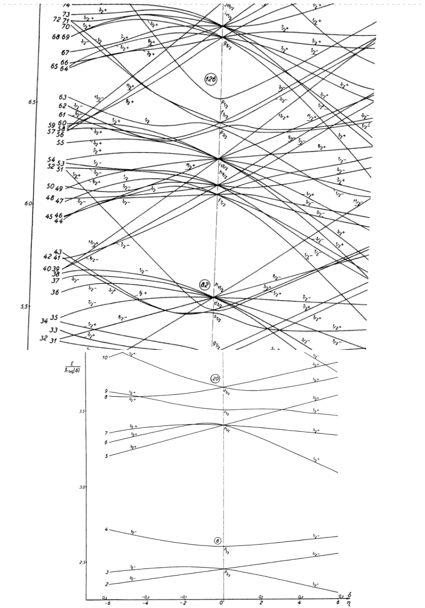
1) μ in independent particle model answers

	J^π	$\mu_{\mathrm{experiment}}$	orbital	μ_{sp}
$^{209}_{83}Bi^{126}$	$9/2^-$	4.1106(2)	$\pi(h_{9/2})$	2.62
$^{209}_{82}Pb^{127}$	$9/2^+$	-1.4735(16)	$\nu(g_{9/2})$	$+\mu_{\mathrm{neutron}}$
$^{213}_{87}Fr^{126}$	$9/2^-$	4.02(8)	$\pi(h_{9/2})$	like ^{209}Bi



Consider Mayer's shell diagram on p. 7 of Lecture 12-13; Wong Eq. 4-53 or p.4-5 of Lecture 12-13 for $\mu_{\mathrm{single-particle}}$. Assuming the configuration is 1 unpaired nucleon, state likely orbital, and compare $\mu_{\mathrm{single-particle}}$ to $\mu_{\mathrm{experiment}}$ for:

a) ^{209}Bi

$$\frac{\mu_{\mathrm{s.p.}}}{\mu_{\mathrm{nm}}} = j\left(g_l \pm \frac{g_s - g_l}{2l+1}\right) \text{ for } j = l \pm \frac{1}{2} \quad \text{W4-53}$$

g_l=1, $g_s = 5.586$ for proton, l=5 so $j = l - 1/2$.

9/2*(1-(5.586-1)/11) = 2.62

much lower, even doubly magic $^{208}_{82}Pb^{126}$ does not forbid more complex configurations

b) ^{209}Pb $\nu(g_{9/2})$ + ^{208}Pb core, l=4, $g_l = 0$, $j = l + 1/2 \Rightarrow$

$\mu_{sp} = 9/2(+\frac{-3.826}{9}) = +\mu_{\mathrm{neutron}}$. Note $|-1.4735| < |-1.9130427(5)|$,

μ_{sp} neutron also has non-single particle configurations

c) ^{213}Fr, particularly compared to ^{209}Bi same prediction

Goeppert Mayer's figure shows lines for μ_p=1 ('to emphasize division into 2 groups') agree with ^{209}Bi $h_{9/2}$ single-particle prediction, but this was not a physics prediction.

1c continued: Blin-Stoyle wrote a paper in 1954 explaining the discrepancy between ^{209}Bi and the single-particle magnetic moment by including configurations with other unpaired nucleons.

Very recently, there were interesting differences between this measured ^{209}Bi magnetic moment and a measurement from hydrogen-like ^{209}Bi in a storage ring. This was resolved by careful molecular corrections and measurements that lowered this 2005 experimental value by about 1%.

d) The diagram from Sven Gösta Nilsson, Dann Mat Fys Medd 29 1 (1955)
shows a Nilsson model calculation with $f_{7/2}$ orbital lower in energy than $h_{9/2}$ orbital. Would deformation favor any $9/2^-$ states other than the $h_{9/2}$ orbital?
(Expand the diagram to see the Ω values to left and right of center, the total angular momentum of the state.)

$i_{13/2}$ has + parity, so can't explain spin parity $9/2^-$.
So if Nilsson's level ordering is right, then only $h_{9/2}$ has a $9/2^-$ state dipping below undeformed $f_{7/2}$ in energy,
and $h_{9/2}$ at larger negative deformation also wins over $f_{7/2}$ at the same deformation.
A negative deformation parameter is consistent with oblate deformation.
It does turn out ^{209}Bi and ^{207,209,211,213}Fr have experimentally negative electric quadrupole moments consistent with small oblate deformation.

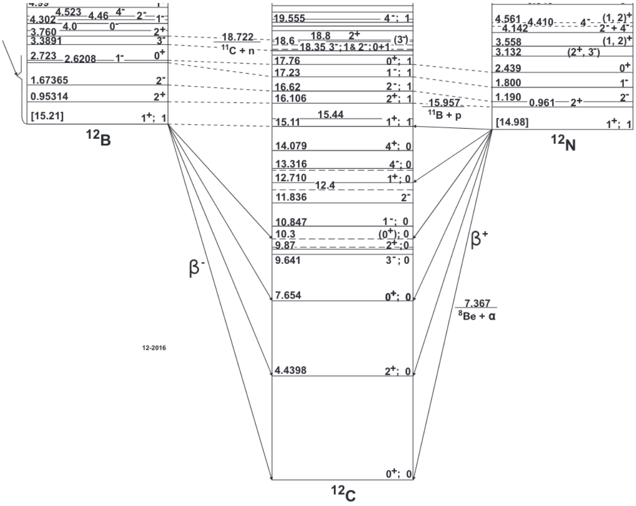
2. Considering ^{12}C as closed core ^{8}Be with 4 p-shell nucleons, similar to 8Be

on p. 26-27 of L12-13 (do not re-derive), what features of ^{12}C can be explained?

a) Assuming ψ_{space} is symmetric, what is *L* for the $0^+, 2^+, 4^+; T = 0$ states in ^{12}C?

b) What does this *L* tell you qualitatively about the α decay rate of the $4^+; T = 0$ state?

c) Can the negative parity states of ^{12}C be explained with 4 p-shell nucleons?



Consider the $0^+; T = 0$ state at 7.654 MeV, the state predicted by Hoyle to help produce ^{12}C by two reactions:

$^{8}\text{Be} + \alpha \rightarrow ^{12}\text{C}$ and 3 α's $\rightarrow$ ^{12}C

d) State $^{2s+1}L_J$ for the two possible $J^\pi = 0^+; T = 0$ configurations for 4 p-shell nuclei, including the one in part a).

e) Besides the one in part a), is there a configuration with ψ_{space} symmetric?

f) If $\psi_{7.654}$ includes the other $0^+; T = 0$ configuration, how would that change Hoyle's rates?

Solutions:

2a) L=0,2,4 from the symmetric space partitions (given S=T=0 for the $\psi_{\text{spin}-\text{isospin}}$ antisymmetric partition)

2b) JB guessed naively this state with large L would have α emission suppressed, but it needs a calculation to quantify.
E.g. the isobar diagram for ^{8}Be indicates α decay widths (the width of the band of diagonal lines) which are larger for the 4^+ than the 2^+, and the energetics are similar to ^{12}C.

2c) Parity is given by the product of the parity of the single-particle configurations, $(-1)^4$ = +1, so no, whatever the space permutation partition.

2d) $^1S_0$ and $^3P_0$ (from the mixed partition on page 27)

2e) no, not completely symmetric.

2f) The mixed symmetry partition is in our lowest-order simple picture not expected to be part of the α ground state. Reaction and decay rates involving α's will drop with the fraction of this configuration.

3) Consider Moshinsky's H-F calculation on p. 29 of Lecture 12-13.

a) Harmonic oscillators are infinite at infinity. Does this cause an issue with the HF integrals in this toy system?

No. The wf's have to fall fast enough towards infinity. Plus, Moshinsky is using an analytic solution to the H-F equations.

b) By inspection, can Moshinsky's exact solution be written as an antisymmeterized product of Moshinsky's single-particle wf's? Why or why not?

No, although $R^2 = r_1^2 + r_2^2$, the coefficient in front of R^2 in the exponential is different.

c) Qualitatively and briefly, what does that tell you about the accuracy of the HF wf's?

JB has added the "c" label explicitly to organize grading?

The exact wf breaks the H-F ansatz of a product of single-particle wavefunctions, so it's unclear that the HF-derived wf's will be accurate.
More generally, the variational principle used to derive them implies the resulting energy will be the minimum available with the ansatz for the interactions used, but the information condensation into density integrals and other integrations implies that the H-F equations may not guarantee much about the accuracy of the wavefunctions to use with other more complex operators.

4. Do not repeat this derivation from O&S. Please read it through for a few minutes to appreciate the clean result, then answer by inspection 2 simple questions at the bottom. t_0 is a parameter, not an isospin projection.

Problem

3.3 Calculate the matrix element of the two-body contact interaction $t_0\delta(\mathbf{r}_1 - \mathbf{r}_2)$ with anti-symmertized two-particle wave function for an even-even nucleus with $N = Z$.

3.3 The anti-symmetrized matrix element is expressed as

$$\langle \widetilde{V} \rangle \equiv \frac{1}{2}\sum_{i,j}^{A} \langle ij|\widetilde{V}(\mathbf{r}_i, \mathbf{r}_j)|ij\rangle$$
$$= \frac{1}{2}t_0 \sum_{i,j}^{A} \int\int \phi_i(\mathbf{r}_1)^*\phi_j(\mathbf{r}_2)^*\delta(\mathbf{r}_1 - \mathbf{r}_2)(1 - P_r P_\sigma P_\tau)\phi_i(\mathbf{r}_1)\phi_j(\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2,$$

where P_r, P_σ and P_τ are the space, the spin and the isospin exchange operators, respectively. Since the δ interaction acts on the relative S-wave between two particles, the space exchange operator has the sign, $P_r = (-)^l = +1$. The spin and isospin operators can be expressed as $P_\sigma = (1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)/2$ and $P_\tau = (1 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)/2$, the matrix element is evaluated as

$$\langle \widetilde{V} \rangle = \frac{1}{2}\sum_{i,j}^{A} \langle ij|\widetilde{V}(\mathbf{r}_i, \mathbf{r}_j)|ij\rangle$$
$$= \frac{1}{2}t_0 \sum_{i,j}^{A} \int\int \phi_i(\mathbf{r}_1)^*\phi_j(\mathbf{r}_2)^*\delta(\mathbf{r}_1 - \mathbf{r}_2)$$
$$\times \left(\frac{3}{4} - \frac{1}{4}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \frac{1}{4}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \frac{1}{4}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right)\phi_i(\mathbf{r}_1)\phi_j(\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2. \quad (3.248)$$

For even-even $N = Z$ nucleus, the spin and isospin dependent terms have no contributions to the matrix elements since the sum of the diagonal matrix elements cancels out. Consequently, we have only a finite contribution from the first term in the equation which is given as

$$\langle \widetilde{V} \rangle = \frac{3}{8}t_0 \int \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2$$
$$= \frac{3}{8}t_0 \int \rho(\mathbf{r})^2 d\mathbf{r}. \quad (3.249)$$

a) How would the answer have changed if O&S had simply ignored the exchange term, i.e. ignored the antisymmetry?

$P_r P_\sigma P_\tau \rightarrow 0$. By inspection, the constant 3/4 in Eq. 3.248 is instead 1, so $\langle \tilde{V} \rangle = \frac{1}{2} t_0 \int \rho(\mathbf{r})^2 d\mathbf{r}$, not $\frac{3}{8}$. Uncontrolled approximation, big change

b) For many years, theorists working in HF and KS theory would only calculate N even, Z even nuclei. By inspection, list 1 reason why.

τ_i terms nonzero for $N \neq Z$, σ_i terms nonzero for N and/or Z odd: significant complications even for this simplest Skyrme force term

Details on 4a)

Instead of writing out $\phi(r)$'s with indices swapped, or using annihilation/creation operators, Sagawa uses a tweener approach, including 'exchange operators' $P_r P_\sigma P_\tau$. Then he writes out each one in terms of appropriate Pauli matrices (JB could have referenced or asked for a derivation of these but instead wanted us all to just accept it), and rewrites in eq. 3.248:

$$(1 - P_r P_\sigma P_\tau) = \frac{3}{4} - Function(this \cdot that)$$

Note the striking change in the 'not-explicitly-exchanged' constant from 1 to 3/4, which comes from including the exhange term.

Then he notes that all the 'this dot that' terms vanish for N=Z, leaving that constant term. A clean result for N=Z for this formalism and this simple contact part of the Skyrme force.

If you intentionally left out the exchange operator term from the start, the final answer isn't scaled by 3/4 (as in red on the previous page), which is terrible.

Leaving out the exchange term is sometimes done and called a "Hartree" calculation– people only do this if they have some knowledge that term is small in similar systems, because otherwise it's an uncontrolled approximation that can make a large error.

Koonin did a Thomas-Fermi model for chemistry as a "Hartree" calculation (I've included it as a 2nd set of hf notes). I was not following the logic, so I did his toy HF homework problem without the Fock term. My classmates, including a present physical chemist at PNNL, teased me that I'd missed out.

5) ^{18}F has 2 nucleons in sd shell ($2s_{1/2}$, $d_{3/2}$, $d_{5/2}$) $\rightarrow$

Don't use jj coupling, which produces more than one unphysical state

Consider ψ_{space} | s | s |

a) Using the A=6 argument (page 9, Lecture 13):

What (S, T) are possible? What $(J^\pi; T)$ in ^{18}F can you explain?

2 antisym nucleons: $(S; T) = (1; 0), (0; 1) \Rightarrow$

$J^\pi; T = 0^+; 1$ and $1^+; 0$ only, like the deuteron

Consider another symmetric ψ_{space} configuration | s | d |

b) What is the only possible total L (by inspection)? $|\vec{2} \pm \vec{0}|=2$

What $J^\pi; T = 1$ are possible? $2^+; T = 1$ (8th excited state)

(i.e. $\vec{J} = \vec{L} + \vec{S} = \vec{L} + \vec{0} = \vec{L} \Rightarrow$ for T=1, J=L only!)

What $J^\pi; T = 0$ are possible ? $|\vec{2} \pm \vec{1}| \Rightarrow J^\pi = 1, 2, 3; T = 0$

Consider ψ_{space} | d | d |

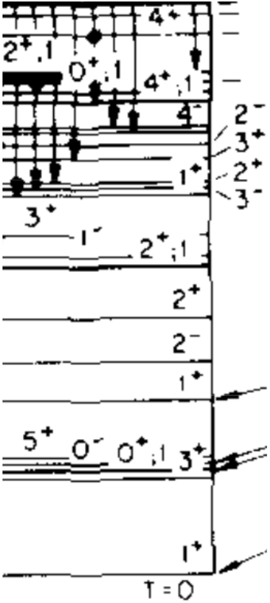
Only even L's are symmetric on the M-scheme table (next page).

c) Using only the even-numbered L's, what T=1 states do you predict this way (note they are all clustered together)? T=0?

$J^\pi = 0^+, 2^+, 4^+; T = 1$ and $J^\pi = 1^+, 2^+, 3^+, 4^+, 5^+; T = 0$

These account for all the $\pi = +$ states in the level diagram ☺

The $\pi = -$ states include excitations from the 1p shell



$^{18}_{9}$F$_9$

(So mixing of the 0^- T=0 and 0^+ T=1 states by the weak interaction is complex to calculate)
Only 'antisymmetric for 2 particles' (S,T)= (1,0) and (0,1) are allowed for the symmetric space configurations considered. Some states from naive jj coupling (e.g. T=1 J odd) are not seen– lacking symmetric space configuration under permutation, they lie much higher.

an M-scheme table for dd configuration (used on previous page)