Qualitative info for Nucleon-Nucleon Interaction

- Historical motivation for N-N forces
- Classical 3-body forces: using 3-body forces does not make QCD special
- Renormalization and $V_{rmlow\ k}$

  Integrate out high-momentum components: an example
- Chiral Effective Field Theory

  A controlled expansion in a small parameter

  $\rightarrow$ one derivation of One-Pion-Exchange-Potential

(• Other topics like lattice QCD and Witten's 1/Nc expansion will also be mentioned below qualitatively)

Motivation to understand Nucleon-Nucleon interaction

Bethe 1953 Sci. Am.

strength of interaction between nucleon and meson required to explain nuclear forces is about the same as that required to explain the scattering of mesons by protons. *About* the same—actually, the two numbers differ by approximately 50 per cent, but probably this difference is simply a measure of our mathematical ineptness in dealing with large forces. Thus the indications are that physicists are on the right track in explaining nuclear forces by transfer of mesons. But it will be a long time before our mathematical tools are developed sufficiently to determine whether the meson theory *really* explains the forces in all details.

Dirac 1962 ("The Strangest Man" G. Farmelo)

1958–1962

FEYNMAN: I am Feynman.
DIRAC: I am Dirac. [*Silence*]
FEYNMAN (*admiringly*): It must have been wonderful to be the discoverer of that equation.
DIRAC: That was a long time ago. [*Pause*]
DIRAC: What are you working on?
FEYNMAN: Mesons.
DIRAC: Are you trying to discover an equation for them?
FEYNMAN: It is very hard.
DIRAC (*concluding*): One must try.[36]

Scattering theory extension to multiple particles (S-matrix) has elegant symmetries and some powerful results, yet we're being encouraged 'Don't give up on understanding the interactions' ☺.

Classical 3-body forces

- For pointlike masses, gravity has only 2-body forces

Moon raises tides on Earth, which change its interaction with the Sun: one could treat full gravity exactly numerically, but pointlike bodies + a tidal 3-body force is helpful computationally and for understanding.

Intuitive features of 3-body tidal force include:

- Moon raising tides on Earth also changes the Moon-Earth interaction, so you just need 2 bodies (JB: is the deuteron affected by 3-body N-N forces?).
- Tides come from the difference in the Moon's pull of the closer and farther parts of the Earth, so you take one derivative and the tide-induced interaction $\propto 1/r^2$ potential, not $1/r$ $\rightarrow$ a 'derivative' term in the Lagrangian
- Similarly, forces in a molecular dimer come from one atom exciting an electric dipole (virtually) in the other, making $1/r^3$ and $1/r^6$ potentials.
- So the actual potential $\sim 1/r$, but phenomenological 3-body forces have different dependence. This does not mean anything special about gravity or E&M, i.e. no weird extra particles exchanged or anything like that. These analogies go over to the N-N interactions, which are less perturbative so 3-body terms are more complex.

Renormalization and $V_{\mathrm{low\ k}}$

Consider the renormalization handout from Shuryak's book

Because of the hard core repulsion of the N-N force and the need to consider momentum dependence,
how one treats the 2-body interaction changes the strength of the 3-body interactions. JB may not be able to clarify this much.

Chiral Effective Field Theory

After the community worked another 4 decades after Bethe 1953,

Weinberg in 1991 started something very helpful, developed by Van Kolck and others. There had already been at least one theory, quantum hadrodynamics, considering nucleons and pions as the fields. (See papers by Walecka et al. Walecka was very proud at an NSF summer school around 1988 to be getting the sign of the spin-orbit coupling!)

Weinberg noticed such an EFT could declare a QCD scale of about 1 GeV and do a controlled expansion in ('momenta'/1 GeV). The resulting EFT includes symmetries of the underlying QCD, including a rather subtle one termed 'chiral symmetry.'
This EFT organizes the usual invariant terms of 'this dot that,' organizing into orders of the expansion parameter, and provides relations between terms of same order in the expansion parameter.
Hammer RMP p. 3: Chiral EFT is challenging to renormalize. "pionless (i.e. contact) EFT" is easier that way, and can be applied for problems with momenta much less than the pion mass (Hammer RMP...)

Culture: Chiral EFT at textbook level

Obertelli and Sagawa "Modern Nuclear Physics" 2021 has a textbook systematic consideration of chiral EFT https://link.springer.com/book/10.1007/978-981-16-2289-2

- **Lays out, developing QFT and diagrams as they go, chiral EFT for the N-N interaction. Writing down explicit Lagrangians that are 1↔1 with diagrams. [• They introduce for their Lagrangian $U(x) = exp(\frac{i}{f_\pi}\tau \cdot \pi(x))$ as their 'chiral field.' This naturally gives many powers of the π interactions, yes, but I'm unable to find where this comes from. (I expected powers of the pion interactions as a consequence of higher-order perturbation theory...)]**
- **Problem 2.6 derives, from the lowest-order π and nucleon terms in the Lagranginan in momentum space, the expression for OPEP in position space. They provide a solution, and the Fourier transforms needed are instructive. This constitutes a derivation of OPEP. It might help your future understanding to read through it and the equations referenced that lead to it.**
- **The next order nonvanishing order, ('momenta'/'1GeV')^2, produces spin-orbit coupling, and eq. 2.80 has most or all of the normal terms I'll lay out in the older approach (Wong). It takes another power to get 3N forces.**

Lecture 6 Nucleon-Nucleon interaction; Effective Field Theories

- Yukawa potential
- Nucleon-nucleon interaction: phenomenological
- Terms constructed from symmetry
- Non-central terms motivated by exp.:
- Spin-orbit term, 'Tensor' term
- $\rightarrow$ One pion exchange potential; heavier mesons, shorter ranges

Constants determined by experiment, e.g. AV18 from pp, np, nn scattering and d binding (what about π-N scattering?)

- 3-body force considered small

- Effective Field Theories
- Weinberg: Controlled expansion in (low energies)/(QCD scale)
- Chiral EFT: extra symmetry of QCD preserved formally
- A predictive formalism for 3-body forces (which trade off with features of 2-body interactions) with exp. consequences.

Constants determined by experiment

- Uncertainty estimates from higher-order terms: a major advance

- There are attempts to calculate constants based on Lattice QCD
- "Vlowk" Renormalization group quantifies potentials used at low momenta lacking spatial resolution to see the hard core

Wong Ch. 3; Obertelli Sagawa "Modern Nuclear Physics..." Springer; Hammer, König, van Kolck, Rev Mod Phys 92 1 (2020)

Yukawa model of nuclear forces 1934 (Frauenfelder and Henley)

- Scalar potential A_0 produced by charge distribution $q\rho(\vec{r})$ satisfies wave equation (massless particle):

$$\nabla^2 A_0 - \frac{1}{c^2}\frac{\partial^2 A_0}{\partial t^2} = -4\pi q\rho;\ \text{for } A_0 \text{ t} - \text{independent}:$$

$$\nabla^2 A_0 = -4\pi q\rho \Rightarrow A_0(\vec{r}) = \int d^3r' \frac{q\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} \quad \text{For point charge } A_0 = q/r$$

- Yukawa forced a more rapid decrease (using Klein-Gordon eq.):

$$(\nabla^2 - k^2)\Phi(\vec{r}) = 4\pi\frac{g}{\sqrt{\hbar c}}\rho(\vec{r}) \quad \Rightarrow \Phi(\vec{r}) = \frac{-g}{\sqrt{\hbar c}}\int \frac{e^{-k|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\rho(\vec{r})d^3r'$$

For a hadronic point source at $\vec{r}'$=0: $\Phi(r) = \frac{g}{\sqrt{\hbar c}}\frac{e^{-kr}}{r}$

What is 'range' k? Get K-G from energy-momentum and canonical substitution:

$$E^2 = (pc)^2 + (mc^2)^2;\quad E \to i\hbar\frac{\partial}{\partial t}\quad p \to -i\hbar\vec{\nabla}$$

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial^2 t} - \nabla^2 + (\frac{mc}{\hbar})^2\right]\Phi(\vec{r}) = 0$$

comparing $\Rightarrow$ $k = mc/\hbar$ inverse of Compton wavelength

- So mass$\propto$ 1/range of potenial

Nuclear Force: descriptive properties

- **Attractive (nuclei exist)**
- **Range $\sim$ fm's**
- **Repulsive at short distances**
- **Depends on spin**
- **Independent of isospin (almost)**

- **Non-central: 'tensor force', spin-orbit force.**

These are motivated further by experimental observations

- **A detailed model of pion exchange: 'One Pion Exchange Potential' (OPEP):**

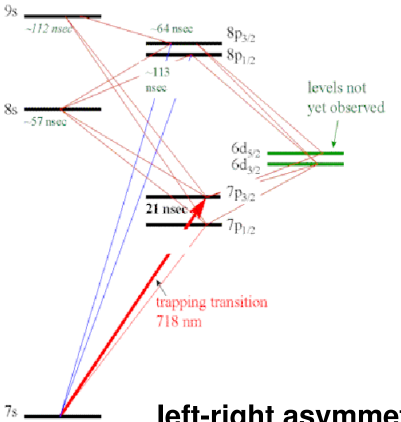
$$V(r) = \frac{g_\pi^2 (m_\pi c^2)^3}{3(Mc^2)^2\hbar^2}\left[\vec{s}_1 \cdot \vec{s}_2 + S_{12}\left(1 + \frac{3R}{r} + \frac{3R^2}{r^2}\right)\right]\frac{e^{-r/R}}{r/R} \quad R = 1/k, \text{ M is nucleon mass}$$

OPEP has some success describing long-range part of interaction from the relatively light π. More complete potentials include more massive short-range mesons and also two-pion exchange ('σ' term)

Strong spin-orbit term: opposite sign from atomic physics

Atomic "fine structure" energy splitting:

$H = -\vec{\mu}_e \cdot \vec{B}_e \propto \vec{S} \cdot \vec{L}$



notation
nL

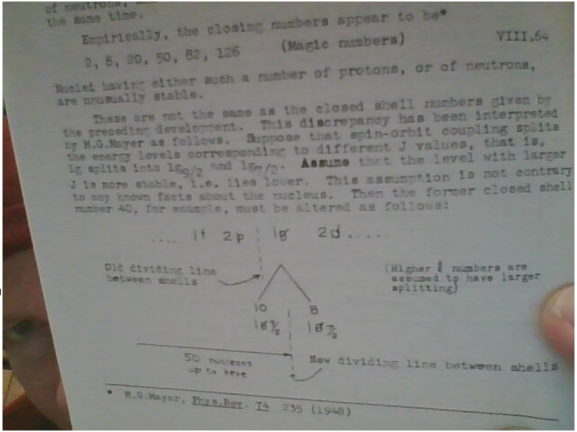
nL_J

with

$\vec{J} = \vec{L} + \vec{S}$

the total angular momentum, the good quantum number

"Nuclear Physics," Fermi, 1949 U. Chicago Press, compiled Orear, Rosenfeld, Schluter



of neutrons, ...
the same time.

Empirically, the closing numbers appear to be* VIII.64

2, 8, 20, 50, 82, 126 (Magic numbers)

Nuclei having either such a number of protons, or of neutrons, are unusually stable.

These are not the same as the closed shell numbers given by the preceding development. This discrepancy has been interpreted by M.G.Mayer as follows. Suppose that spin-orbit coupling splits the energy levels corresponding to different J values, that is, 1g splits into $1g_{9/2}$ and $1g_{7/2}$. **Assume** that the level with larger J is more stable, i.e. lies lower. This assumption is not contrary to any known facts about the nucleus. Then the former closed shell number 40, for example, must be altered as follows:

..... 1f 2p | 1g 2d ....

Old dividing line between shells

(Higher ℓ numbers are assumed to have larger splitting)

10 8

$1g_{9/2}$ $1g_{7/2}$

50 nucleons up to here

New dividing line between shells

* M.G.Mayer, Phys.Rev. 74 235 (1948)

left-right asymmetries of longitudinally polarized beams: much physics, including systematics in parity-violating weak interaction exps

Non-Central forces: Spin-orbit

- **Wong's argument spin-orbit term does not make D-state of deuteron:**

$\langle LS|\vec{L}\cdot\vec{S}|L'S'\rangle = 0$ for $L \neq L'$

L carries only 1 unit of orbital angular momentum, so 0 for $|L - L'| > 1$

Parity of state is $(-1)^L$, so state parity changes if $L' = L \pm 1$, but $\vec{L}$, $\vec{S}$ are even under parity, so 0 for $L' = L \pm 1$.

So spin-orbit term is zero between states of different L, so it does not mix in any D state. So the 'tensor force' is needed.

- **Why are the higher angular momentum states, with intrinsic spin in same direction as orbital, of lower energy in nuclei?**

There seems to be no simple intuitive answer

Density functional theory– treating nucleons in a mean field produced by all the others– sees difference in relativistic vs. non-relativistic treatments: Bender et al. Rev Mod Phys 75 121 (2003)

Non-Central forces: Tensor force

D-state probability in the deuteron ground state is produced by this interaction. (previous slide: not induced by spin-orbit.)
see Wong for quadrupole moment of deuteron in full detail

- **Non-central: 'tensor force'**

Tensor force between classical magnetic dipoles,

$$E_{12} = \frac{1}{r^3}\left[\vec{m}_1 \cdot \vec{m}_2 - 3(\vec{m}_1 \cdot \hat{r})(\vec{m}_2 \cdot \hat{r})\right]$$

is analogous to tensor operator for 2 spins:

$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Symmetries in a general 2-body potential: Wong Ch. 3

- **Want an isospin scalar**

Want $[H, T^2] = 0$ for isospin invariance

$\vec{T} = \frac{1}{2}(\vec{\tau_1} + \vec{\tau_2})$

consider

$\vec{T}^2 \equiv \vec{T} \cdot \vec{T} = \frac{1}{4}(\tau_1^2 + \tau_2^2 + 2\vec{\tau_1} \cdot \vec{\tau_2})$ **we want to know expectation value, so rearrange:**

$\vec{\tau_1} \cdot \vec{\tau_2} = 2\vec{T}^2 - \frac{1}{2}(\tau_1^2 + \tau_2^2)$

in Ch 2 $t(t+1) = (\frac{1}{2})(\frac{3}{2}) = 3/4$

(Pauli matrices are constructed from 'unity', so there are factors of 1/2)

$\vec{\tau}^2 = 4t(t+1) = 3$

so $\vec{\tau_1} \cdot \vec{\tau_2}$ is an isospin scalar, and a two-body operator, that distinguishes T=0 and T=1 states: $\langle T|\vec{\tau_1} \cdot \vec{\tau_2}|T\rangle$ = -3 for T=0, 1 for T=1

- **Translational, Galilean invariance**
- **invariance under: rotation, parity, and permutation between 2 nucleons**

Two-body nuclear potential with all allowed terms from general principles

$$\begin{aligned} V(r;\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\boldsymbol{\tau}_1,\boldsymbol{\tau}_2) &= V_0(r)+V_\sigma(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_\tau(r)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+V_{\sigma\tau}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\ &+V_{LS}(r)\boldsymbol{L}\cdot\boldsymbol{S}+V_{LS\tau}(r)(\boldsymbol{L}\cdot\boldsymbol{S})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\ &+V_T(r)S_{12}+V_{T\tau}(r)S_{12}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\ &+V_Q(r)Q_{12}+V_{Q\tau}(r)Q_{12}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\ &+V_{PP}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{p})(\boldsymbol{\sigma}_2\cdot\boldsymbol{p})+V_{PP\tau}(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{p})(\boldsymbol{\sigma}_2\cdot\boldsymbol{p})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \end{aligned} \tag{3-51}$$

isospin dependence
different meson mass
different ranges

with $S_{12} = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$ **and**

$$Q_{12} = \frac{1}{2}\left[(\vec{\sigma}_1\cdot\vec{L})(\vec{\sigma}_2\cdot\vec{L}) + (\vec{\sigma}_2\cdot\vec{L})(\vec{\sigma}_1\cdot\vec{L})\right]$$

Determine the terms from different types of experiments,
mostly pp, np, nn scattering,
and polarized observables, to explore spin and isospin dependence.

Then use many-body physics techniques to calculate in heavier nuclei

Quadratic spin-orbit term; 'last 2 terms'

$$Q_{12} = \frac{1}{2}\left[(\vec{\sigma_1}\cdot\vec{L})(\vec{\sigma_2}\cdot\vec{L}) + (\vec{\sigma_2}\cdot\vec{L})(\vec{\sigma_1}\cdot\vec{L})\right]$$

Wong: these enter where then is momentum dependence of the potential. This is in Argonne V18 (Wiringa, Stoks, Schiavilla et al. PRC 51 38 (1995))

$$V_{PP}(r)(\vec{\sigma_1}\cdot\vec{p})(\vec{\sigma_2}\cdot\vec{p}) + V_{PP_\tau}(r)(\vec{\sigma_1}\cdot\vec{p})(\vec{\sigma_2}\cdot\vec{p})(\vec{\tau_1}\cdot\vec{\tau_2})$$

Wong: For elastic scattering, these can be expressed as a linear combination of other terms. Since we get most of our info from elastic scattering of the nucleons, the info doesn't distinguish these terms. So these are often dropped

More modern 'effective field theory' approach

We can't get strong interaction from QCD (yet)

Put symmetries of QCD into an effective field theory

(Weinberg 'Nuclear Forces from chiral lagrangians' Phys. Lett. B 251 (1990) 288;Nucl. Phys. B363 (1991) 3; Von Kolck Prog. Phys. 43 (1999) 337)

Sonic Bacca, U. Mainz, has a helpful pedagogical overview talk online https://www.triumf.ca/sites/default/files/bacca_Lecture_NucleonNucleonForce.pdf

Ruprecht Machleidt U. Idaho https://abinitio.triumf.ca/2013/TRIUMF2013_machleidt.pdf e.g. this mentions "spin-orbit force! ... from 3NF contacts at N4LO " from Girlanda, Kievsky, Viviani, PRC 84, 014001 (2011) [Though Sagawa mentions a spin-orbit force at lower order... this must be a refinement.] People have carried out these EFT's to quite high order now, to check convergence. Convergence is quite slow, and not guaranteed at all the energy scales desired.

(Bacca's and Machleidt's .pdf's now uploaded to Canvas Files: links obsolete)

Demonstrative example: 'effective field theory' for Rayleigh scattering

(Holstein Prog. Part. Nucl. Phys. 50 (2003) 203)

Rayleigh scattering: $\lambda >> r_{\rm atom}$: photon sees a neutral blob

As wavelength decreases, scattering occurs: Hamiltonian must satisfy

1) quadratic in the vector potential, since we are describing Compton scattering;

2) must be a scalar under rotations;

3) must be gauge invariant;

4) must be Parity and Time reversal-even $\Rightarrow$

$$H_{\rm eff} = 2\pi \left(\alpha_{\rm electric} \vec{E}^2 + \beta_{\rm magnetic} \vec{H}^2\right)$$

Atomic physics is stuffed into α, β, to be determined from experiment. In the meantime, one can immediately conclude:

1) $\alpha_{\rm electric}, \beta_{\rm magnetic}$ have units of volume: λ is large that volume must be the atom

2) $\vec{E}, \vec{H} \sim \omega\hat{\epsilon}$ for plane waves

$\frac{d\sigma}{d\Omega} \propto |< f|H_{\rm eff}|i>|^2 \propto a_0^6\omega^4$ **'So the sky is blue' Q.E.D.**

In the case of Rayleigh scattering we could have done the whole calculation, but in QCD we can't, so this kind of approach becomes useful...

EFT should preserve symmetries of QCD; many are more general

E.g. Piarulli and Tews doi: 10.3389/fphy.2019.00245

Nuclear potentials V need to be:

- **Hermitian, because the Hamiltonian is hermitian,**
- **Symmetric under the permutation of identical particles, i.e., $V_{ji} = V_{ji}$**
- **Translationally and rotationally invariant,**
- **Invariant under translations in time, i.e., time- independent,**
- **Lorentz invariant (for non-relativistic interactions $\rightarrow$ Galilean invariance),**
- **Invariant under parity transformations and time reversal,**

(θ_{QCD} term is $\not{T}$. Exp. $\Rightarrow < 10^{-9}$. Theory mechanism for tiny θ_{QCD} has an extra global U(1) $\rightarrow$ axion. deVries et al. work in EFT's with $\not{T}$ in same volume)

- **Conserve baryon and lepton number.**

Most are pretty general requirements. Specific to QCD:

- **Approximately isospin symmetric and charge independent**
- **Include the properties of spontaneously and explicitly broken chiral symmetry $\rightarrow$**

Chiral symmetry of QCD Lagrangian: what does this mean?

Rewrite $$L_{QCD} = \bar{q}(i \not{D} - m)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

in terms of left- and right- handed chirality components of the quark fields

$$q_L = \frac{1}{2}(1+\gamma_5)q; \quad q_R = \frac{1}{2}(1-\gamma_5)q :$$

$$L_{QCD} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L m q_R - \bar{q}_R m q_L - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

This is equivalent to pointing out that for a massive particle with chirality $\hat{s}\cdot\hat{k}$, you can boost to a frame where $\hat{k} \to -\hat{k}$, so the particle appears to have opposite chirality. So you need $m = 0$ to have definitive chirality.

Since $m_u \sim$ 5 MeV and $m_d \sim$ 9 MeV $<< \Lambda \sim$1 GeV (some QCD scale, hadron mass) the mass term is small, so L_{QCD} has approximate 'chiral' symmetry $SU(3)_L$ X $SU(3)_R$ which gets spontaneously broken, producing e.g. 3 π's and the η

The π's are then pseudo-Goldstone bosons and acquire a small mass, but of course since the π's are spin-0 they are not the things acquiring chirality...

Effective field theory 'Nuclear forces from chiral langrangians'

Weinberg Phys Lett B251 288 (1990) then uses effective field theory:

- **Assign fields (not necessarily physical ones) that keep QCD symmetries: chiral symmetry, respect isospin**

- **keep terms to given powers of m/Λ and derivatives**

Result is the following Lagrangian and 2-body potential:

$$\mathscr{L} = -\tfrac{1}{2}D^{-2}\,\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \tfrac{1}{2}D^{-1}m_\pi^2\boldsymbol{\pi}^2$$
$$-\bar{N}[\not\partial + m_N + 2iD^{-1}F_\pi^{-1}\gamma_5 g_A \boldsymbol{t}\cdot\not\partial\boldsymbol{\pi}$$
$$+2iD^{-1}F_\pi^{-2}\boldsymbol{t}\cdot(\boldsymbol{\pi}\times\not\partial\boldsymbol{\pi})]N - (\bar{N}\Gamma_\alpha N)(\bar{N}\Gamma^\alpha N)$$

$$V_{\text{2-nucleon}} = 2(C_S + C_T\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\delta^3(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$
$$-\left(\frac{2g_A}{F_\pi}\right)^2(\boldsymbol{t}_1\cdot\boldsymbol{t}_2)(\boldsymbol{\sigma}_1\cdot\nabla_1)(\boldsymbol{\sigma}_2\cdot\nabla_2)Y(|\boldsymbol{x}_1 - \boldsymbol{x}_2|)$$
$$-(1'\leftrightarrow 2')\,, \qquad (8)$$

where $Y(r) \equiv \exp(-m_\pi r)/4\pi r$ is the usual Yukawa potential. [Throughout it should be understood that these are local potentials, containing a delta function factor like $\delta^3(\boldsymbol{x}'_1 - \boldsymbol{x}_1)$ for each nucleon.]

Effective field theory 'Nuclear forces from chiral langrangians'

Expanding this to more orders must be checked for convergence (Entem and Machleidt Phys Rev C 68 041001R (2003) for 4th-order 'next-to-next-to-next-to-leading order' 'N^3LO' in m/Λ and derivatives),

That σ term for 2-pion exchange e.g. follows naturally in this formalism

Produces similar results to more phenomenological potentials of older vintage (Bonn potential, Argonne V18) but on more systematic ground.

These theories can fit the large body of NN and Nπ scattering data, fixing the coupling constants... then you predict other observables

They also explicitly include isospin breaking terms. I mentioned different observables having different % effects: there are natural explanations in EFT's based on 'power counting,' yielding e.g. what power of $(m_u - m_d)/\Lambda_{\rm QCD}$ contributes Machleidt and Entem, Phys Rep 503 (2011) 1.

Phenomenological 3-body forces

For pointlike masses, gravity has only 2-body forces

Moon raises tides on Earth, which change its interaction with the Sun: a 3-body force

198 Hans-Werner Hammer, Andreas Nogga, and Achim Schwenk

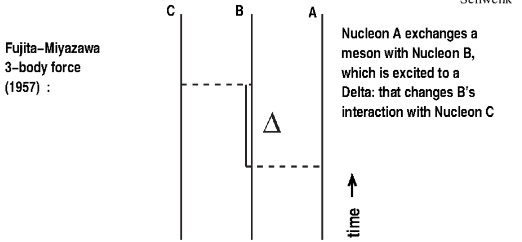


FIG. 1. Three-nucleon force arising from virtual excitation of a Δ(1232) degree of freedom. Solid (dashed) lines indicate nucleons (pions).

Hammer, Nogga, Schwenk, Rev Mod Phys 85 (2013) 197

A phenomenological 3-body force. Wong 1999 says these were thought small... in modern EFT's, 3-body forces are predictable consequences of approximate 2-body interactions, and are critical in certain systems →

Features of 3-body forces in EFT (Hammer... Rev Mod Phys 85 (2013) 197)

theory that is made of two particle species, a light and a heavy one with $M_{\rm low} \ll M_{\rm high}$. We focus on soft processes in which the energies and momenta are of the order of the light particle mass (the so-called soft scale). Under these conditions, the short-distance physics related to the heavy-particle exchange cannot be resolved. However, it can be represented systematically by contact interactions between light particles. It can be expanded in powers of $q^2/M_{\rm high}^2$ as

$$\frac{g^2}{M_{\rm high}^2 - q^2} = \frac{g^2}{M_{\rm high}^2} + \frac{g^2 q^2}{M_{\rm high}^4} + \cdots . \qquad (1)$$

This expansion can be represented in the EFT. At low momentum transfer q^2, the effects of the pole from the heavy-particle exchange in Eq. (1) are captured by a series of local momentum-dependent interaction terms reproducing the expansion in Eq. (1) term by term. This idea is closely related to

$m_\Delta - m_{\rm nucleon} \sim$ 300 MeV $\sim 2\, m_\pi$: is it surprising changing the EFT field scale can change a 3-body force strength?

We use a simple EFT model to illustrate how field redefinitions can be used to shift strength from off-shell two-body interactions to on-shell three-body interactions (Hammer and Furnstahl, 2000):

$$\mathcal{L} = \psi^\dagger \mathcal{D}\psi - g_2(\psi^\dagger\psi)^2 - \eta[\psi^\dagger(\psi^\dagger\psi)\mathcal{D}\psi + \psi^\dagger\mathcal{D}(\psi^\dagger\psi)\psi], \qquad (4)$$

where $\mathcal{D} = i\partial_t + \vec{\nabla}^2/(2m)$ is the free Schrödinger operator. The model has a two-body contact interaction with coupling constant g_2 and an off-shell two-body contact interaction with coupling η which we assume to be small. Now consider a field transformation

$$\psi \to [1 + \eta(\psi^\dagger\psi)]\psi, \qquad \psi^\dagger \to [1 + \eta(\psi^\dagger\psi)]\psi^\dagger. \qquad (5)$$

Performing this transformation and keeping all terms of order η we obtain a new Lagrangian:

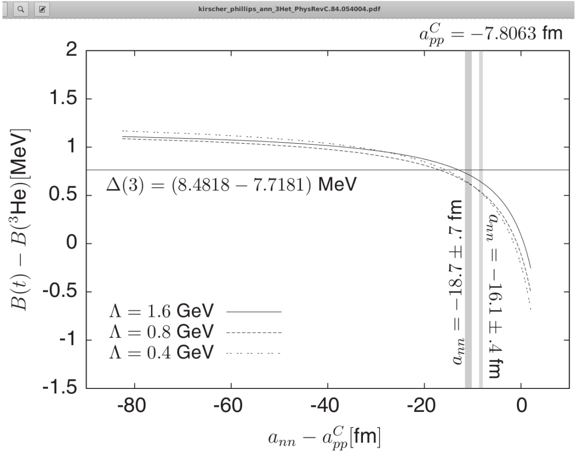
$$\mathcal{L}' = \psi^\dagger\mathcal{D}\psi - g_2(\psi^\dagger\psi)^2 - 4\eta g_2(\psi^\dagger\psi)^3 + \mathcal{O}(\eta^2), \qquad (6)$$

where the off-shell two-body interaction has been traded for a three-body interaction. This is illustrated in Fig. 2. Off-shell interactions always contribute together with many-body forces and only the sum of the two is meaningful.

EFT formulations typically need 3N fixed by experiment

(an exception is 'π-less' chiral EFT, useful at very low momenta where even the π is short range: then each 3N follows once 2N is fixed)

E.g.: Kirscher, Phillips PRC 84 054004 (2011) related scattering length differences to A=3 binding energies. The EFT includes 3N forces, Coulomb, and isospin breaking terms
The calculated $a_{nn} = -22.9 \pm 4.1\,\mathrm{fm}$ agrees with experiment, but with 20% uncertainty "due to the use of pionless EFT"



König et al. J. Phys. G 43 055106 (2016) includes more perturbative corrections, inputs a_{nn} and calculates binding energy difference with 170 keV accuracy in agreement with experiment.

2020 Summary reviews for EFT approaches in this volume:

frontiers in Physics

REVIEW
published: 18 March 2020
doi: 10.3389/fphy.2020.00057

Check for updates

Nucleon-Nucleon Scattering Up to N^5LO in Chiral Effective Field Theory

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During the past few decades a large effort has been made toward describing the *NN* interaction in the framework of chiral Effective Field Theory (EFT). The main idea is to exploit the symmetries of QCD to obtain an effective theory for low energy nuclear systems. In 2003, the first accurate charge-dependent *NN* potential in this scheme was developed and it has been applied to many ab-initio calculations, opening the possibility to study nuclear systems in a systematic and accurate way. It was shown that the fourth order (N^3LO) was necessary and sufficient to describe the *NN* scattering data with a χ^2/d.o.f on the order of so-called high precision potentials. However the systematics of chiral EFT also allow to relate two- and many-body interactions in a well-defined way. Since many-body forces make their first appearance at higher order, they are substantially smaller than their two-body counterparts, but may never-the-less be crucial for some processes. Thus, there are observables where they can have a big impact and, for example, there are indications that they solve the long standing A_y puzzle of *N-d* scattering. The last few years, have also seen substantial progress toward higher orders of chiral EFT which was motivated by the fact that only three-body forces of rather high order may solve some outstanding issues in microscopic nuclear structure and reactions. In this chapter we will review the latest contributions of the authors to development of chiral EFT based potentials up to N^4LO as well as first calculations conducted for *NN* scattering at N^5LO.

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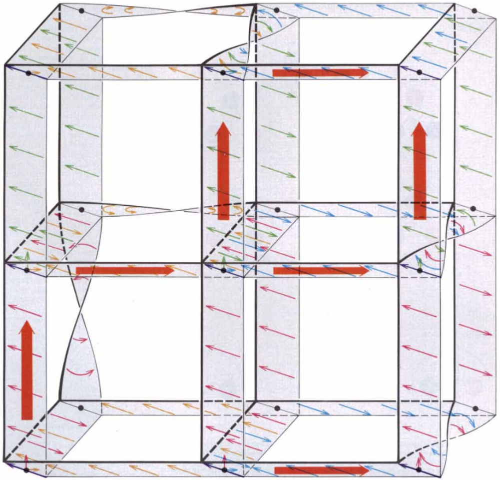
Lattice Gauge Theory of QCD cartoon

- consider Feynman path integrals, sum over all possible paths between two times
- Spacetime discretized:
  Quarks on vertices.
  Gluons travel along lines between vertices
  m_q taken nonphysically large (expressed as large m_π)
  Want lattice spacing $\to$ 0, times, volume $\to \infty$, $m \to$ physical
- Numerical results include
  quark confinement,
  m_{hadrons},
  axial vector coupling g_A of n,
  π decay constant f_π



Rebbi Sci Am 248 54 (1983)
Obertelli and Sagawa Chapter 10.2 "Lattice QCD" develops path integral version of QCD, then discretizes it

Nucleon-nucleon interaction from Lattice QCD

Ishii et al. PRL 99 022001 (2007); Iritani et al. PRD 99 014514 (2019)

Qualitative features reproduced

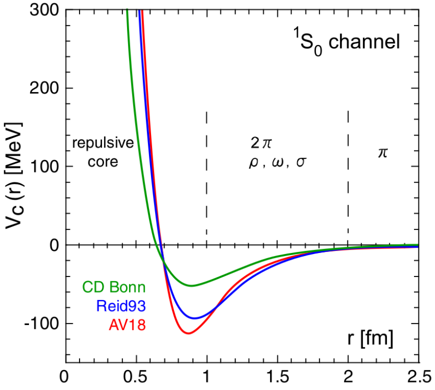


FIG. 1 (color online). Three examples of the modern *NN* potential in the $^1S_0$ (spin singlet and *s*-wave) channel: CD-Bonn [17], Reid93 [18], and AV18 [19] from the top at $r = 0.8$ fm.

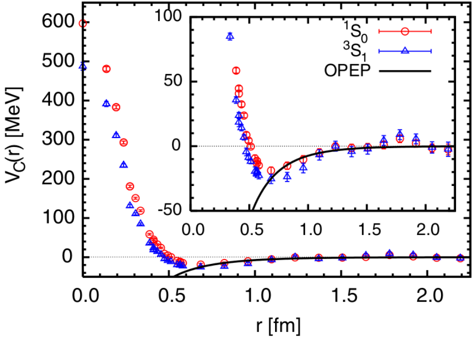


FIG. 3 (color online). The lattice QCD result of the central (effective central) part of the *NN* potential $V_C(r)$ $[V_C^{\rm eff}(r)]$ in the $^1S_0$ $(^3S_1)$ channel for $m_\pi/m_\rho = 0.595$. The inset shows its enlargement. The solid lines correspond to the one-pion exchange potential (OPEP) given in Eq. (5).

Witten Nucl Phys B160 (1979): QCD large-N expansion vs. QED

Just for the sake of comparison, let us ask why perturbation theory is successful in QED. It is not enough to say that "the electric charge is small." In fact, normalized in the usual way so that the interaction vertex is just $e\gamma^{\mu}$, the electric charge is approximately $e \doteq 0.302$. Perturbation theory is a good approximation in QED because when one carries out perturbative expansions, one finds that the typi-

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cal expansion parameter is really $e^2/4\pi$. If the typical parameter had turned out to be $4\pi e^2$, perturbation theory would not have been very successful for e as large as 0.302. And if we had not yet learned how to do perturbative calculations, we would have been unable to judge, just from the fact $e = 0.302$, whether an expansion in e^2 would be a good approximation.

If, for instance, as is perfectly possible, the characteristic parameter in the non-planar diagrams is really not $1/N^2$ but $1/4\pi N^2 = 1/113$, then non-planar diagrams in QCD are almost as tiny as electromagnetic corrections. While this is only an extreme possibility, there is no reason to be surprised that phenomenology seems to show that the $1/N$ expansion is a good approximation.

1/Nc expansion: an analytic calculation in QCD ☺

G. 't Hooft, NPB 72 461 (1974); E. Witten, NPB 160 57 (1979); Donoghue Golowich Holstein "Dynamics of the Standard Model"

Allows organization of quark-gluon graphs by complexity.
Nc=3, so 1/Nc converges slowly
Relations between different quantities can be calculated.

- **The weak nucleon-nucleon interaction, which can be measured because it violates parity, has isoscalar and isovector parts.**

Phillips, Samart, Schat PRL 114 062301 (2015) **calculated ratio of isoscalar to isovector interaction which is $\sin^2\theta_W/N_c \approx 1/12$, the level of suppression seen.**

- Samart, Schat, Schindler, Phillips PRC 94 024001 (2016) **calculated relations between isoscalar, isovector, and isotensor $\not{T}$ in the interaction between nucleons**
- **The actual calculations are technically very difficult and well beyond the scope. But it's good to realize one can calculate something analytically in QDC, compared to numerically in lattice QDC.**

1/Nc can fail in ways difficult to understand: e.g. $K \rightarrow \pi\pi$ favors small isospin changes by 20 experimentally ("$\Delta I = 1/2$ rule" of thumb) but by $\sqrt{2}$ in 1/Nc