1) μ in independent particle model

	J^π	$\mu_{\text{experiment}}$
${}^{209}_{83}\mathrm{Bi}^{126}$	$9/2^-$	4.1106(2)
${}^{209}_{82}\mathrm{Pb}^{127}$	$9/2^+$	-1.4735(16)
${}^{213}_{87}\mathrm{Fr}^{126}$	$9/2^-$	4.02(8)

Consider Mayer's shell diagram on p. 7 of Lecture 13-14; Wong Eq. 4-53 or p.4-5 of Lecture 13-14 for $\mu_{\text{single-particle}}$. (Remember the μ_p=1 'Dirac particle' lines are speculative)

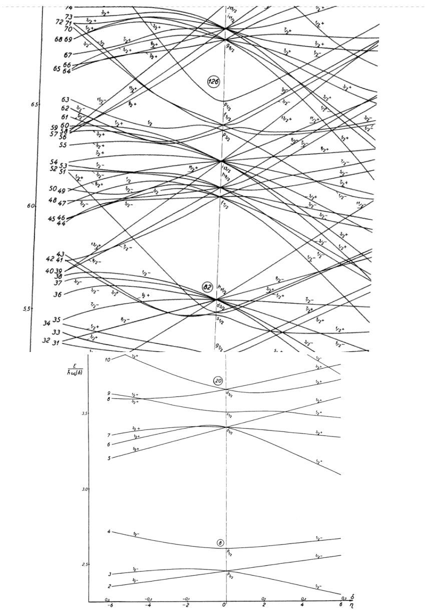
Assuming the configuration is 1 unpaired nucleon, state likely orbital, and compare $\mu_{\text{single-particle}}$ to $\mu_{\text{experiment}}$ for:

a) ^{209}Bi

b) ^{209}Pb

c) ^{213}Fr, particularly compared to ^{209}Bi

d) This diagram from Sven Gösta Nilsson, Dann Mat Fys Medd 29 1 (1955) **shows a Nilsson model calculation with $f_{7/2}$ orbital lower in energy than $h_{9/2}$ orbital. Would deformation favor any $9/2^-$ states other than the $h_{9/2}$ orbital? (Expand the diagram to see the Ω values to left and right of center, the total angular momentum of the state.)**



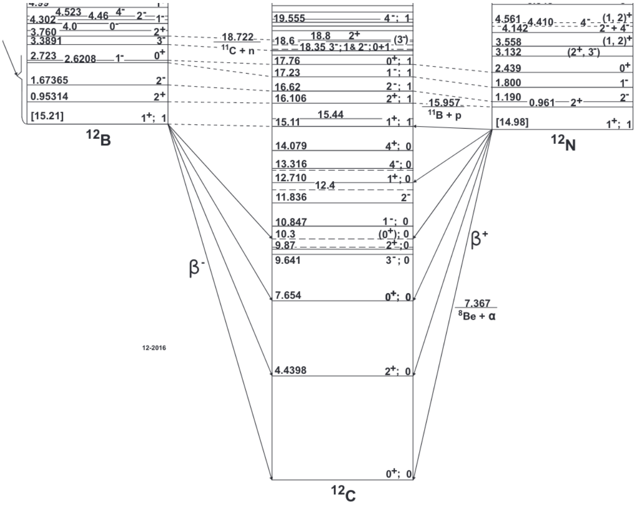
2. Considering ^{12}C as closed core ^{8}Be with 4 p-shell nucleons, similar to 8Be

on p. 26-27 of L12-13 (do not re-derive), what features of ^{12}C can be explained?

a) Assuming $\psi_{\rm space}$ is symmetric, what is L for the $0^+, 2^+, 4^+$; $T = 0$ states in ^{12}C?

b) What does this L tell you qualitatively about the α decay rate of the 4^+; $T = 0$ state?

c) Can the negative parity states of ^{12}C be explained with 4 p-shell nucleons?



Consider the 0^+; $T = 0$ state at 7.654 MeV, the state predicted by Hoyle to help produce ^{12}C by two reactions:

$^{8}\mathrm{Be} + \alpha \rightarrow ^{12}\mathrm{C}$ **and 3 α's $\rightarrow$ ^{12}C**

d) State $^{2s+1}L_J$ for the two possible $J^\pi = 0^+$; $T = 0$ configurations for 4 p-shell nuclei, including the one in part a).

e) Besides the one in part a), is there a configuration with $\psi_{\rm space}$ symmetric?

f) If $\psi_{7.654}$ includes the other 0^+; $T = 0$ configuration, how would that change Hoyle's rates?

3) Consider Moshinsky's H-F calculation on p. 29 of Lecture 12-13.

a) Harmonic oscillators are infinite at infinity. Does this cause an issue with the HF integrals in this toy system?

b) By inspection, can Moshinsky's exact solution be written as an antisymmeterized product of Moshinsky's single-particle wf's? Why or why not? Qualitatively and briefly, what does that tell you about the accuracy of the HF wf's?

4. Do not repeat this derivation from O&S. Please read it through for a few minutes to appreciate the clean result, then answer by inspection 2 simple questions at the bottom. t_0 is a parameter, not an isospin projection.

Problem

3.3 Calculate the matrix element of the two-body contact interaction $t_0\delta(\mathbf{r}_1 - \mathbf{r}_2)$ with anti-symmertized two-particle wave function for an even-even nucleus with $N = Z$.

3.3 The anti-symmetrized matrix element is expressed as

$$\langle \widetilde{V} \rangle \equiv \frac{1}{2}\sum_{i,j}^{A} \langle ij|\widetilde{V}(\mathbf{r}_i, \mathbf{r}_j)|ij\rangle$$
$$= \frac{1}{2}t_0 \sum_{i,j}^{A} \int\int \phi_i(\mathbf{r}_1)^*\phi_j(\mathbf{r}_2)^*\delta(\mathbf{r}_1 - \mathbf{r}_2)(1 - P_r P_\sigma P_\tau)\phi_i(\mathbf{r}_1)\phi_j(\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2,$$

where P_r, P_σ and P_τ are the space, the spin and the isospin exchange operators, respectively. Since the δ interaction acts on the relative S-wave between two particles, the space exchange operator has the sign, $P_r = (-)^l = +1$. The spin and isospin operators can be expressed as $P_\sigma = (1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)/2$ and $P_\tau = (1 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)/2$, the matrix element is evaluated as

$$\langle \widetilde{V} \rangle = \frac{1}{2}\sum_{i,j}^{A} \langle ij|\widetilde{V}(\mathbf{r}_i, \mathbf{r}_j)|ij\rangle$$
$$= \frac{1}{2}t_0 \sum_{i,j}^{A} \int\int \phi_i(\mathbf{r}_1)^*\phi_j(\mathbf{r}_2)^*\delta(\mathbf{r}_1 - \mathbf{r}_2)$$
$$\times \left(\frac{3}{4} - \frac{1}{4}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \frac{1}{4}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \frac{1}{4}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right)\phi_i(\mathbf{r}_1)\phi_j(\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2. \qquad (3.248)$$

For even-even $N = Z$ nucleus, the spin and isospin dependent terms have no contributions to the matrix elements since the sum of the diagonal matrix elements cancels out. Consequently, we have only a finite contribution from the first term in the equation which is given as

$$\langle \widetilde{V} \rangle = \frac{3}{8}t_0 \int \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2$$
$$= \frac{3}{8}t_0 \int \rho(\mathbf{r})^2 d\mathbf{r}. \qquad (3.249)$$

a) How would the answer have changed if O&S had simply ignored the exchange term, i.e. ignored the antisymmetry?

b) For many years, theorists working in HF and KS theory would only calculate N even, Z even nuclei. By inspection, list 1 reason why.

JB thought about trying to relate t_0 to the 1st semiempirical mass term $\alpha_{\text{Volume}}A$, i.e. setting $\int \rho(r)^2 d^3r \propto (\int \rho(r) d^3r)^2$. This only works for uninteresting cases with ρ=constant.

5) ^{18}F has 2 nucleons in sd shell ($2s_{1/2}$, $d_{3/2}$, $d_{5/2}$) $\rightarrow$

Don't use jj coupling, which produces one unphysical state

Consider ψ_{space}

s	s

a) Using the A=6 argument (page 9, Lecture 13):

What (S, T) are possible? What (J^π; T) in ^{18}F can you explain?

Consider another symmetric configuration for ψ_{space}

s	d

b) What is the only possible total L (by inspection)?

What J^π; $T = 1$ are possible?

What J^π; $T = 0$ are possible ?

Consider ψ_{space}

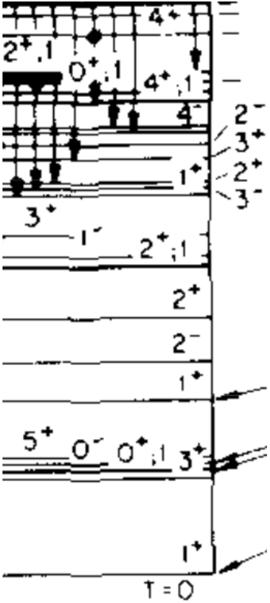
d	d

Only even L's are symmetric on the M-scheme table (next page).

c) Using only the even-numbered L's, what T=1 states do you predict this way (note they are all clustered together)? T=0?

These account for all the $\pi = +$ states in the level diagram ☺

The $\pi = -$ states include excitations from the 1p shell



$^{18}_{9}$F$_9$

an M-scheme table for dd configuration (used on previous page)

$M = 4$	2 | 2				
$M = 3$	2 | 1	2 1			
$M = 2$	2 | 0	2 0	1 | 1		
$M = 1$	2 | -1	2 -1	1 | 0	1 0	
$M = 0$	2 | -2	2 -2	1 | -1	1 -1	0 | 0
	L=4	L=3	L=2	L=1	L=0