Nucleons and their Structure; Hadrons and Isospin

- **quark content of baryons and mesons**
- **Isospin symmetry and some consequences**
- **the pion as the pseudo-Goldstone boson of broken chiral symmetry**
- **Antisymmetry of wavefunctions under exchange of fermions and examples**

Existence of excited-nucleon Δ
$\Rightarrow$ "Color" (QCD 'charge') is a needed degree of freedom
(μ of *p*, *n* in constituent quark model supports this)

- References:

Wong ch. 2
Halzen and Martin, Quarks and Leptons, Ch. 2: "HM"

quark content of n, p; isospin

ignoring antisymmeterization for now

$|p\rangle = |uud\rangle$ $|n\rangle = |udd\rangle$

u **has electric charge q = 2/3,** ***d*** **has -1/3**

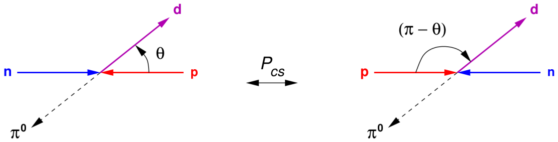
Consider the nucleon, a spin-1/2 fermion with isospin t**=1/2,**
and isospin projection along the quantization axis
$t_0 = t_3 = +1/2$ **for proton and -1/2 for the neutron**
Then for nuclei, isospin projection just counts protons and neutrons
$T_3 = (Z-N)/2$
But there is a lot more physics in the total isospin T
(We may switch to the typical nuclear sign convention later, but we often study Z>N ☺)

Heisenberg invented isospin (Zeit. für Physik 77 1 (1932)) more as a label than anything else. Wigner (Phys Rev 51 106 (1937)) and others turned it into a powerful symmetry in nuclear physics:

good Isospin symmetry: qualitative informal comments

This field was in progress for Wong 1999:

The strong interaction respects isospin symmetry well (for 1st generation)



See e.g. Opper... Hutcheon... Yen et al. PRL 91 212302 (2003) charge symmetry breaking in front-back asymmetry of n p → d π^0

$$A_{fb} = 1.7 \pm 0.8 \pm 0.6 \times 10^{-3}$$

Such effects all traceable to $m_u \neq m_d$ so far: review Miller et al. Ann Rev Nucl Part Sci 56 253 (2006).

Coulomb and other E&M can usually be treated perturbatively. Many interesting phenomena in nuclear structure and reactions from isospin symmetry: an active field testing the strong interaction into the 1980's and now ☺ Isospin Symmetry Breaking ISB in strong interaction is naturally in chiral EFT N-N interaction (Lecture ∼6) and is now important calculating beta decay wf's for Vud extraction (Lecture ∼21)

Isospin breaking: N-N scattering lengths, binding energies

- **n p mass difference is about 1 MeV out of 1000 MeV.**

consider scattering lengths for n,p $\sigma_{\text{elastic}} \stackrel{k\to 0}{\rightarrow} = 4\pi a^2$

- **p. 93 and 94 of Wong mention reasons for the higher value of a_{np} compared to a_{nn} and a_{pp} (e.g. in terms of mesons of different mass and range). This is thought to be understood, and not breaking isospin symmetry in the strong interaction.**

Indirectly Chen et al PRC 77 054002 (2008) measured in $d(\pi^-, n\gamma)n$
a_{nn} = -18.9$\pm$0.4 fm
compared to a_{pp} = -17.3 $\pm$ 0.4 fm (raw value is 7.8063 fm, corrected for E&M).
The difference is 1.6$\pm$0.5 fm different from a_{pp}.

This effect is enhanced by 10 by the physics compared to A_{fb} (Gardestig JPG 36 (2009) 053001; Machleidt and Entem Phys Rep 503 (2011) 1), and it's all consistent with the u d mass difference.

- The 'Nolen-Schiffer anomaly' in binding energies of T=1/2 nuclei needs more strong interaction isospin breaking ☺ ? Konieczka PRC 105 065505 (2022) Sagawa PRC 109 L011302 (2024)

Isospin symmetry: treat formally as spin, like SU(2)

We can write the nucleon as a 2-component column matrix:

$$|p\rangle = |t=\tfrac{1}{2}, t_0=+\tfrac{1}{2}\rangle = \begin{pmatrix}1\\0\end{pmatrix}_t$$

$$|n\rangle = |t=\tfrac{1}{2}, t_0=-\tfrac{1}{2}\rangle = \begin{pmatrix}0\\1\end{pmatrix}_t$$

with formal operators like Pauli spin matrices

$$\tau_1 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \quad \tau_2 = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \quad \tau_3 = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

and commutation relation

$$\tau_i \tau_j = \delta_{ij} I + i\epsilon_{ijk}\tau_k$$

Construct raising and lowering operators:

$$\tau_+ = \tfrac{1}{2}(\tau_1 + i\tau_2) = \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix} \quad \tau_- = \tfrac{1}{2}(\tau_1 - i\tau_2) = \begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}$$

Just like spin, $\tau_\pm$ changes t_3 (i.e. t_0) without affecting total isospin t

$$\tau_\pm |t, t_0\rangle = \sqrt{t(t+1) - t_0(t_0 \pm 1)}\, |t, t_0 \pm 1\rangle$$

This formal expression, from raising and lowering operators, we'll need to understand beta decay rates of isobaric analog decays (beta decays really changing one proton to one neutron, almost.)

Isospin of quarks

$\tau_-|p\rangle \equiv \tau_-|uud\rangle = |n\rangle \equiv |udd\rangle$

similarly

$\tau_+|udd\rangle = |uud\rangle$

Also, t_0 just counts ('is a scalar quantity') so

$t_0|uud >= +1/2$ and $t_0|udd >= -1/2$

$\therefore$ u has $t_0 = +1/2$ and d has $t_0 = -1/2$, a doublet ☺ in 'strong isospin'

Wong p. 35 justifies why c, s, t, b quarks are not in isospin doublets. They have isospin 0. ☹. I was hoping this was related to their much heavier masses, but apparently not.

K mesons $u\bar{s}$ etc. have t=1/2, and are not found with t=3/2 (quoted as an experimental observations) $\Rightarrow$ s has isospin zero, and not isospin 1.

The Standard Model's quark doublets we see on charts all the time

$\binom{u}{d}$ $\binom{c}{s}$ $\binom{t}{b}$

are under SU(2)$_{\mathrm{L}}$ weak interaction ('weak isospin') [though the eigenstates of weak interaction differ: see CKM matrix]

We will just be considering 1st-generation 'strong isospin' consequences

Isospin of antiparticles

Since $p + \bar{p} \rightarrow \gamma + \gamma$, and γ has isospin 0,
$\bar{p}$ has t=1/2 like p, and $t_0[\bar{p}] = -1/2$, $t_0[p] = +1/2$

Under charge conjugation C,
Wong (and references therein) uses 2nd-quantized operators to show that there's a phase to keep track of:

$$|p\rangle \xrightarrow{C} (-1)^{(t+t0)}|\bar{p}\rangle = -|\bar{p}\rangle$$

and similarly we may need

$$|n\rangle \xrightarrow{C} (-1)^{(1/2-1/2)}|\bar{n}\rangle = +|\bar{n}\rangle$$

$$|u\rangle \xrightarrow{C} -|\bar{u}\rangle$$

$$|d\rangle \xrightarrow{C} +|\bar{d}\rangle$$

Spin-0 pseudoscalar mesons from up, down quarks

π has spin 0, isospin 1; π^+ $t_3 = 1$; π^- $t_3 = -1$; π^0 $t_3 = 0$
Parity of π: (+ q parity)*(- $\bar{q}$ parity)* $(-1)^l$ for spatial wf; for $l = 0$ parity is -

- **To get electric charge -1, there's a unique combination:**

$|\pi^-\rangle = |\bar{u}d>$
Wong formally constructs the neutral π from the isospin raising operator:
$|\pi^0\rangle = \frac{1}{\mathrm{Norm}}\tau_+|\pi^-\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle)$ **there's also a phase for the + one:**
$|\pi^+\rangle = -|u\bar{d}\rangle$ **(I would have to study these phases to use them ☹)**

- **masses: π^0 134.9766(6) MeV; π^+ 139.57018(35) MeV; π^+ can β decay to π^0.**

mean τ_e: π^0 8.4×10^{-17}s; π^+ is 2.6×10^{-8}s $\Leftarrow$ The $\pi^0 \to \gamma + \gamma$, much faster.

Can m(π^+) $\neq$ m(π^-) ? poll

By inspection, there's another $t_3 = 0$ meson orthogonal to the π^0:
$|\eta\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$ **spin 0, $t = 0$, m_η = 550 MeV** **and $s\bar{s}$ content? say Wikipedia**

These "wf's" are missing exchange terms. poll

Spin-1, negative parity, mesons from up, down quarks

ρ, ω mesons are also virtually exchanged between nucleons– one of the ways to think about the strong interaction between nucleons.
one can couple the quark-antiquark together to spin 1 instead of 0.

ρ^- is $d\bar{u}$; ρ^0 is $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$; ρ^+ is $u\bar{d}$
ρ^0 mass 775.3 MeV, mean lifetime 4.45 $\times 10^{-24}$s

ω is $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$,
mass 782.7 MeV, lifetime 7.8$\times 10^{-23}$s

Lifetimes shorter than π, because they decay by strong interaction

ρ^0 and ω mix because of isospin symmetry breaking (and differently if they are virtual Iqbal PLB386 (1996)) so do π^0 and η but mass splitting bigger is this clear?
This changes the nucleon-nucleon interaction a little– e.g. there's a 5 keV extra mass difference between ^{3}He and ^{3}H that is unaccounted for by the Coulomb interaction (G. Miller arXiv:1810.05239)

Most mass is from where? poll

	m	m/m_p	q's	Long-range pairs A(A-1)/2	ratio pairs	Short-range ratio # of quarks
p,n	938 MeV	1	uud	3	1	3/3
ρ	775	0.83	$u\bar{u},d\bar{d}$	1	1/3	2/3
ω	783	0.83	$u\bar{u},d\bar{d}$	1	1/3	2/3
η	550	0.59	$u\bar{u},d\bar{d},s\bar{s}$	1	1/3	2/3
π	135	0.15	$u\bar{u},d\bar{d}$	1	1/3	2/3

The π is special.

The π mass is much smaller than one would guess

It turns out at least 3 physics terms help determine the π mass

Goldstone's theorem drags m_π down

the π is the pseudo-Nambu-Goldstone boson of broken chiral symmetry(!?)

People say this a lot. The derivation mechanics seem simple and helpful if you accept some QM.

Goldstone's theorem A. Zee QFT in a nutshell M. Schwartz QFT Goldstone Salam Weinberg PR127 965 (1962):

Continuous symmetry **spontaneously** *broken* $\Rightarrow$ *massless fields emerge*

- Some continuous symmetry is conserved $[H, Q] = 0$ and has conserved charge Q (where $Q = \int d^3x J_0(x)$ is the integral of a current so is an operator)

  Let g.s. be $|0\rangle$

- Let $H|0\rangle = E_0|0\rangle$ (Prof.Z. sets E_0=0 and calls $|0\rangle$ "the vacuum")

Usually g.s. is invariant under the symmetry transformation $e^{i\Theta Q}|0\rangle$ i.e. $Q|0\rangle = 0$

But **suppose the symmetry is "spontaneously" broken**, not in the Hamiltonian, but by the g.s.'s geometry or something so $Q|0\rangle \neq 0$. **Keeping red assumptions still true!**

Consider the state $Q|0\rangle$, and find its energy, $HQ|0\rangle$:

Given $[H, Q] = 0 \Rightarrow (HQ - QH)|0\rangle = 0$ i.e. $HQ|0\rangle = QH|0\rangle = E_0 Q|0\rangle$

so we have another state $Q|0\rangle$ with same energy as the g.s. $|0\rangle$

Now we can construct a state which acts like a massless state $\to$

→ Consider our conserved charge $Q = \int d^3x J_0(x)$

Textbooks concur we can construct a state with 3-momentum $\vec{p}$ from the vacuum

$|s(\vec{p})\rangle = \int d^3x\; e^{i\vec{p}\cdot\vec{x}} J_0(x)|0\rangle \overset{\vec{p}\to 0}{\to} Q|0\rangle$ which has energy $E(\vec{p}) + E_0 \overset{\vec{p}\to 0}{\to} E_0$

so $|s\rangle$ satisfies a massless dispersion relation so has mass=0

(This seems clearer if, like Zee, we set E_0=0, though it should not matter.)

Whenever a continuous symmetry spontaneously broken, massless fields

"Nambu-Goldstone bosons" **emerge** ☺ **extremely general, originated in condensed matter physics...**

"Broken chiral symmetry"? We will look at chiral EFT for the N-N interaction.

The quark Lagrangian mass term has mass $\sim$ a few MeV $\ll m_N$.

Massless quarks would have a well-defined spin direction with respect to motion, i.e. chirality, so massless quarks are termed 'chiral.'

Breaking chiral symmetry (but is this 'spontaneous'?) naturally generates a massless boson (!?), which can be identified as the π. → L6

π is "pseudo-Nambu-Goldstone"? **At least 3 mass contributions to m(π) $\neq$ 0.**

Structure of π also reflects this. **The QCD Lagrangian has m_{quark} so looks non-spontaneously broken to JB... → L6**

Sagawa: "the bulk of p and n masses comes from the chiral symmetry breaking which is called the "spontaneous" symmetry breaking." (!?)

→ L6

2023 errors in L21: "...since the π is both a Nambu-Goldstone boson and a $q\bar{q}$ bound state, it holds a unique position in nature" Horn, Roberts Jour Phys G 43 2016 073001

Continuous symmetry leaving a Lagrangian invariant + what? → spin-0 $m = 0$ boson poll
Goldstone Salam Weinberg PR 127 965 (1961)

Breaking that symmetry + other things! generates a pseudo Nambu Goldstone boson with $m \neq 0$ other m contributions ⇒"pseudo":

- The π can be treated as a Goldstone boson acquiring m from broken chiral symmetry.

(Remember chiral symmetry means m_q's = 0, so q's then have well-defined handedness.)

Remembering that $m_\rho \approx 4/5\ m_{\text{nucleon}}$, and constituent q's work for μ_ρ, but $m_\pi \approx 1/7\ m_{\text{nucleon}}$

Gell-Mann Oakes Renner PR 175 2195 (1968)
(note this pre-dates QCD)
$m_\pi^2 \propto \langle\pi|U_{ECSB}|\pi\rangle$
U_{ECSB} is interactions breaking chiral symmetry
(also $m_\pi^2 + 3m_\eta^2 - 4m_K^2 = 0$)

π as a $q\bar{q}$ bound state of constituent q's leads instead to $m_\pi \propto \langle\pi|U_{ECSB}|\pi\rangle$
(to get $m_\pi = 135$ MeV needs fine tuning)

- If m_q's = 0, then for a Goldstone boson π, m_π = 0, and its interactions also vanish (!)
  Restoring chiral symmetry in nuclei was a possible solution to the Gamow-Teller strength deficit (below).

- We saw that chiral EFT of N-N interaction is based on ≈ chiral symmetry of the q's.
  So is chiral perturbation theory, which quantifies a QCD-induced weak decay (below).
  The axion is another Goldstone boson from $\not{T}$ in QCD (below)

Qualitative comments on spontaneous symmetry breaking

Nambu and Jona-Lasinio had a model in 1961 (before quarks) with "... an illustrative theory in which, with some drastic approximations, a suitable chiral symmetry was found to be spontaneously broken, and in consequence the **light pion appeared as a bound state of a nucleon and antinucleon**" Weinberg CERN Courier 2008 https://cerncourier.com/a/from-bcs-to-the-lhc/

The general simple proof we showed of a massless boson occuring in a spontaneously broken symmetry doesn't say anything about its structure or interactions between its constituents. So your model using the Goldstone boson mechanism to calculate $\boldsymbol{m_\pi}$ and other properties will depend on those details– this is good, because it might tell you something about QCD.

(This Weinberg article goes through why the standard model Higgs is not a Goldstone boson, though it also has a paragraph of how it was possible something acting like the Higgs could have been produced by spontaneous symmetry breaking if it were a composite particle bound by a new interaction– such "technicolor" models have not worked out, at least not yet.)

Problem 6 in HW2 addresses parity and spontaneous symmetry breaking.

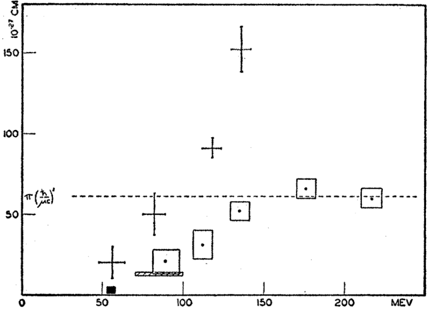


FIG. 1. Total cross sections of negative pions in hydrogen (sides of the rectangle represent the error) and positive pions in hydrogen (arms of the cross represent the error). The cross-hatched rectangle is the Columbia

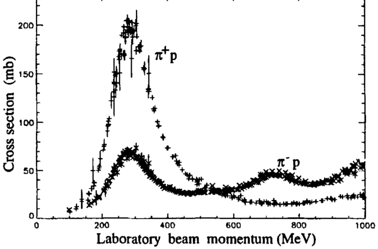


Figure 2-3: Total cross section of charged pions scattering off protons. The strong P_{33}-resonance in $\pi^+ + p$ reaction occurs in the $(J^\pi, T) = (\frac{3}{2}^+, \frac{3}{2})$ channel with $\ell = 1$. The $\pi^- + p$ cross section at the same energy is much smaller, as the

Δ resonance excitation and isospin

Anderson, Fermi, Long, Nagle Phys Rev 85 935 (1952)

$$\sigma(p + \pi^+ \to \Delta^{++}) \propto$$

$$|\langle t_p\, t_3(p)\; t_\pi t_3(\pi^+)|t_\Delta t_3(\Delta^{++})\rangle|^2 =$$

$$|\langle \tfrac{1}{2}\,\tfrac{1}{2}\;1 1|\tfrac{3}{2}\;\tfrac{3}{2}\rangle|^2 = 1$$

$$\sigma(p + \pi^- \to \Delta^{0}) \propto$$

$$|\langle t_p\, t_3(p)\; t_\pi t_3(\pi^-)|t_\Delta t_3(\Delta^{0})\rangle|^2 =$$

$$|\langle \tfrac{1}{2}\,\tfrac{1}{2}\;1\;-1|\tfrac{3}{2}\;-\tfrac{1}{2}\rangle|^2 = 1/3$$ ☺

Though Fermi et al. pointed out the exit channel branches matter, too (“K.A. Bruckner private comm.”)

unitarity limit $\sigma \propto \pi\bar{\lambda}^2$ independent of interaction strength (!!)

What else am I assuming?

Δ^{++} and need for 'color' quantum number

Using u and d, here are four spin 3/2, isospin $t = 3/2$ baryons:
Δ^{++} uuu
Δ^{+} uud, excited states of proton with spins all aligned
Δ^{0} udd, excited states of neutron with spins all aligned
Δ^{-} ddd

Consider antisymmetry. Δ's have noninteger spin, so are fermions, so wf's must be antisymmetric under exchange of particles.
Writing $\Delta^{++} = uuu \uparrow\uparrow\uparrow \psi_{\text{spatial}}$,
noting parity is known positive so ψ_{spatial} is symmetric
Δ^{++} would be completely symmetric under exchange !! ☹ ☹

We need another degree of freedom, the 'color' charge of QCD. A very powerful and fundamental statement, the need for a new quantum number to satisfy antisymmeterization i.e. Pauli exclusion principle

$$\Delta^{++} = uuu \uparrow\uparrow\uparrow \sqrt{\tfrac{1}{6}}(RGB - RBG + BRG - BGR + GBR - GRB)$$

This asymmetric wf also has zero color, a 'color singlet.'

Nucleon resonances PDG 2019:

Excited states of the nucleon, all from u and d quarks only. All unbound!

Δ's are t=3/2, N's are t=1/2

$\Delta(1232)$ Γ_{FWHM} = "114 to 120 MeV"

$N(1440)$ Γ_{FWHM}= "250 to 450 MeV" ("Roper" resonance, quarks perhaps in an excited spatial state with N=2? arXiv:1909.13732.v2)

Widths $\sim$ 10% of their mass: $\therefore$ unbound to strong interaction decays.

atomic excited states have $\frac{\Gamma}{E} \sim 10^{-6}$

Does the p wf have admixture of N? Δ?

$$|p'\rangle = |p\rangle + \frac{|\Delta\rangle\langle p|H_{Coul}|\Delta\rangle}{\Delta E} + \frac{|\Delta\rangle\langle p|H_{strongISB}|\Delta\rangle}{\Delta E}$$

$$|p'\rangle \overset{?}{\approx} |p\rangle + 10^{-2\text{ to }-3}|\Delta\rangle$$

Particle	J^P	overall
$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**

**** Existence is certain.

N	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

**** Existence is certain

Exchange symmetry Example/Review: coupling 2 spin-1/2 particles

Can couple two spin-1/2 particle to total S=1 or S=0:

$\frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}$	**S=0**	**Antisymmetric under 1$\leftrightarrow$2**
$\uparrow\uparrow$, $\frac{\uparrow\downarrow + \downarrow\uparrow}{\sqrt{2}}$, $\downarrow\downarrow$	**S=1** **m_S=1, 0, -1**	**Symmetric under 1$\leftrightarrow$2**

mantra: "the stretched state is always symmetric"

antisymmetric under exchange: nucleon wavefunction

One ansatz: $\psi = \psi_{\text{flavour}}\psi_{\text{spin}}\psi_{\text{color}}$

Using the antisymmetric color singlet we used for the Δ^{++}, then $\psi_{\text{flavour}}\psi_{\text{spin}}$ must be symmetric under exchange

Both ψ_{flavour} and ψ_{spin} can be symmetric;

both can be antisymmetric; or,

one can break from the ansatz and have terms symmetric under exchange of the first pair and antisymmetric in the second pair.

Young tableaux i.e. Young diagrams are a good way to organize terms of mixed symmetry. I will use those later: the mixed symmetry terms are needed for antisymmeterizing nuclear wf's

the proton wf:

Assuming the same RGB antisymmetric wf for color we used for the Δ^{++}, here is the isospin symmetric wf multiplying the spin symmetric wf :

NOTE: It is possible to write a completely antisymmetric nucleon wavefunction without color! A problem constructs this, showing it gives wrong experimental μ.

1st 2 quarks S=0: $|\tfrac{1}{2},+\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}\Big(|q(1)\uparrow\rangle|q(2)\downarrow\rangle - |q(1)\downarrow\rangle|q(2)\uparrow\rangle\Big)|q(3)\uparrow\rangle$

Let us start by giving the first two quarks different flavors. Equation (2-44) becomes

$$|\tfrac{1}{2},+\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}\Big(|u(1)\uparrow\rangle|d(2)\downarrow\rangle - |u(1)\downarrow\rangle|d(2)\uparrow\rangle\Big)|u(3)\uparrow\rangle$$

The combination of spin and flavor may be symmetrized in two stages. First we shall carry out the process only for the first two quarks and obtain

$$\begin{aligned}|\tfrac{1}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{2}\Big(|u(1)\uparrow\rangle|d(2)\downarrow\rangle - |u(1)\downarrow\rangle|d(2)\uparrow\rangle \\ &\quad +|d(1)\downarrow\rangle|u(2)\uparrow\rangle - |d(1)\uparrow\rangle|u(2)\downarrow\rangle\Big)|u(3)\uparrow\rangle \qquad (2\text{-}45)\end{aligned}$$

Next, we shall generate the others by applying permutations P_{31} and P_{32} on each of the four terms in Eq. (2-45). This gives us a total of 12 terms. On grouping identical terms together, we obtain the quark wave function for a proton with spin orientations of all the quarks indicated explicitly,

$$\begin{aligned}|p\rangle &= \frac{1}{\sqrt{18}}\Big\{2\Big(|u\uparrow\, u\uparrow\, d\downarrow\rangle + |u\uparrow\, d\downarrow\, u\uparrow\rangle + |d\downarrow\, u\uparrow\, u\uparrow\rangle\Big) \\ &\quad -\Big(|u\uparrow\, u\downarrow\, d\uparrow\rangle + |u\uparrow\, d\uparrow\, u\downarrow\rangle + |d\uparrow\, u\uparrow\, u\downarrow\rangle \\ &\quad +|u\downarrow\, u\uparrow\, d\uparrow\rangle + |u\downarrow\, d\uparrow\, u\uparrow\rangle + |d\uparrow\, u\downarrow\, u\uparrow\rangle\Big)\Big\} \qquad (2\text{-}46)\end{aligned}$$

from Wikipedia: μ_{proton} and color

The anomalously large magnetic moment of the proton was discovered in 1933 by Otto Stern in Hamburg.[4][5] Stern won the Nobel Prize in 1943 for this discovery.[6]

By 1934 groups led by Stern, now in Pittsburgh, and I. I. Rabi in New York had independently measured the magnetic moments of the proton and deuteron.[7][8][9] While the measured values for these particles were only in rough agreement between the groups, the Rabi group confirmed the earlier Stern measurements that the magnetic moment for th proton was unexpectedly large.[10][11] Since a deuteron is

Both groups achieved 10% accuracy.
Stern was low by 1 σ.
Rabi was high by 2 σ: Nobel 1944 ☺

The value is much too big for a Dirac particle $\Rightarrow$ *p* has structure

Direct evidence for *p* substructure from e^- scattering at SLAC: 1967-73.

In one of the early successes of the Standard Model (SU(6) theory), in 1964 Mirza A. B. Beg, Benjamin W. Lee, and Abraham Pais theoretically calculated the ratio of proton to neutron magnetic moments to be −3/2, which agrees with th experimental value to within 3%.[20][21][22] The measured value for this ratio is −1.459 898 06(34).[23] A contradiction the quantum mechanical basis of this calculation with the Pauli exclusion principle led to the discovery of the color charge for quarks by Oscar W. Greenberg in 1964.[20]

Greenberg credits 'color as a gauge symmetry' to Nambu

20. ^ a b Greenberg, O. W. (2009), "Color charge degree of freedom in particle physics", *Compendium of Quantum Physics, ed. D. Greenberger, K. Hentschel and F. Weinert, (Springer-Verlag, Berlin Heidelberg P*: 109–111, arXiv:0805.0289, CiteSeerX 10.1.1.312.5798, doi:10.1007/978-3-540-70626-7_32, ISBN 978-3-540-70622-9, S2CID 17512393
21. ^ Beg, M.A.B.; Lee, B.W.; Pais, A. (1964). "SU(6) and electromagnetic interactions". *Physical Review Letters*. **13** (16): 514–517, erratum 650. Bibcode:1964PhRvL..13..514B. doi:10.1103/physrevlett.13.514.
22. ^ Sakita, B. (1964). "Electromagnetic properties of baryons in the supermultiplet scheme of elementary particles". *Physical Review Letters*. **13** (21): 643–646.

Magnetic moment of p in general

$\vec{\mu} = (g_l\vec{l} + g_s\vec{s})\frac{q\hbar}{2mc}$ **Experimentally,** $\mu_p = (1 + 1.79)\frac{e\hbar}{2mc}$

The deviation from the pointlike Dirac value told people very early on the proton is not a pointlike Dirac particle

Since all particles have similar quantized angular momenta $\sim \hbar$, note the $1/m$ scaling. A problem guides you through the classical derivation of the orbital angular momentum part, to try to provide intuition.

$m_{\rm proton}$ **= 938 MeV**

$\therefore$ **If the quark mass were the 'current' mass in QCD, a few MeV, magnetic moments of u, d quarks would be much larger than for the nucleon**

Instead, we get close to the experimental answer iff we tune $m_{\rm quark} \approx m_{\rm nucleon}/3$, the "constituent" mass. People do calculate the 'constituent mass' from QCD binding energy...

Some textbooks are taking ratios of n and p magnetic moments, not comparing the absolute values:

magnetic moment of nucleon, quark model

$$\vec{\mu} = (g_l \vec{l} + g_s \vec{s}) \frac{q\hbar}{2mc}$$

$$\mu \stackrel{def}{=} \langle J, M = J | \vec{\mu} | J, M = J \rangle$$

$$\equiv \langle JJ | \mu_z | JJ \rangle$$

So μ operator is μ_z, does nothing to wf but project out the value.
So each term of these wf's remains orthogonal– you don't mix terms.
(A good test of calculated wf's in many physical systems.)
For nucleons, sum over the 3 constituents:

$$\mu_{\mathrm{p}} = \sum_{i=1}^{3} \mu_i$$

$$|p\rangle = \frac{1}{\sqrt{18}}\Big\{2\Big(|u\uparrow u\uparrow d\downarrow\rangle + |u\uparrow d\downarrow u\uparrow\rangle + |d\downarrow u\uparrow u\uparrow\rangle\Big)$$
$$-\Big(|u\uparrow u\downarrow d\uparrow\rangle + |u\uparrow d\uparrow u\downarrow\rangle + |d\uparrow u\uparrow u\downarrow\rangle$$
$$+|u\downarrow u\uparrow d\uparrow\rangle + |u\downarrow d\uparrow u\uparrow\rangle + |d\uparrow u\downarrow u\uparrow\rangle\Big)\Big\}$$

Square the coefficients, notice the $\pm$ spin projections:

$$\frac{1}{18}\{4((\mu_u + \mu_u - \mu_d)$$
$$+(\mu_u - \mu_d + \mu_u) + (-\mu_d + \mu_u + \mu_u))$$
$$+\frac{1}{18}(0\mu_u + 6\mu_d)\ \}$$

$$\Rightarrow \mu_{\mathrm{p}} = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$

for neutron, switch all u's to d's:

$$\Rightarrow \mu_{\mathrm{n}} = = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$$

Compare μ from quark model to experiment

$\mu_{\rm p} = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$
$\mu_{\rm n} = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$

- an individual magnetic moment
$\mu^i_z = s^i_z \frac{q^i}{e}\frac{e\hbar}{m_q c}$
scales with charge/mass

In 'constituent quark model' (?!)
Assume same mass (?!) so $\mu_u = -2\mu_d$ from the ratio of charges, then
μ_p = -3 μ_d and μ_n = 2 μ_d, and

- μ_p/μ_n = -1.5. Measured ratio is -1.46 ☺.

Further assuming $m_q \approx m_{\rm nucleon}/3$ (?!)

- μ_p = 3 $\mu_{\rm nucleon}$ Measured value is 2.79 ☺
- μ_n = - 2 $\mu_{\rm nucleon}$ Measured value is -1.91 ☺

This builds a consistent picture. Wong Table 2-4 shows similar results for other baryons including *s* quark.
But 'current' mass of the quarks in QCD is a few MeV, and $m_d > m_u$. If these classical ideas were the whole story, μ_u and μ_d should be 100x bigger.
This is part of the 'spin puzzle' where the spin of the nucleon also has components from the gluons (although a great deal of this is studying the spin structure at small distances by high momentum transfer). Also, people do work on calculating the 'constituent mass' by QCD binding energy.

μ of baryons

Table 2-4: Magnetic dipole moment of baryon octet.

Octet member	Quark content			Best fit	Observed
	u	d	s	μ_N	μ_N
p	$\frac{4}{3}$	$-\frac{1}{3}$	0	2.793	2.792847386(63)
n	$-\frac{1}{3}$	$\frac{4}{3}$	0	−1.913	−1.91304275(45)
Λ	0	0	1	−0.613	−0.613(4)
Σ^+	$\frac{4}{3}$	0	$-\frac{1}{3}$	2.674	2.458(10)
Σ^-	0	$\frac{4}{3}$	$-\frac{1}{3}$	−1.092	−1.160(25)
Ξ^0	$-\frac{1}{3}$	0	$\frac{4}{3}$	−1.435	−1.250(14)
Ξ^-	0	$-\frac{1}{3}$	$\frac{4}{3}$	−0.493	−0.6507(25)
$\Sigma^0 \to \Lambda$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{3}}$	0	−1.630	−1.61(8)
Ω^-			3	−1.839	−2.02(5)
u	1			1.852	
d		1		−0.972	
s			1	−0.613	

- **Fit parameters μ_u, μ_d, μ_s to all the measured baryon magnetic moments, which are reproduced pretty well. Consistent with our nucleon-only numbers, but adding redundancy.**
- **Note μ_d and μ_u are pretty close to the factor of -2 from known charge ratio assuming same mass. Note μ_s is not close.**
- **These effective magnetic moments of the quarks while bound into hadrons seem to be guided by their 'constituent' masses. Martinelli PLB 116 434 (1982) lattice QCD for μ_p, μ_n It might be interesting to compare μ_ρ = 2.1 $\pm$ 0.5 $\frac{e}{2m_\rho}$ (BABAR 2015) in a quark model.**

Electromagnetism alters μ_e in many-body systems

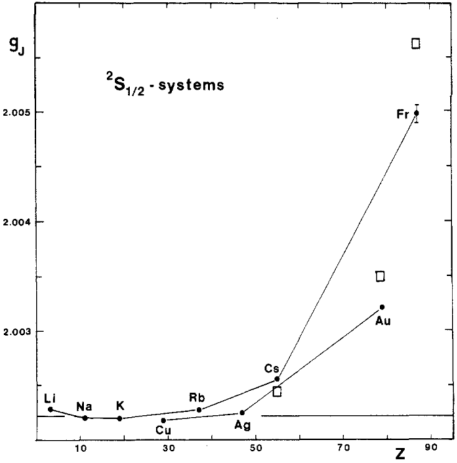


Fig. 2. Experimental g_J-values of the electronic ground states $ns\ ^2S_{1/2}$ in the alkali elements and in the noble metals. The result in francium is from the present work. A strong deviation from the horizontal line, representing the Landé value $g_J = 2.002319$, is observed with increasing atomic number. The theoretical results from Refs. [12, 13] are given by squares.

Ekström, Robertsson, Rosén, Phys Scripta Vol. 34 624 (1986)

- **The magnetic moment of the electron changes in high-Z atoms by 0.15%.**
- **Maybe it's unsurprising for μ_u, μ_d to be altered from 'free quark' values in a strongly-interacting many-body system.**

Summary Phys 505 Lectures 2,3

• Isospin is a slightly broken symmetry in the strong interactions, providing understanding of reactions and decays.

• Antisymmetry of fermion wf's (implementing Pauli exclusion) $\psi(1,2) = -1\,\psi(2,1)$ has many phenomenological consequences. To construct quark wf's for baryons that match experiment, one needs the quantum number 'color'

• Together, isospin symmetry and antisymmetry will have many consequences for nuclear structure that don't depend much on other details of the interaction.

• The magnetic moment of baryons is reproduced in the constituent quark model, a major success historically. This is likely trying to tell us that the constituent mass that comes as QCD binds the quarks into hadrons is the important driver of this physics, not the 'current' mass of the quarks in the QCD Lagrangian. Hence the 'spin puzzle' is being addressed at smaller scales probed at higher momentum transfer, eventually at the new EIC, which will also address Goldstone boson physics and the π structure