = 0$



[• The other logical possibility: there are 2 states, with opposite sign of the EDM, and *T* just formally changes one state to the other.
For most fundamental particles, we know there aren't 2 states
Why do we know the electron doesn't have 2 states?
E.g. some polar molecules have a dipole moment listed in tables, which produces degenerate states and does not break *T* ...]

Schiff's Theorem: does a nuclear EDM make an atomic EDM?

Schiff's Theorem PR 132 2194 (1963): The nuclear electric dipole moment $d_{\rm nuclear} = \sum q_i r_i \hat{r}_i$ causes the atomic e^-'s to rearrange themselves so they develop an opposite dipole moment. In the limit of nonrelativistic e^-'s and a point nucleus, the e^-'s dipole moment exactly cancels the nuclear moment, so that the net atomic dipole moment vanishes.

(For the e^-'s EDM, there is 'antiscreening,' and $d_{\rm atom} \overset{Z\gg 1}{\gg} d_{e-}$ Sandars Phys Lett 14 194 (1965))

The Schiff moment S involves $\sum q_i r_i^2 \hat{r}_i$ does not get screened completely:
$\langle S\rangle = \sum q_i (r_i^2 - \frac{5}{3}\langle R_{\rm ch}^2\rangle) \approx R^2_{\rm nucleus} d_{\rm nucleus}$, so $d_{\rm atom}/d_{\rm nucleus} \sim R^2_{\rm nucleus}/R^2_{\rm atom} \sim 10^{-8}$
Combination of Large Z and relativistic wf's offset by $10\, Z^2 \approx 10^5$, with overall suppression of $d_{\rm atom} \sim 10^{-3} d_{\rm nucleus}$

Best measurements in diamagnetic (atomic total angular momentum 0) ^{199}Hg constrain strong interaction $\not{T}$ competitive with neutron EDM.

A nuclear magnetic quadrupole moment is also $\not{T}$. This also produces an observable atomic EDM, yet with no screening Haxton+Henley PRL 51 1937 (1983), so it's more accurate to interpret experiments. (The total atomic angular momentum must be nonzero, so stray Larmor precession of 1000x greater μ makes experiments challenging.)

$\not T$ in QCD and nucleon-nucleon interactions

$\mathcal{L}_{\not{CP}} = \theta_{QCD} \frac{g^2}{32\pi^2} F^{\mu\nu}_{\alpha} F^{*}_{\alpha\mu\nu}$

Neutron EDM bounds $\Rightarrow \theta_{QCD} \lesssim 0.5x10^{-10}$

Peccei-Quinn mechanism drives SM $\not{CP}$
$\bar\theta = \theta_{QCD} - \mathrm{arg det}(Y_u Y_d)$ zero by a global U(1):
breaking the U(1) produces a 0^- axion p-GB with
m $\propto$ (symmetry-breaking scale)/(coupling).
Null experiments drive that scale high.

Nelson-Barr models keep $\bar\theta$ small though CKM angle is large (no axions)
Im(det(CKM)) $\propto \eta \sin^6(\theta_C) \sim 3x10^{-4}$ (two numbers same to ppm)

Other $\not T$ physics in the N-N potential is parameterized by isoscalar, isovector, and isotensor terms, with a separate set for whether or not they break P.
(Terms can be related by chiral EFT deVries fphy.2020.00218 or the $1/N_c$ expansion Samart PRC 94 024001 (2016))
θ_{QCD} is isoscalar

The QCD and effective nucleon-nucleon $\not T$ physics produces:

- $\not T$ nuclear Schiff and magnetic quadrupole moments,
- $\not T$ asymmetries in polarized beam experiments (Simonius PRL 78 4161 (1997))
- $\not T$ asymmetries in polarized neutron experiments on polarized targets ($\lesssim 10^{-5}$ Huffman et al. PRC 55 2284 (1997), with plans to improve these at next-generation neutron sources enough to complement n and ^{199}Hg EDM experiments.)

If one sees these asymmetries, they are from $\not T$: unlike decays, they are free of 'final-state interaction' false effects.

$\not{T}$ EDM measurements, Schiff moments, and octupole enhancment

Outline from Engel arXiv:2501.02744 **Ann Rev Nucl Sci 2025:**
A more formal argument for why EDM's are $\not{T}$

Why the SM CKM phase makes tiny EDM's beyond reach of present experiments

(see article for Engel Friar Hayes' general proof of Schiff shielding))

Chiral N-N EFT extension to a $\not{T}$ interaction and Calculation of Schiff moment

Simple single-particle mean-field model and why it fails

- **why the difficult $\sigma \cdot p$ effective interaction appears there**
- **Core excitation in HF and HF Bogoliobov models**
- **Need for SRG and CC calculations $\rightarrow \psi$'s of predictable accuracy**

Jon Engel arXiv:2501.02744 makes no appeal to the dipole moment being aligned with spin as QM vectors nor formal use of Wigner-Eckart theorem as in Atomic Physics by Kimball Budker DeMille

The argument goes as follows: The time-reversal operator T, because it reverses angular momenta, takes normalized states with well defined angular momentum J and projection M into normalized states with J and $-M$. Thus, if time-reversal symmetry is conserved, one must have within a rotational multiplet $|J,M\rangle$ of definite energy,

$$\begin{aligned}\langle J,M|D_z|J,M\rangle &= \langle J,M|T^{-1}TD_zT^{-1}T|J,M\rangle \\ &= \langle J,-M|TD_zT^{-1}|J,-M\rangle \\ &= \langle J,-M|D_z|J,-M\rangle\ ,\end{aligned} \qquad 3.$$

where the last equality holds because $\boldsymbol{D}$, which depends only on positions, is even under time reversal. The operator R_π that rotates around the x axis by π also takes $|J,M\rangle$ to a phase times $|J,-M\rangle$, but but $\boldsymbol{D}$ is odd under this operation. Thus

$$\begin{aligned}\langle J,M|D_z|J,M\rangle &= \langle J,M|R_\pi^{-1}R_\pi D_z R_\pi^{-1}R_\pi|J,M\rangle \\ &= -\langle J,-M|D_z|J,-M\rangle\ .\end{aligned} \qquad 4.$$

Equations 3 and 4 together imply that $\langle J,M|D_z|J,M\rangle = 0$. The argument breaks down if time-reversal symmetry is violated because in that case the state $|J,-M\rangle$ in the second and third lines of Eq. 3 need not belong entirely to the same rotational multiplet as the state $|J,M\rangle$, and thus need not be the same state as $|J,-M\rangle$ in Eq. 4.

detectable EDM will have to be caused by $\mathcal{L}_{\bar{\theta}}$ or physics beyond the Standard Model, even with the amazing experimental sensitivity that is already possible. The reason is that the CKM phase causes a change of flavor and so flavor-diagonal quantities such as EDMs require Feynman diagrams with several loops to produce a non-zero result. **Figure 1** below shows one of the leading diagrams (12) in the expression for the neutron EDM, which the result of a full calculation reveals (13) to be about $10^{-32}e$ cm. Experiments looking for a new flavor-conserving source of CP violation will have to increase their sensitivity by several orders of magnitude before background from the CKM phase becomes an issue.

New physics needs more than 1 phase, but then often makes an EDM in 1 loop (sensitive to 50 TeV scale) or 2 loops (2 TeV)

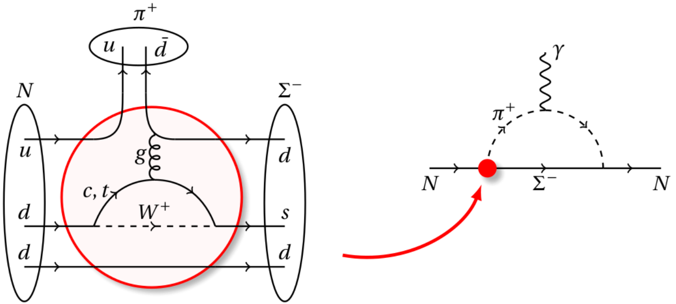


Figure 1

A leading diagram in the Standard Model for the neutron EDM caused by the CKM phase.

The N-N $\not{T}$ interaction is like chiral EFT. Note the nucleon spins and the gradient

At leading order in χEFT, the usual strong nuclear potential contains one-pion-exchange and contact nucleon-nucleon interactions. The same is often true of the leading-order P- and T-violating potential V_{PT} (19, 20), though exactly which terms are leading depends on the underlying sources of CP violation. With the use of the strong pion-nucleon coupling $g \approx 13.3$ in the definition instead of $g_A m_N / f_\pi$, which is equal to g to within a few percent, the pion-exchange part (always occurring at leading order) is

$$V^{\pi}_{PT}(\boldsymbol{r}_1 - \boldsymbol{r}_2) = \frac{g}{2m_N}\Big\{ [\bar{g}_0\, \vec{\tau}_1 \cdot \vec{\tau}_2 + \bar{g}_2(3\tau_{1z}\tau_{2z} - \vec{\tau}_1 \cdot \vec{\tau}_2)]\,(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) - \bar{g}_1\,(\boldsymbol{\sigma}_1\tau_{1z} - \boldsymbol{\sigma}_2\tau_{2z}) \Big\} \cdot \boldsymbol{\nabla} Y(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)\,, \quad 5.$$

We know OPEP comes from chiral EFT. This is OPEP with TRV couplings.

where

$$Y(r) = \frac{e^{-m_\pi r}}{4\pi r}\,, \quad 6.$$

Note the sigma and gradient terms

and the $\bar{g}_i$ are unknown CP-violating versions of g that depend on the underlying source of the violation. For special sources, e.g., the $\bar{\theta}$ term in Eq. 1, theorists have used lattice QCD to compute the constant $\bar{g}_0$ (21, 22), obtaining the value $\bar{g}_0 = (15.5 \pm 2.6) \times 10^{-3}\bar{\theta}$. The other couplings are harder to calculate, though Ref. (23) used resonance saturation to conclude, again for the $\bar{\theta}$ source, that $\bar{g}_1/\bar{g}_0 \approx -0.2$.

Pions are the pseudo-Goldstone bosons associated with the spontaneous breaking of chiral symmetry, and for sources of CP violation that conserve chiral symmetry at the quark and gluon level — i.e. in Standard-Model effective field theory — pion-exchange potentials of the form in Eq. 5 are suppressed. For such sources, a contact interaction with two parameters contributes at the same order as the suppressed pion exchange:

The usual contact term

$$V^{\delta}_{PT} = \frac{1}{2}\left[\bar{C}_1 + \bar{C}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\right] (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \boldsymbol{\nabla}\delta^3(\boldsymbol{r_1} - \boldsymbol{r_2})\,. \qquad 7.$$

There are other contact interactions, not shown here, that never contribute at the same order as pion exchange. In addition, according to Standard-Model EFT, $\bar{g}_2$ is suppressed compared to $\bar{g}_0$ and $\bar{g}_1$, no matter what the underlying source of CP violation. For a review of EFT for P- and T-violating interactions and operators, see Ref. (20). deVries et al. Frontiers in Physics 8 (July) (2020)

The most important result of all these considerations for the interpretation of experiments on atoms or molecules is that we can proceed to compute the effects of nuclear CP violation on EDMs as functions of a few important χEFT parameters, without worrying about the underlying source of CP violation. We will see how to do so shortly.

(Does using chiral EFT mean the underlying $\not{T}$ physics must be at $<$ GeV? Or $\gg$ GeV?)

$$S = \sum_n \frac{\langle g|S_z|n\rangle_{J,J}\,\langle n|V_{PT}|g\rangle_{J,J}}{E_g - E_n} + c.c\,,$$

Perturbation theory gives: **with Schiff moment *S*:**

$$S = a_0 g\bar{g}_0 + a_1 g\bar{g}_1 + a_2 g\bar{g}_2 + A_1\bar{C}_1 + A_2\bar{C}_2 + a_p d_p + a_n d_n\ ,$$

Simplest assumptions: mean field, single valence particle, no core excitations, zero-range pions, exchange terms unimportant ⇒

$$U_{PT} = \frac{\varepsilon}{M_N m_\pi^2}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\rho\tau_z\,,\qquad \varepsilon = \frac{g}{2}\left[\left(\frac{N-Z}{A}\right)(\bar{g}_0 + 2\bar{g}_2) - \bar{g}_1\right].$$

perturbing Hamiltonian is proportional to $[\boldsymbol{\sigma}\cdot\boldsymbol{p}, U_0] = -i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}U_0$ to analytically evaluate the sum in the perturbation-theory expression in Eq. 21 for the state of the last (valence) nucleon, obtaining

$$|\tilde{\psi}_{lj}\rangle = \left(1 + i\varepsilon\frac{\rho(0)}{M_N m_\pi^2 U_0(0)}\boldsymbol{\sigma}\cdot\boldsymbol{p}\right)|\psi_{lj}\rangle\,, \qquad 25.$$

this leads to crude estimate *S* = 0 for n's (!? not so in better calculations),

$$S^{\text{ch}} \approx |e|\frac{\left[1 \pm \left(j+\frac{1}{2}\right)\right]}{j+1}A^{2/3}\times 10^{-2}\varepsilon\ \text{fm}^3\,,$$

and for p's .

For the odd-proton ^{199}Hg, this independent particle estimate agrees pretty well with large-scale shell-model calculations, though RPA (HF + 1-particle excitation uncorrelated excitations) and QRPA (HFB +2-particle excitations, i.e. adding phenomenological pairing to the Hamiltonian to be minimized) undershoot by large factors.

This is where the general form $\sigma\cdot p$ comes from for the $\not{T}$ N-N interaction. This 'spin hedgehog' operator has no benchmark observable and needs good wf tails with spin knowledge to calculate.

For octupole-deformed nuclei, the perturbation theory calculation *S*

$$S \approx \frac{\langle g|S_z|\bar{g}\rangle_{J,J}\,\langle \bar{g}|V_{PT}|g\rangle_{J,J}}{E_g - E_{\bar{g}}} + c.c$$
$$= -2\frac{J}{J+1}\frac{\langle S_z\rangle_{\rm int}\,\langle V_{PT}\rangle_{\rm int}}{\Delta E},$$

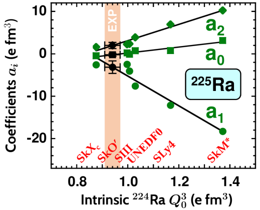


Figure 4

Coefficients a_i, in units of $|e|$ fm^3, in ^{225}Ra for six Skyrme functionals and propagated to the measured octupole moment in ^{224}Ra

is dominated by one low-lying state of opposite parity with similar nuclear structure, and deformation that enhances the r^2 weighting of the *S*. The $\langle g|S_z|\bar{g}\rangle$ is close to the classical Schiff moment of the pear-shaped charge distribution.

$\langle\bar{g}|V_{PT}|g\rangle$ is the tough part, e.g. because $\sigma \cdot p$ needs good ψ's. RPA and QRPA predict two to three orders of magnitude more sensitivity to microscopic $\not{T}$ for ^{225}Ra than ^{199}Hg, though varying the NN interactions produces linear correlations with observable strength of even-even 0+ to 3- transitions that actually pass through zero for a_0, a_1, a_2. This is pretty scary, so Engel wants a calculation where he can estimate uncertainties. He mentions near-term SRG variations (including that diagonal+nondiagonal generator mentioned by Sagawa) that also evolve the interaction to include high-energy intermediate states. He also mentions Coupled Cluster as ways to get the sigma dot p operator with reliable matrix elements.

Hergert said at APS Anaheim there is a ^{225}Ra calculation working that he couldn't tell us about.

Octupole deformation **P. Butler review**

J. Phys. G: Nucl. Part. Phys. **43** (2016) 073002 Topical Review

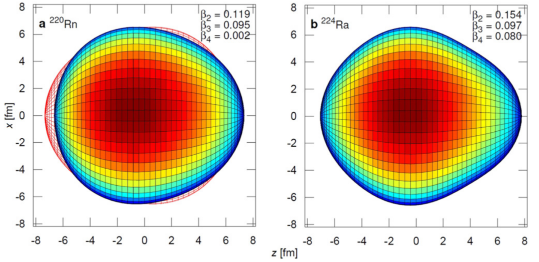


Figure 2. Graphical representation of the shapes of ^{220}Rn and ^{224}Ra. (a), ^{220}Rn; (b), ^{224}Ra. Panel (a) depicts vibrational motion about symmetry between the surface shown and the red outline, whereas (b) depicts static deformation in the intrinsic frame. Theoretical values of β_4 are taken from [3]. The colour scale, blue to red, represents the y-values of the surface. The nuclear shape does not change under rotation about the z axis. Figure reprinted from [4]. Copyright 2013, Rights Managed by Nature Publishing Group.

J. Phys. G: Nucl. Part. Phys. **43** (2016) 073002 Topical Review

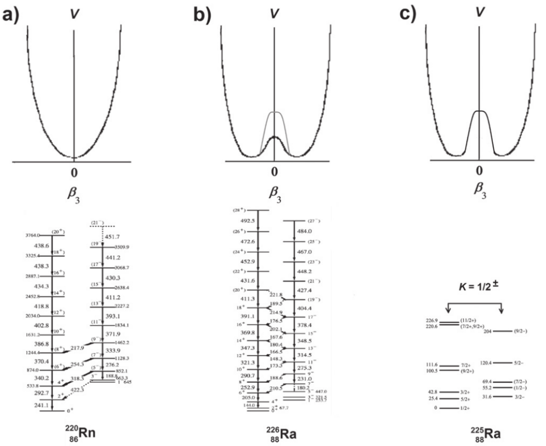


Figure 3. Nuclear potential energy as a function of octupole deformation β_3 for an octupole vibrator (a) and a system having permanent octupole deformation (b) and (c). For nuclei having permanent octupole deformation, the barrier increases with angular momentum as pairing is reduced. Figure (c) represents a nucleus where the barrier is high in the ground state, which would be the case for odd-A nuclei. The three cases (a)–(c) are illustrated by actual nuclei ^{220}Rn [8], ^{226}Ra [8] and ^{225}Ra (adapted from [9]). For the last case, the decoupling parameters a for the $K = \frac{1}{2}^{+}$ and $K = \frac{1}{2}^{-}$ bands have opposite sign [11, 12].

Single-particle mean field picture:

One way to generate octupole deformation is with nearly degenerate pair of single-particle orbitals differing by orbital angular momentum Δj=Δl=3, e.g. $\pi(h_{11/2}), \pi(d_{5/2})$ for $Z > 50$ and $\nu(i_{13/2}), \nu(f_{7/2})$ for $N > 82$

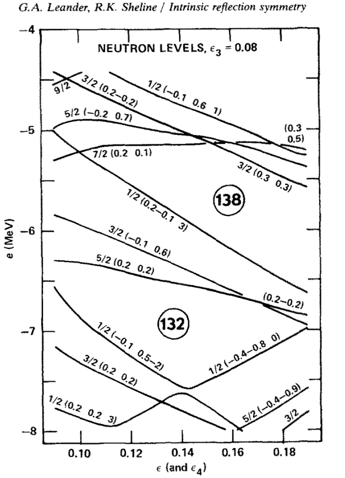
Such states approach each other and the Fermi surface when either Z or N $\approx$ 34, 56, 88, 134, i.e. at values just greater than spherical magic numbers

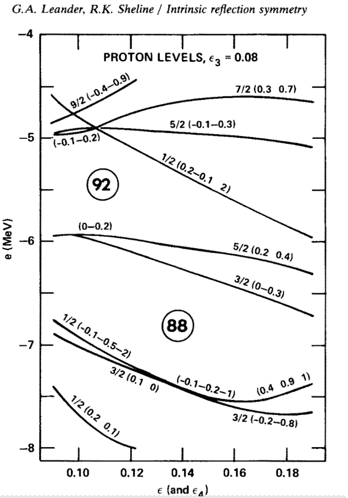
$^{225}_{88}Ra^{137}$ $^{223}_{88}Ra^{135}$ $^{227}_{90}Th^{137}$ $^{223}_{87}Fr^{136}$ $^{223}_{86}Rn^{137}$ $^{221}_{86}Rn^{135}$ $^{229}_{90}Th^{139}$

Simple pictures are helpful for orientation, but it for collective physics one has to excite the core

"reflection asymmetric" $\beta_3 \neq 0$ changes shell gaps

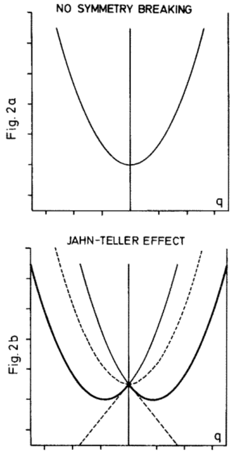
Leander Sheline NPA413 1984 375



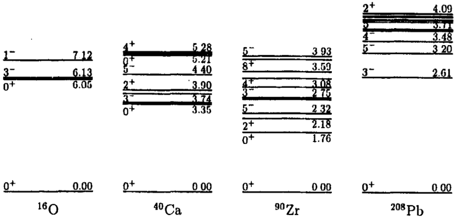


Octupole physics

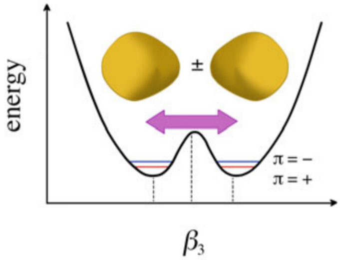
A good e.g. of the Jahn-Teller effect



Octupole ΔJ^{π}=3^{-} vibrations can be excited in many nuclei:



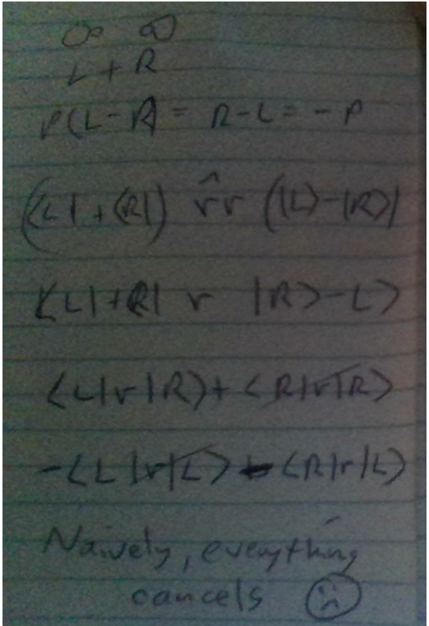
Wong Fig 6.2



Obertelli and Sagawa Fig. 7.33

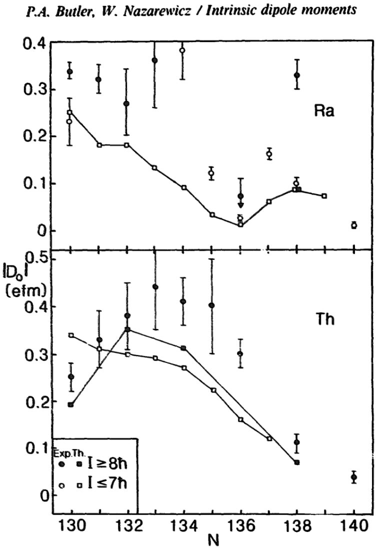
- Pear-shaped nucleus is $\not{P}$. Must construct good-*P* wf's: $\psi_{\pm} = (|a\rangle \pm |b\rangle)/\sqrt{2}$ A parity doublet of otherwise nearly identical states, each with a band of same *P*.

- Relatively large E1's between the parity-doubled states can indicate octupole phenomena, e.g.: Bohr Mottleson NP4 529 (1957) citing R. Christy phenomenologically get $D \propto (\frac{e^2}{R_0}\frac{A}{C_1})Z\beta_2\beta_3 eR_0$ The resulting E1 rate works for either $\langle\beta_3\rangle^2$ (static octupole deformation) or $\langle\beta_3^2\rangle \neq 0$ for octupole vibrations. But the converse is not so: Bohr Mottleson NP9 687 (1958), Strutinski Nucl Energy 4 (1957) 523 E1 size depends on details like surface n, p distributions



Butler Nazarewicz NPA522 1991 249

Naive cancellation might be under control in macroscopic models (with “Strutinsky shell corrections”) but note the absolute sign



Oc Def Schiff moment enhancement

The low-lying parity doublet:

- **enhances mixing of opposite-parity states**
- **enhances the resulting Schiff moment because of the octupole and quadrupole deformations.**

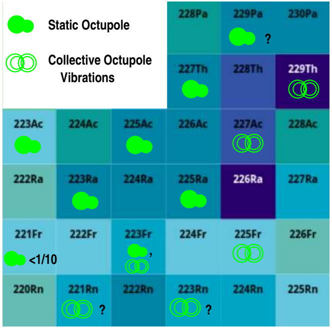
Schematic model estimates typically:

$$S \propto \frac{J}{J+1}\beta_2\beta_3^2 Z A^{\frac{2}{3}} \frac{1}{E^- - E^+} e\eta$$ Spevak PRC 56 1357 (1997)

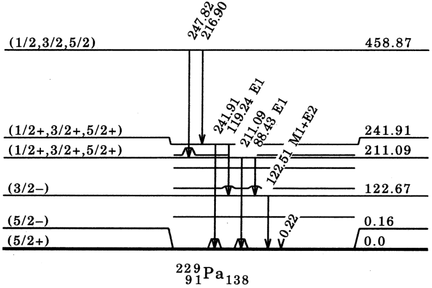
Result is 100-1000 x enhancement over ^{199}Hg Graner PRL 116 161601 (2016), **$\sim$ restoring the full effect of the nuclear EDM, and in one case 10^4 or 10^5 enhancement going beyond (if a low-lying state is really the same J with opposite π).**

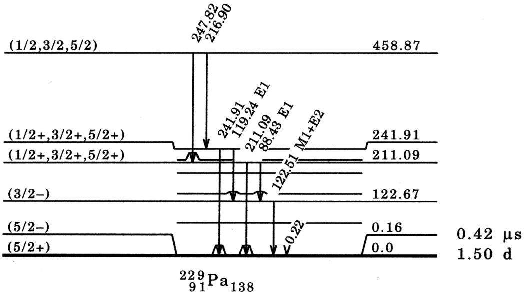
- These models treat critical effective $\not{T}$ NN interaction $\langle f|\sigma \cdot p|i\rangle$ macroscopically, an unresolved order-of-magnitude uncertainty in the best self-consistent mean field calculations Dobaczewski PRL 121 232501 (2018)
- Macroscopic models can't say what % of ψ is the octupole configuration. Spevak (Auerbach) estimate this effect, which greatly dilutes Schiff moment enhancement in nuclei without octupole collectivity
- As with E1's, there are similar enhancements from $\langle\beta_3\rangle^2$ and $\langle\beta_3^2\rangle$, though the calculations are more complex for octupole vibrations

clear treatment: Engel Friar Hayes PRC61 035502 (2000)



All demonstrated static octupole deformation is in radioactive nuclei Behr arXiv 2203.06758 resource letter





Berman Fultz RMP 47 713 (1975):

The Thomas–Reiche–Kuhn (TRK) sum rule (see Levinger, 1960) is an expression giving the total integrated cross section for electric dipole photon absorption, in the absence of exchange forces, and is given by

$$\int_0^\infty \sigma(E)\, dE = \frac{2\pi^2 e^2 \hbar}{Mc}\frac{NZ}{A} = 60\left(\frac{NZ}{A}\right) \text{MeV mb},$$

^{229}Pa could have an atomic EDM enhanced by its Schiff moment by $\sim 10^5$ times, because of its tiny parity doublet splitting and octupole phenomena

However, the 5/2$^-$ state has not been identified. The splitting is known to be 60 $\pm$ 50 eV

Ahmad PRC 024313 (2015) "with this large uncertainty the existence of the parity doublet is not certain"

J. Singh from MSU proposes measuring that photon directly

Very little is known about atomic levels of Pa

If that is a parity doublet, what is known about the E1 strength in ^{229}Pa suggests the E1 between them would take up the entire Thomas-Reiche-Kuhn electric dipole sum rule

Haxton, MSU/FRIB EDM 2019 workshop.

Nuclear nearest-level spacing and $\not{T}$

Bohr and Mottleson 2C-2:
Assume a Hamiltonian matrix with random values, the Gaussian Orthogonal Ensemble (GOE). Diagonalizing the Hamiltonian produces a statistical distribution of level spacings ϵ in terms of average spacing D (the "Wigner distribution")

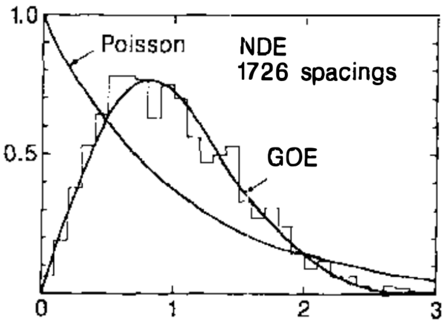
$$P(\epsilon) = \frac{\pi}{2D^2}\epsilon e^{-\frac{\pi}{4}\frac{\epsilon^2}{D^2}}$$

$$P(\epsilon) \overset{\epsilon\to 0}{\propto} \epsilon$$

This was for time-reversal invariant interactions. If you allow for $\not{T}$, you have unitary matrices instead, the Gaussian Unitary Ensemble (GUE) with twice as many elements, because they're complex. Then

$$P(\epsilon) \overset{\epsilon\to 0}{\propto} \epsilon^2$$

Bohigas+Weindenmüller Ann Rev Nucl Part Sci 38 421 1988



More sophisticated statistical measures extract an upper limit for the amount of $\not{T}$ in nuclear interactions $\alpha \lesssim 2\times 10^{-3}$ (J.B.French Ann. Phys. 181 235 (1988)). It's treated as an upper bound, since nuclear level spacings are not necessarily random ☺

SM 2nd-order weak $\nu\nu\beta\beta$ vs $\not{SM}$ $0\nu\beta\beta$ decay

We've already seen *SM* $\beta\beta\nu\nu$ decay. 1st measured geochemically, then directly in very-low-background experiments. In $0\nu\beta\beta$ all energy is captured, a distinctive signature.

Kayser Journal of Physics: Conference Series 173 (2009) 012013;
Primakoff and Rosen 1959 Rep. Prog. Phys. 22 121

Particle physics for $0\nu\beta\beta$ to happen:

- Lepton number must not be conserved
- ν's have mass ?

Cleanest statement: The non-SM physics that produces $0\nu\beta\beta$ generates a Majorana mass term.

A mass term for a Dirac ν_L needs a ν_R:
$m\bar{\nu}\nu = m(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L)$
(because $\bar{\nu}_L\nu_L = \bar{\nu}_R\nu_R = 0$ 'not its own antiparticle')

Majorana particles are their own antiparticles, making natural a mass term $\propto m\overline{\nu_L^C}\nu_L$
(blithely ignoring much interesting physics formalism!)

Diagonalizing the Dirac+Majorana mass matrix then economically generates the light ν_L mass observed and very heavy ν_R: seesaw mechanism but is not the only possibility

Schechter, Valle PRD 25 2951 (1982)

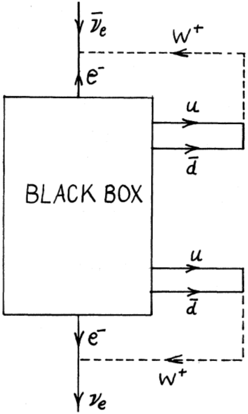


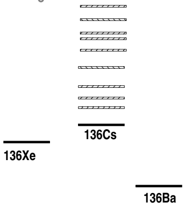
FIG. 2. Diagram showing how any neutrinoless double-β decay process induces a $\bar{\nu}_e$-to-ν_e transition, that is, an effective Majorana mass term.

Schechter and Valle qualify in text: some other physics could precisely cancel this diagram and keep the Majorana mass term 0.
All variations (e.g. photon exchange between u and $\bar{d}$) must be cancelled, "extremely unlikely."

$0\nu\beta\beta$ rate, if due to exchange of light $\nu' s$:
$\Gamma = G|M|^2(\sum_i U_{ei}^2 m_i)^2$ if due to exchange of light ν's,
where G is the lepton phase space factor (trivial),
U_{ei} the ν mass mixing matrix,
M the nuclear matrix element (hard to calculate).
Suhonen Front. Phys. 5 art 55 p.1 (2017)

- **0^+ parent, progeny, $\nu\nu\beta\beta$ dominated by 1^+ intermediate states, GT transitions.**
- **$0\nu\beta\beta$ has contributions from forbidden operators and more spins, so $\nu\nu\beta\beta$ is not a complete benchmark for theory.**
- **Formally, two-γ emission from an excited state also sums over virtual states, QRPA developed for these experiments** Schirmer PRL 53 1897 (1984) **but operators and states are different.**
- **A variety of approximate many-body answers vary by 2-4$\times$. ^{48}Ca calculable by complete many-body methods $\rightarrow$**

(Boehm+Vogel "Physics of massive ν's" **crudely set non-rel $F(Z,E) \sim \frac{E}{p}\frac{2\pi\alpha Z}{1-e^{-2\pi\alpha Z}}$ (sort of ok for spectrum, poor for rates) to allow analytic phase space integrals $\sim E_0^{11}$ for $\nu\nu\beta\beta$ and $\sim E_0^5$ for $0\nu\beta\beta$)** So lower E_0 suppresses $\nu\nu\beta\beta$ 'background' but increases natural bkg wrt 2.6 MeV γ's. This is part of SNO+ and nEXO isotope choices



A contact term from chiral EFT changes the nuclear calculation

Cirigliano PRL 126 172002 (2021)

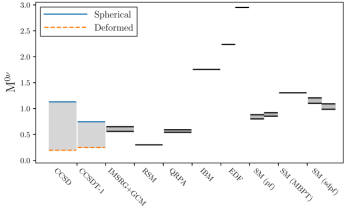


FIG. 1. Comparison of the NME for the $0\nu\beta\beta$ decay of ^{48}Ca,

Novario PRL 126 182502 (2021)

β-ν correlation from recoil momentum spectrum Kofoed-Hansen Dan. Mat. Fys. Medd. 28 nr9 (1954)

The recoil momentum spectrum is straightforward and analytic:
If we write angular distribution in terms of E (β total energy), θ (β-ν angle), p (β momentum), q (ν momentum) (it's understood we have to evaluate q to conserve energy-momentum; it's not a free parameter)

$$P(E,\theta)dEd\Omega_\theta =$$

$$F(Z,E)pEq^2\left(1+\frac{b}{E}+a\frac{p}{E}\cos\theta\right)dEd\Omega_\theta$$

Then if the recoil momentum is r, energy conservation E+q=E_0 (E_0=Q+m_β), then we just use law of cosines:

$$p^2+q^2+2pq\cos\theta = r^2$$

differentiate θ with respect to r:

$$|\sin\theta d\theta| = 2d\Omega_\theta = \frac{r}{pq}dr$$

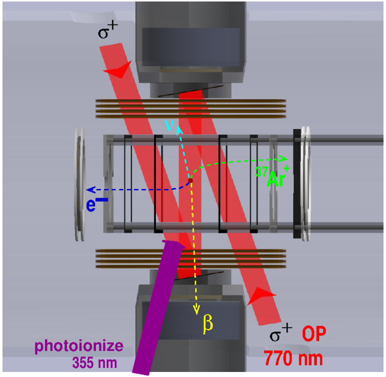
we immediately get the recoil momentum spectrum
$P(E,r) =$

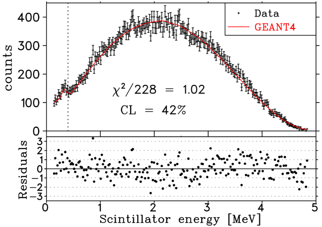
$$\frac{1}{2}F(Z,E)\left(rEq+brq+r\frac{a}{2}(r^2-p^2-q^2)\right)dEdr$$

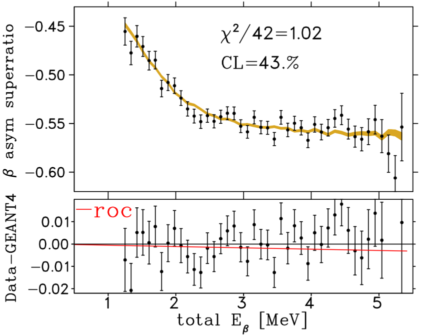
at fixed E, it's linear in recoil energy R
$P(E,R)dEdR =$

$$\frac{M}{2}F(Z,E)\left(Eq+bq+\frac{a}{2}(2MR-p^2-q^2)dEdR\right)$$

β^+ asymmetry ^{37}K data







Fenker et al. Phys Rev Lett 120, 062502 (2018)

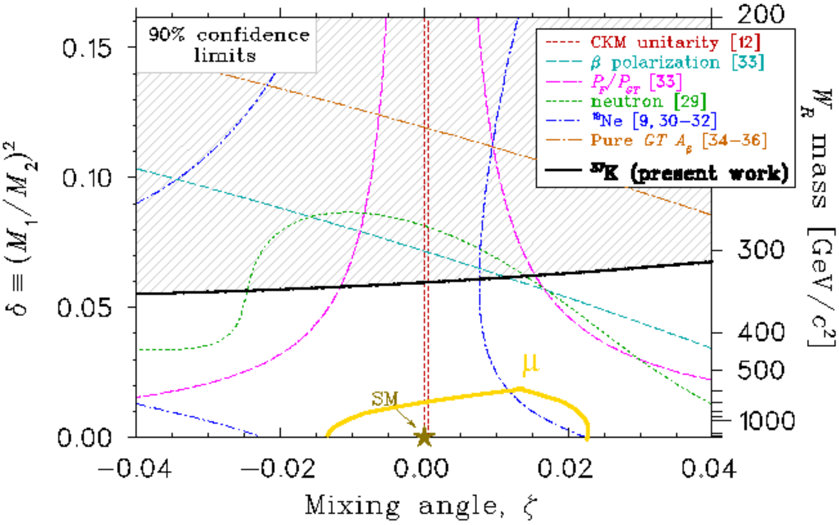
A_β[experiment]= -0.5707 $\pm$ 0.0019

A_β[theory] = -0.5706 $\pm$ 0.0007

theory prediction needs GT/F ratio from $t_{1/2}$

The best fractional accuracy achieved in nuclear or neutron β decay

Still no wrong-handed ν's



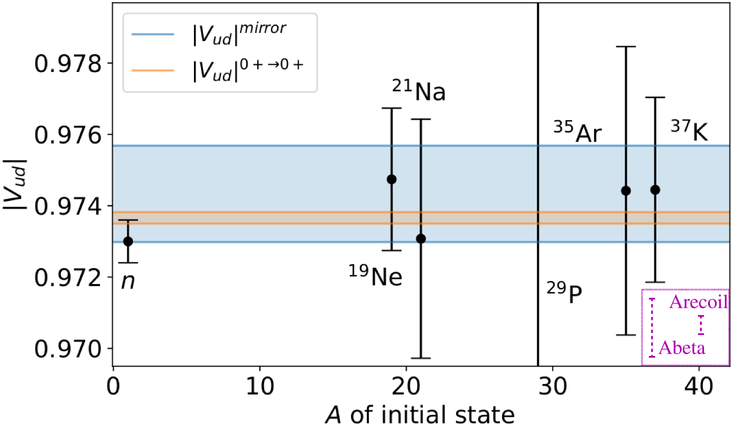
Extra W' with heavier mass, couples to wrong-handed ν_R

We can evade TWIST limits by assuming the muon ν_R is heavy

LHC $M'_W > 3.7$ TeV 90%

Weak interaction: same strength, all nuclei?

TEXAS A&M UNIVERSITY



Hayen and Severijns, arXiv:1906.09870 (June 2019)

Deduced V_{ud} from mirror decays

Are people overestimating their uncertainties? We aren't ☺

We project to reach 0.0005 accuracy, as good as any $0^+ \to 0^+$ except ^{26m}Al.

Assumes 5% isospin breaking calculation.