

1) ^{18}F has 2 nucleons in sd shell ($2s_{1/2}$, $d_{3/2}$, $d_{5/2}$) \rightarrow

Consider ψ_{space}

s s

a) Using the $A=6$ argument (page 10, Lecture 13):

What (S, T) are possible? What $(J^\pi; T)$ in ^{18}F can you explain?

2 antisym nucleons: $(S; T) = (1; 0), (0; 1) \Rightarrow$

$J^\pi; T = 0^+; 1$ and $1^+; 0$ only, like the deuteron

Consider another symmetric configuration for ψ_{space}

s d

b) What is the only possible total L (by inspection)? $|\vec{2} \pm \vec{0}| = 2$

What $J^\pi; T = 1$? $2^+; T = 1$ (8th excited state)

(i.e. $\vec{J} = \vec{L} + \vec{S} = \vec{L} + \vec{0} = \vec{L} \Rightarrow$ for $T=1, J=L$ only!)

What $J^\pi; T = 0$? $|\vec{2} \pm \vec{1}| \Rightarrow J^\pi = 1, 2, 3; T = 0$

Consider ψ_{space}

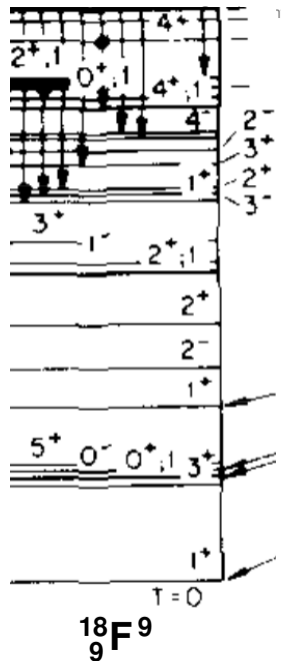
d d

Only even L 's are symmetric on the M-scheme table (next page).

c) Using only the even-numbered L 's, what $T=1$ states do you predict this way (note they are all clustered together)? $T=0$?

$J^\pi = 0^+, 2^+, 4^+; T = 1$ and $J^\pi = 1^+, 2^+, 3^+, 4^+, 5^+; T = 0$

These account for all the $\pi = +$ states in the level diagram ☺



(The $\pi = -$ states include excitations from the 1p shell) Only 'antisymmetric for 2 particles' (S,T)= (1,0) and (0,1) are allowed for the symmetric space configurations considered. Some states from naive jj coupling (notably T=1 J odd) are not seen– they don't have symmetric space configuration under permutation, so are expected to lie at much higher excitation.

an M-scheme table for dd configuration (used on previous page) HowTo →

$M = 4$	<table border="1"><tr><td>2</td><td>2</td></tr></table>	2	2												
2	2														
$M = 3$	<table border="1"><tr><td>2</td><td>1</td></tr></table>	2	1	<table border="1"><tr><td>2</td></tr><tr><td>1</td></tr></table>	2	1									
2	1														
2															
1															
$M = 2$	<table border="1"><tr><td>2</td><td>0</td></tr></table>	2	0	<table border="1"><tr><td>2</td></tr><tr><td>0</td></tr></table>	2	0	<table border="1"><tr><td>1</td><td>1</td></tr></table>	1	1						
2	0														
2															
0															
1	1														
$M = 1$	<table border="1"><tr><td>2</td><td>-1</td></tr></table>	2	-1	<table border="1"><tr><td>2</td></tr><tr><td>-1</td></tr></table>	2	-1	<table border="1"><tr><td>1</td><td>0</td></tr></table>	1	0	<table border="1"><tr><td>1</td></tr><tr><td>0</td></tr></table>	1	0			
2	-1														
2															
-1															
1	0														
1															
0															
$M = 0$	<table border="1"><tr><td>2</td><td>-2</td></tr></table>	2	-2	<table border="1"><tr><td>2</td></tr><tr><td>-2</td></tr></table>	2	-2	<table border="1"><tr><td>1</td><td>-1</td></tr></table>	1	-1	<table border="1"><tr><td>1</td></tr><tr><td>-1</td></tr></table>	1	-1	<table border="1"><tr><td>0</td><td>0</td></tr></table>	0	0
2	-2														
2															
-2															
1	-1														
1															
-1															
0	0														
	L=4	L=3	L=2	L=1	L=0										

The goal of an M -scheme table is to assign permutation symmetry to each possible value of L . (I left this implicit.) Here is how to make them (easier with a blackboard or a pencil):

- Write all the possible combinations of m 's that add up to a given M , with all allowed symmetries under permutation following the Young diagram rules.

(When I line them up in a nice table? I'm ignoring the actual work:)

- Grouping them by permutation symmetry, assign them to an L . (There is likely a formal proof that all configurations for a given L must have the same permutation symmetry— it seems plausible.) Handy tricks:

(Ignore negative M — these are obvious from nonnegative M and don't add info.)

First consider “the stretched state is always symmetric” and assign the max ℓ symmetric configuration to $L = \max M$. Find the rest of the M 's needed, with same symmetry, to account for that max L . (Then I line them up in the nice table— traditionally one just crosses them off on a blackboard).

Continue to gather all the M 's one needs for each L , all with given permutation symmetry. The rest usually shake down from there.

One gets an orphan single $M=0$ state that one assigns to $L=0$.

This is just a plausibility argument. There is likely a formal proof that this procedure gives you the correct permutation symmetry for each L .