solutions HW4

1a) Ground state is spin 1, so we want the $^{2s+1}L_J = {}^3S_1$ channel. Extrapolating from the data here gives greater than 145 degrees, perhaps $\pm$ 20, $\Rightarrow n_0 \approx 0.8 \pm 0.1$. Doesn't rule out 1.
(Note we have no $J = 0$ bound state. The $^1S_0$ state has phase shift difference 120 degrees, considerably less than 180, though the extrapolation to zero energy needs more data than shown here.)

1b) The phase shift at infinite energy looks to be no less than -40. The phase shift at lower energy needs to be 140 degrees– it seems possible that could be so.

These comparisons suggest Levinson's theorem is an elegant math physics result, but when n_ℓ is 0 or 1 it is difficult to interpret.

Addendum: I interpret the $^1S_0$ channel as relating to the $J^\pi; T = 0^+; 1$ resonance unbound at 80 keV. The phase shift difference has a substantial deficit from π. Although that seems intuitively reasonable for a slightly unbound resonance, one would have to look carefully at the math physics proof to check if that is a rigorous conclusion.

Solutions HW4 continued

2a) ^{6}Li 3.563 MeV state has $J^\pi; T = 0^+; 1$. Since deuteron and α have g.s. with $T = 0$, the decay has to take place by an isospin-breaking interaction, or the g.s. has to have an admixture of $T = 1$ from an isospin-breaking interaction.
(One could in principle emit d or α in an excited state with $T = 1$, though these are all unbound to nucleon emission. People have tried to measure temperatures in compound nuclei by looking at the ratio of d^*/d and using a Boltzmann relation including the extra energy needed to create d^*, indentifying d^* by correlation n,p emission.)

2b) Such a decay would also need L=1, so the outgoing state has parity -1. So this decay would also break parity symmetry.

Published calculations using the parameterized weak nucleon-nucleon interaction– its isovector component– predict about 10^{-4} for this branch. A measurement would be an interesting test of our understanding, though I'm aware only of upper limits on the branch.

HW4 Problem 3 page 1

Thanks to some students for trying to use Wong Eq. 4-54 and the TA [and one student in particular] for figuring out the discrepancy and bringing to my attention the very different answer. Both my notes and Wong in principle lay out a 2-level mixing problem the same way, with the same notation.

I should be using tan(theta), not sin(theta), for (matrix element)/(energy difference), and this matters at 10-20%. However, I suspect Wong is using another small-angle approximation incorrectly, as Eq. 4-54 is failing badly at large angles, while Eq. 4-54 is consistent with my relations at small angles. I don't recommend trying any harder to prove or verify Eq. 4-54– I respect those who have tried, and I find experimental literature using tan(theta) = (matrix element)/(energy difference).

Extra info: I skipped a physics step in the notes: for ^{8}Be experiment:

$$\frac{\Gamma_\alpha(16.9)}{\Gamma_\alpha(16.6)} = \tan^2(\theta) = 0.69$$

which leads to the angle of 40 degrees I wrote down. This relation assumes α 'penetrabilities' are the same. Note that would not work for the very different energies in ^{12}C, where a full technical review of α and M1 γ decay (isovector + isoscalar) is in Adelberger et al. PRC 15 484 (1977) Section III.

HW4 Problem 3 page 2 (no changes)

3a) L07, p. 22/23 for ^{8}Be: $\sin\theta = 0.64 \Rightarrow$

$\langle T = 1 | H_{\mathrm{Coul}} | T = 0 \rangle = \sin\theta \times (16.922 - 16.626) = 0.19$ MeV

3b) scaling by Z, 6/4, gives estimate 0.27 MeV for ^{12}C states.
(This is naive, ignoring details of the wf's in the analog and antianalog states; more detailed consideration in the McDonald and Adelberger paper has ^{12}C's matrix element about 10% larger than ^{8}Be)

$\sqrt{Admixture}$ = 0.27 MeV/2.41 MeV = 0.11,
so Admixture $\sim$ 0.012, much smaller than ^{8}Be simply because two-level mixing amplitude scales with 1/(energy splitting)

3c) No. To mix states, one needs nonzero $\langle J = 1 | H_{\mathrm{Coulomb}} | J = 2 \rangle$.
The Coulomb interaction inside a uniform charged sphere (then $H_{\mathrm{Coulomb}} \sim r^2$) is spherically symmetric, so it can't change the angular momentum, so this matrix element vanishes.

Solutions HW4 continued

4a) ^{34}Cl

4b) Likely depends on compilation. NNDC presently has ^{70}Br, and lists higher-A odd N = odd Z g.s. with "(0^+)" indicating inconclusive evidence. [Note NNDC is out-of-date for at least ^{74}Rb: the beta decay absolute rate, limited excited-state feeding, and mass of parent and progeny (measured at TRIUMF and ISOLDE) imply it is a $0^+ \rightarrow 0^+$ decay to the ground state of ^{74}Sr, while all N even Z even nuclei are 0^+. The compilers have a lot of info to keep track of– if you need the answer, check the original references and input yourself.]

Levinson's theorem, np scattering

1) There are challenges deducing info about a potential from the phase shifts. One tool is "Levinson's Theorem,":

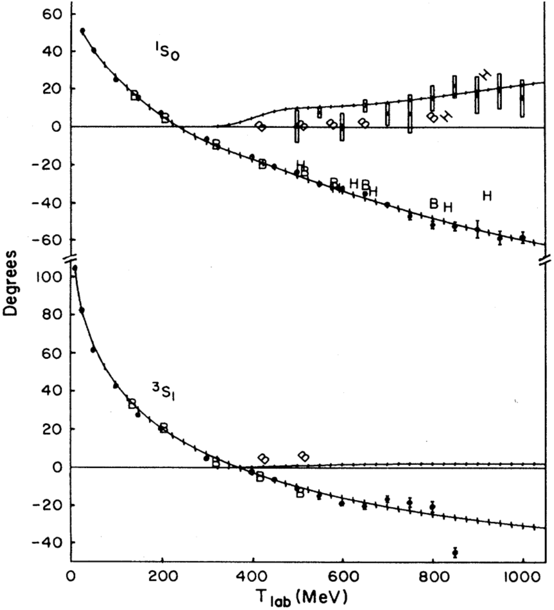
• The difference in the ℓ-wave phase shift of a scattered wave at zero energy, $\phi_\ell(0)$, and infinite energy, $\phi_\ell(\infty)$, for a spherically symmetric potential $V(r)$, is related (!) to the number of bound states n_ℓ of the potential by:

$$\phi_\ell(0) - \phi_\ell(\infty) = n_\ell \pi$$

(Wellner, American Journal of Physics 32, 787 (1964); https://doi.org/10.1119/1.1969857 .)

1a) Using only data plotted here for the phase shifts, determine the effective number of bound states (a fraction) and estimate its uncertainty. Is this a meaningful constraint on the number of bound states of the deuteron?

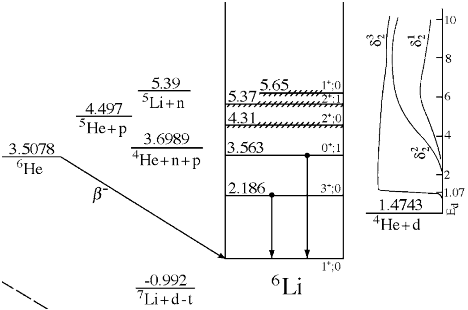
1b) Instead, assume n_0=1, which it is. What can be said about the values of the phase shift extrapolated to 0 and infinite energy?



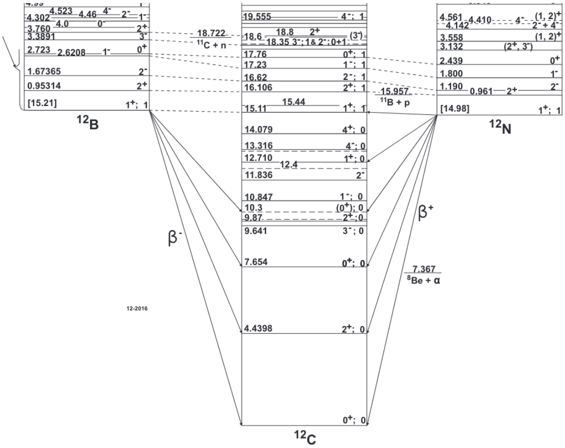
Arndt et al. Phys Rev D 28 97 (1983)

Decay of excited 3.563 MeV J^π=0^+; T=1 state of 6Li

This state, the isobaric analog of the ^{6}He ground state, is energetically allowed to decay to d + α (and not be emitting neutron or proton).
Yet it has only been observed to γ decay.
2a) Would the α decay obey isospin symmetry? (Isospin-'forbidden' particle decay is routinely observed– the decay rate is much slower than if isospin is allowed.)
2b) Assuming total angular momentum is conserved (smooth rotations are good!), what orbital angular momentum would be needed for the final *d* + α system? Would this decay preserve parity symmetry? (Note: the weak interaction between nucleons can break parity, and has both isoscalar and isovector components.)



Isospin mixing in ^{12}C



See pages 20-21 of Lecture 7:

3a) Assuming 1st-order perturbation theory, what size matrix element of $H_{\rm Coul}$ is needed to produce $\theta \sim 40^{\rm o}$ between ^{8}Be 16.6 and 16.9 MeV states?

3b) Scaling this matrix element by Z, guesstimate the size of the Coulomb matrix element in ^{12}C between the $J^\pi = 1^+$ 12.7 MeV (mostly T=0 as indicated) and $J^\pi = 1^+$ 15.11 MeV (mostly T=1) states. (This is another case of analog anti-analog mixing.) What prediction results for the amount of T=0 admixture in the 15.11 MeV state's wf?

3c) Assuming the Coulomb operator is spherically symmetric, is there a Coulomb-produced admixture between the 12.7 MeV $J^\pi = 1^+$ and the 16.106 $J^\pi = 2^+$ states?

Reversal of energy level order of deuteron-like states

Note from Problem 2's diagram: the g.s. of ^{6}Li has T=0, $J \neq 0$, $\pi+$ like the deuteron. (The valence orbital is $p_{1/2}$ and the spins couple ("jj coupling") with same permutation symmetry, so that total J=1 is somewhat accidental.)
Note the 2nd excited state of ^{6}Li we considered above has the same J^π, T as the excited (unbound) state of the deuteron, with the same permutation symmetry (symmetric $T = 1$, antisymmetric $J = 0$.)
These 2 deuteron-like states exist for odd-N = odd-Z to quite high A.
4a) What's the lightest odd-N = odd-Z nucleus where these deuteron-like states invert their energy order? I.e. with J^π=0^+ g.s. ?
4b) What's the heaviest known odd-N = odd-Z nucleus with g.s. having this paired $J^\pi = 0^+$?
(If you don't have a wall chart handy, used www.nndc.bnl.gov, click on the wall chart, pick a nucleus, Zoom about 3, and mouse over to get g.s. properties).
I've been asked why the deuteron g.s. configuration has the lower E– this inversion suggests to me that the full answer may be somewhat complicated.