Levinson's theorem, np scattering

1) There are challenges deducing info about a potential from the phase shifts. One tool is "Levinson's Theorem," :

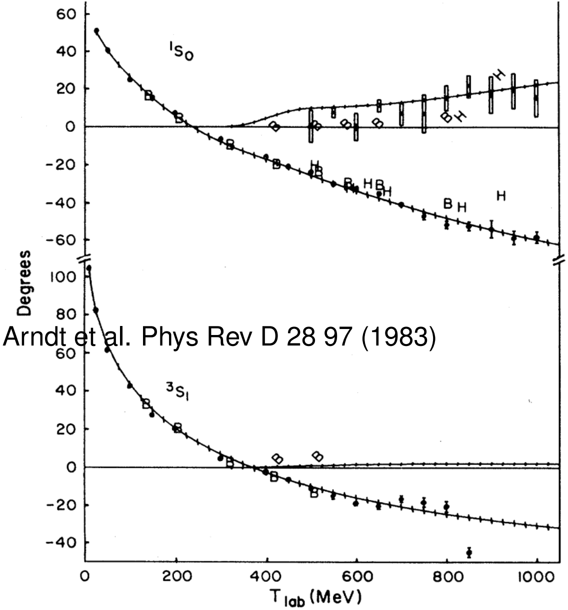
• The difference in the ℓ-wave phase shift of a scattered wave at zero energy, $\phi_\ell(0)$, and infinite energy, $\phi_\ell(\infty)$, for a spherically symmetric potential $V(r)$, is related to the number of bound states n_ℓ with angular momentum l of the potential by:

$$\phi_\ell(0) - \phi_\ell(\infty) = n_\ell \pi$$

(Wellner, American Journal of Physics 32, 787 (1964); https://doi.org/10.1119/1.1969857 .)

1a) The d g.s. is $^3S_1$. There is 1 bound state. Extrapolate the phase shifts shown, estimating uncertainty, and comment on consistency with L's theorem.

1b) The np scattering resonance is $^1S_0$, i.e. there are no bound states with this character. Extrapolate the phase shifts and comment.

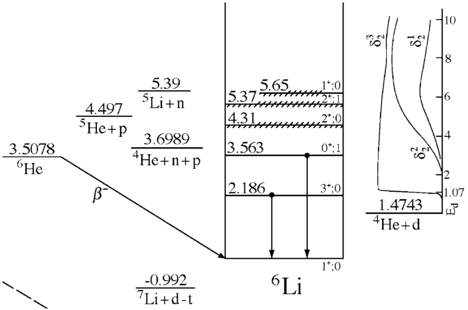


Arndt et al. Phys Rev D 28 97 (1983)

JB assumes the theorem holds separately for each J for same L:
this formulation is for spinless particles, and it's unclear to JB whether one should somehow 'average' over the different experimental ϕ for different J but same L.

Decay of excited 3.563 MeV J^π=0^+; T=1 state of 6Li

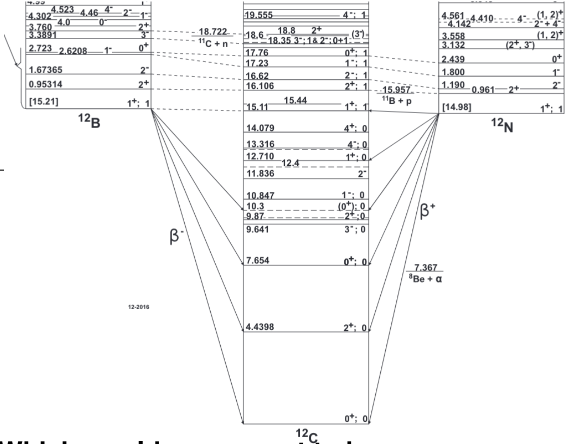
This state, the isobaric analog of the ^{6}He ground state, is energetically allowed to decay to d + α (and not be emitting neutron or proton).
Yet it has only been observed to γ decay.
2a) Would the α decay obey isospin symmetry? (Isospin-'forbidden' particle decay is routinely observed– the decay rate is much slower than if isospin is allowed.)
2b) Assuming total angular momentum is conserved (smooth rotations are good!), what orbital angular momentum would be needed for the final *d* $+ \alpha$ system? Would this decay preserve parity symmetry? (Note: the weak interaction between nucleons can break parity, and has both isoscalar and isovector components.)



Isospin mixing in ^{12}C

For info, see e.g. pages 15, 17, 19, 20 of L07_IsospinInNuclei_JB_2025.pdf.

$^{8}_{4}$Be States	Measured matrix element
16.63 2^+, 16.92 2^+	145 ± 3 keV
17.64 1^+, 18.15 1^+	103 ± 14 keV



Using each of these two Coulomb matrix elements, estimate isospin mixing between the $J^\pi = 1^+$ 12.7 MeV (mostly T=0) and $J^\pi = 1^+$ 15.1 MeV (mostly T=1) states of $^{12}_{6}$C :

3a) First, assume the matrix element scales with Z(Z-1)/R as on p. 15.

What are the two predicted matrix elements? Which would you expect to be more accurate theoretically?

(We will look in more detail later at similarities in space-spin symmetry between some of these states) The experimental value for the ^{12}C states is 110$\pm$30 keV.

3b) Scaling by the energy differences in the denominator of 1st-order perturbation theory, estimate the Coulomb mixing ϵ of T=1 character into the 12.7 MeV state. Comment on the degree of isospin breaking compared to the ^{8}Be 16.6,16.9 pair (discussed in lecture)

3c) Assuming the Coulomb operator is spherically symmetric, is there a Coulomb-produced admixture between the 12.7 MeV $J^\pi = 1^+$ and the 16.106 $J^\pi = 2^+$ states?

Signs Consider the lowest-order chiral EFT Lagrangian in coordinate space:

$$V^{(\mathrm{LO})}(\boldsymbol{r}) = (C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\delta(\mathbf{r}) - \left(\frac{g_A}{2f_\pi}\right)^2 (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \nabla_1)(\boldsymbol{\sigma}_2 \cdot \nabla_2)\frac{e^{-m_\pi r}}{4\pi r}, \quad (2.79)$$

from e.g. Obertelli and Sagawa

Assume all constants are real.

4a) Is the relative strength of the pion exchange and the pointlike interaction determined by chiral EFT, yes or no?

4b) Is the sign of the pion exchange determined from chiral EFT, yes or no?

Although C_S and C_T are determined by experiment, lattice QCD appears to calculate the sign from QCD (Sagawa mentions approximations...)

Consider an alternate universe with the signs of C_S and C_T both flipped.

4d) State the physics change to the nucleon-nucleon interaction

4e) List two possible changes in that universe

problem 4 continued $\rightarrow$

One calculation of the β function of QCD, assuming SU(3) color, is :

$$\beta(\alpha_S) \propto -(11 - \frac{Nc}{6} - \frac{2n_q}{3})\frac{\alpha_S^2}{2\pi}$$

where $Nc = 3$ colors of gluons, and n_q is the number of flavors of quarks, in our case 6 so far. (A more general expression for SU(Nc) was derived by T'Hooft before the meaning of the sign was understood.)

The sign of β is famously negative, implying asymptotic freedom.

4f) Both asymptotic freedom and SU(3) for color are well-established experimentally. What is the maximum n_q possible in our observed universe?

Energies of *d* and *d*-like states

OPEP, generated by Problem 4's Lagrangian, includes the "tensor force" that is said to bind the deuteron. By inspection this term scales with $\vec{\tau}_1 \cdot \vec{\tau}_2$.

5a) What is $\vec{\tau}_1 \cdot \vec{\tau}_2$ for the two deuteron states? (Wong Ch. 3; JB p. 12 L06 NN Int: just state the answer.)

However, fully written-out OPEP:

e.g. N. Jelley, Fund Nucl Phys Cambridge free .pdf at publisher

$$V_{\mathrm{P}} = g_s^2 \left(\tfrac{1}{3} \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B + S_{AB} \left[\tfrac{1}{3} + \frac{1}{\mu r} + \frac{1}{(\mu r)^2} \right] \right) \boldsymbol{\tau}_A \cdot \boldsymbol{\tau}_B \frac{\mu^2 e^{-\mu r}}{r}$$

where $S_{AB} = 3(\boldsymbol{\sigma}_A \cdot \mathbf{r})(\boldsymbol{\sigma}_B \cdot \mathbf{r})/r^2 - \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B$.

shows dependence on $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ intertwined in the "tensor force."

5b) What is $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ for the two deuteron states? JB hopes this is obvious from 5a

JB: so these 2 simple quantities flip with the *d*'s states: the full answer for *d* binding requires less handwaving and more radial integration.

Note from Problem 2's diagram: the g.s. of ^{6}Li has T=0, $J \neq 0$, $\pi+$ like the deuteron.

That *J*=1 is the same as in the *d* is coincidental: the same permutation symmetry means these $p_{1/2}$ orbitals add their total angular momenta to maximum. Similarly, the 1st excited J=3 state adds two $p_{3/2}$ orbitals to max total angular momentum.

Also, the 2nd excited state of ^{6}Li has the same J^π, *T* as the excited (unbound) state of the *d*, and same permutation symmetry (symmetric $T = 1$, antisymmetric $J = 0$.) Such 'deuteron-symmetry' states exist for odd-N = odd-Z to quite high A.

5c) What is the lightest odd-N = odd-Z nucleus where these deuteron-like states invert their energy order? I.e. has ground state J^π=0^+ g.s. ?

(mouse over the wall chart at www.nndc.bnl.gov to get g.s. properties once you pick a nucleus and zoom in.)

What drives this inversion (spin-orbit, size, 'isospin dependence of pairing'...) is beyond JB so far

Consider the operator $\vec{r} \cdot \vec{p}$
If a constant times this operator were in the N-N interaction,
6a) Would parity symmetry be respected? I.e., consider what happens if you change $\vec{r} \rightarrow -\vec{r}$. You can think nonrelativistically and classically and consider $\vec{p} \propto d\vec{r}/dt$

6b) Would time reversal symmetry be respected? Similarly, consider $t \rightarrow -t$.

7. Can ρ "exchange currents" contribute to μ_d?

To my transcription of EGA's proof that μ_d does not have contributions from π meson exchange currents, I've added labels a,b,c to the critical steps.
Consider steps a,b,c for ρ meson exchange, and comment on any differences. The ρ has intrinsic spin 1 and isospin 1.

so only $\Delta T=0$ piece of photon contributes $\rightarrow$

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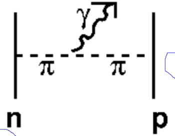

a

J^π of photon is 1^- (ignoring orbital angular momentum)
So the J^π of the two pions at the vertex must also together be 1^-
Because π is a boson,
the two-π state must have symmetric wf under exchange

b π g.s. has S=0, so the two π's need to together have $\vec{J}$=1.
That must be orbital $\vec{L}$=1, which is always antisymmetric.
So to get a symmetric total wf, the isospin wf is also antisymmetric.

c T_π=1, so two pions can have T=2, 1, or 0; of these, only T=1 is antisymmetric
Therefore the γ for this case must also have isospin =1
But we saw above that piece does not contribute
So π exchange currents don't contribute to μ_d

solutions HW4

1a) Ground state is spin 1, so we want the $^{2s+1}L_J = {}^3S_1$ channel. Extrapolating from the data here gives greater than 145 degrees, perhaps $\pm$ 20, $\Rightarrow n_0 \approx 0.8 \pm 0.1$, only consistent with 1 at about 2 σ accuracy.
This to JB illustrates the difficulty of extrapolating to 0 and infinite-energy phase shifts without a model. Likely a model, or more measurements closer to 0 and at higher energy, would help, though both extremes are challenging experimentally or it would have been done already.
1b) Note we have no $J = 0$ bound state. The $^1S_0$ state has phase shift difference 120 degrees, considerably less than 180, though the extrapolation to zero energy needs more data than shown here.)
Addendum: JB is interpreting the $^1S_0$ channel as relating to the $J^\pi; T = 0^+; 1$ resonance unbound at 80 keV. The phase shift difference has a substantial deficit from π. Although that seems intuitively reasonable for a slightly unbound resonance, one would have to look carefully at the math physics proof to check if that is a rigorous conclusion.
Comment: These comparisons suggest Levinson's theorem is an elegant math physics result, but when n_ℓ is 0 or 1 it is difficult to interpret.

Solutions HW4 continued

2a) ^{6}Li 3.563 MeV state has $J^\pi; T = 0^+; 1$. Since deuteron and α have g.s. with $T = 0$, the decay has to take place by an isospin-breaking interaction, or the g.s. has to have an admixture of $T = 1$ from an isospin-breaking interaction.
(One could in principle emit d or α in an excited state with $T = 1$, though these are all unbound to nucleon emission. People have tried to measure temperatures in compound nuclei by looking at the ratio of d^*/d and using a Boltzmann relation including the extra energy needed to create d^*, indentifying d^* by correlation n,p emission.)

2b) Such a decay would also need L=1, so the outgoing state has parity -1. So this decay would also break parity symmetry.

Published calculations using the parameterized weak nucleon-nucleon interaction– its isovector component– predict about 10^{-4} for this branch. A measurement would be an interesting test of our understanding of the weak interaction between nucleons, though I'm aware only of upper limits.

HW4 Problem 3

3a) Z(Z-1)/R with R=1.2$A^{1/3}$
for ^{8}Be is 5.0
for ^{12}C is 10.9
giving 317 and 225 keV.
JB was expecting the 1^{+} states to be closer, but both are far off the experimental value. So either JB's scaling is wrong, or these states have different wavefunctions and the scaling is simply naive.
3b) The experimental value for the mixing would be 110 keV/2400 keV = 0.046 $\pm$ 0.013. This is an order of magnitude smaller than in ^{8}Be, mostly due to the much larger splitting between the two states.
3c) No, because the total angular momentum is conserved.

4a) No (Comment: because there are arbitrary constants in the contact term)
4b) Yes– although the constants are not computed in chiral EFT and the best determinations remain experimental, the combination is positive definite.
4c) JB skipped 4c, sorry.
4d) attractive at short distance instead of repulsive
4e) There are many interesting possibilities. JB had in mind greater binding energies in general, smaller nuclei, Scattering cross-sections at higher energy would change. Stable nuclei could have larger imbalance of Z and N. Energetics of nuclear astrophysics and element abundances would change. Would A=5, A=8 have stable isotopes?– see lectures after the midterm on permutation symmetry, JB thinks the answer is no for A=5 but wants to think about it.
4f)
$\boldsymbol{n_q}$ =15 is ok, $\boldsymbol{n_q}$=16 would flip the sign and alter the observables. So we know we have fewer than 16 quarks. JB is unaware of any other bounds on the number of quarks.

The goal here was to see if simple scaling answers the question of which *d* state should have lower energy, but it doesn't. **At least it doesn't depend on the sign convention for t_3 JB only asked for the answers but for completeness:**

Pasting in notes in Wong Ch 2 $t(t+1) = (\frac{1}{2})(\frac{3}{2}) = 3/4$
(Pauli matrices are constructed from 'unity', so there are factors of 1/2)
$\vec{\tau}^2 = 4t(t+1) = 3$

so $\vec{\tau}_1 \cdot \vec{\tau}_2$ is an isospin scalar, and a two-body operator, that distinguishes T=0 and T=1 states: $\langle T|\vec{\tau}_1 \cdot \vec{\tau}_2|T\rangle$ = -3 for T=0, 1 for T=1

5a) i.e. -3 for the T=0 *d* ground state, +1 for the T=1 *d* excited unbound resonance
$\vec{\tau}_1 \cdot \vec{\tau}_2$ scaling of this potential helps determine which ***d*** state has lower energy

5b) $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ is just reversed, +1 for the J=S=1 *d* ground state, -3 for the excited *d* resonance.

To answer the question of whether either ***d*** state is bound requires evaluating $\vec{\sigma}_i \cdot \vec{r}$, a spatial wavefunction, and radial integrals with this N-N potential term.

5c) ^{34}Cl. Such nuclei break the rule of thumb that ground states have T=T_3

p.s. We may cover that E1 photon multipole is isovector. The giant dipole (E1) resonance excitation of an even-even $N = Z$ $T = 0$ ground state will group a number of T=1 1^- states in the continuum. There are other specific nuclear structures that group states of same T.

6a) yes, two minus signs multiply to a plus
6b) No.
Comment: Time reversal in quantum mechanics also involves changing initial and final states and taking complex conjugates (to keep the Schroedinger equtaion itself invariant e.g.) but this operator would clearly break time-reversal symmetry.

JB realizes it's not as clean as envisioned, and hopes people realized it's easier to prove something is sometimes nonzero than always zero.

7a) ρ is spin 1 so is also a boson, so the two-ρ state must also have symmetric wf under exchange

7b) The ρ has spin 1, not 0. So the two ρ's need to add to spin-1 of the photon, and there are lots of ways to do it.

JB is pretty sure there must still be orbital $\vec{L}$=1 to get negative parity to match the photon (JB did not consider that when he composed this problem and is hoping it doesn't overly complicate things).

That $\vec{L}$ still makes the ρ pair antisymmetric in spatial exchange.

So the ρ pair has to be antisymmetric in spin x isospin (spin is not zero, so spin is available here).

7c) T_ρ=1, so two ρ's can have T=2,1, or 0.

We need T=0 to couple to the photon as above.

Total S can also be 2,1, or 0.

It seems clear we can come up with an S combination to match to $T = 0$ to make the result antisymmetric as needed, and then couple any of the possible total S to L=1 to get total photon spin 1 as required.

So JB concludes that ρ exchange currents can contribute to μ_d according to the symmetries of the problem. JB suspects that the short-range ρ interaction will contribute less to the low-density deuteron than in other nuclei.