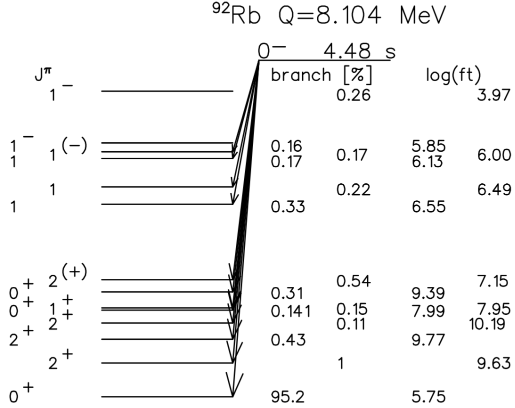
1) G-T and 1st-forbidden competing: Reactor ν's and ^{92}Rb decay (20 points)

$^{92}_{37}\mathrm{Rb}^{55}$ $\sim$10% of reactor ν's 5-7 MeV

Levels from 2012 NDS compilation:



- Remember t = ln(2)/(partial decay rate) = (decay's $t_{1/2}$)/(branch fraction).

a) List one pair of possible orbitals for the odd proton and neutron of the ^{92}Rb $J^\pi = 0^-$ ground state. Note their relative π.

b) for 3 sets of states, comment on:
whether the β^- decay is allowed or forbidden, and on the size of the $log_{10}ft$ values:

i) the J^π=0^+ ground state (branch 95.2%);
ii) the highest $J^\pi = 1^-$ state;
iii) the six other J=2 and J=0 states.

c) β^- decay to the highest $J^\pi = 1^-$ state (at 7.363 MeV excitation) has log(ft)=3.97, much faster than the 0^+ ground state with log(ft)=5.75. Given the maximum β kinetic energy, get 'Fermi integral' *f* (Wong Eq. 5-69) from Wong Fig. 5.7 for these two transitions (assuming $0^- \rightarrow 0^+$ is 'allowed'). Check these listed branch fractions and log(ft)'s for consistency.

d) Re: the other six J=1 states clustered together, state the compiler's reasoning and threshold log(ft) for determining allowed vs. forbidden and $\therefore$ parity

1a) Z=37, 3 short of closed Z=40 suggests f5/2 (parity minus)
N=55 closed N=50 + 5, suggests g7/2 (parity plus)
OK to add 5/2 and 7/2 and get 0. Parity is minus.
(Common in fission products to have ‘valence’ n and p in opposite-parity orbitals)
1b) i) 0- to 0+ changes nuclear parity so is ‘1st-forbidden.’ log(ft) of 5.75 is one of the faster forbidden ones in the histogram Fig. 5-8 from Wong.
ii) log(ft) 3.97 suggests a fast Gamow-Teller. 0- to 1- keeps nuclear parity and changes J by 1, satisfies G-T selection rule.
iii) 0- to these 0+ and 2+ are all positive parity; all are 1st-forbidden. log(ft)’s are more than 10x slower than the strong 0- to 0+ g.s to g.s.
1c) Q=8.104 MeV is highest possible kinetic energy. Since Wong plots down to 0.1 MeV, less than mass of electron, he is clearly plotting maximum kinetic energy on x-axis(despite somewhat nonstandard E0 notation). 0- to 0+ has two operators, though it’s commonly assumed one dominates for these higher-energy transitions, so it’s ok to assume ‘allowed.’ Z=37 Q=8 MeV has log(f)=4.3 to 4.5 or so.
transition to the highest J=1- at 7.363 then has 8.104-7.363= 0.741 MeV, about f=-1.5. Branch should be lower by log(4.5-(-1.5)= log(6) = 10^6 from the momentum integral f alone. The log(ft) is 3.97 instead of 5.75, so that G-T (matrix element)^2 is 10**(1.78) times bigger, and rate scales with (matrix element)^2. 6-1.78= 4.22 or 10^4 times smaller. The actual branch is 300x smaller, so something is not right here.

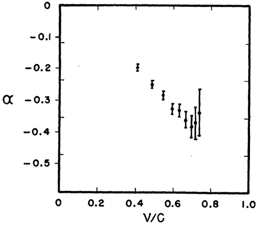
1d) The compiler is saying a log(ft) of smaller than 6 is G-T, which is plausible but might not be perfect. One has to look very carefully at any state where one needs the answer.

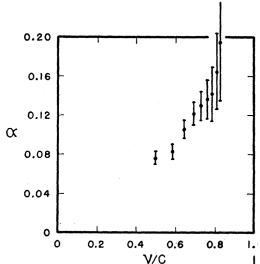
Some qualitative considerations I did not go through:
Many of the 0- to 0+ transitions seem to have log(ft) at 6 or faster, faster than most 1st-forbidden transitions.
One doesn't get opposite-parity states, which are needed to get G-T transitions to bleed strength from the higher-energy nu's, until fairly high excitation. The g.s. to g.s. transition is often then the largest branch, making more energy come out in higher-energy ν's instead of γ's.
That highest 1^- state is likely part of the low-Ex tail of the giant dipole resonance, which has centroid energy (phenomenological from Berman+Fultz RevModPhys 47 713 (1975)) E_{GDR}=31.2$A^{-1/3}$ + 20.6 $A^{-1/6}$ = 16.6 MeV with full width half maximum $\sim$ 5 MeV.) This is one reason for the "pandemonium effect: strong E1 transitions at 5 MeV produce a forest of narrow lines with poor Germanium efficiency that makes them very hard to detect. (If both parent and progeny have same parity, the "Giant Gamow-Teller" resonance produces states with a similar role.) Total absorption spectrometers, 4 pi arrays of high-Z scintillator (with inherently poorer energy resolution)help by absorbing all the gamma energy and measuring beta feeding patterns averaged over many states.

2) β^- vs. β^+ asymmetry with respect to spin (20 points)

E. Ambler, R. W. Hayward, D. D. Hoppes, R. P. Hudson, and C. S. Wu

Phys. Rev. 106, 1361 (1957)





Ambler measured the $2^+ \rightarrow 2^+$ β^+ decay of ^{58}Co to have A_β “roughly one third the magnitude and opposite sign” of ^{60}Co.

a) Why is the “opposite sign” interesting?

b) Explain “one third the magnitude” using the equation for pure G-T on p. 54 of Lecture 16-18.

Which graph is ^{58}Co? Which graph is ^{60}Co?

c) The symbol $\alpha = A_\beta$. Given the angular distribution

$W[\theta] = 1 + AP\frac{v}{c}\cos[\theta]$

extrapolate the data simply (by ruler) to $\frac{v}{c}=1$ and read off the measurement of AP.

Deduce the nuclear polarization P in each case.

(Should the polarizations be similar? Yes. The dilution refrigerator + B field selects the lowest-energy state, given by $\mu_{\text{nuclear}} \cdot B_{\text{effective}}[\text{nucleus}]$. The effective B field is the same for any Co atom, and the μ’s are measured to be within 10%.]

(The Fermi operator contribution to ^{58}Co decay has been measured separately (it changes γ-ray polarization...) and is small enough to ignore at the level of accuracy considered here.)

I didn't explain features of the expression on p. 43 from JTW Phys Rev 106 517 (1957). (You should recognize the allowed decay expression $p_e E_e p_\nu E_\nu$. The Fermi function F(Z,E) is included in the later Coulomb corrections paper from Nucl Phys A quoted on p. 56.) This is the general decay distribution as a function of E_β and β and ν angle. All these correlations appear, including the *a* and *A* terms. If you integrate over ν angles (i.e. if you don't measure the ν), then the *a*, *c*, *B*, *D* terms vanish, and your experiment measures the β asymmetry A term. (bm/E is a normalization that is zero in the S.M.) If you average over the initial spin polarizations of the nucleus, the *c*, *A*, *B*, *D* terms vanish, and you're left with the $\beta - \nu$ correlation *a* term.

2) a) beta- and beta+ are lepton and antilepton and should have opposite helicity. (note though that the lambda on p. 54 depends on J and J', so one has to be careful). One could also say that CP is looking like it's conserved, since C is different and the P breaking is changing sign.
b) beta asymmetry wrt nuclear spin Abeta=A for 2+ to 2+ GT is +J/(J+1)=+1/3 for a positron emitter (compared to -1 for 5+ to 4+ for a beta minus decay).
So bottom graph must be from 58Co and top graph from 60Co.
c) AP is about 2/3 of the calculation in each case, so P is about 2/3 (the experimentalists note this in their paper– they have an independent measurement from the anisotropy of the gamma ray emission that agrees.)

3) ^{131}Cs decays by electron capture, J^{π}= $5/2^{+} \rightarrow 3/2^{+}$. (10 points)
Assuming an angular distribution
$W(\theta) = 1 + A_{\nu} P cos(\theta)$
and polarization P=1, compute A_{ν} assuming the SM left-handed ν helicity.

131Cs + electron $\rightarrow$ 131Something + neutrino
m initial = m final
Assume fully polarized up for 'derivation'
m Cesium + m electron = m (spin 3/2) + m(neutrino)
+5/2 + +- 1/2 = ($\leq$ +3/2) + m(nu)
want to find null of angular distribution, so look for projection to be not allowed by angular momentum conservation.
Consider nu going straight up. Then left-handed m(nu) always has projection -1/2.
At least 2 on left-hand side, no more than 1 on right-hand side, so this is forbidden.
Nu can't go straight up, cos(0)=1 in that case, so Anu= -1.
(If try nu going down, m(nu) = + 1/2, so it's possible to satify the spin projection conservation.)