1) β^- vs. β^+ asymmetry with respect to spin

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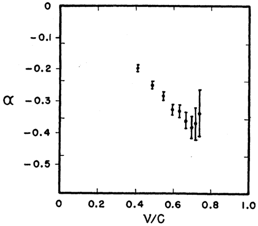
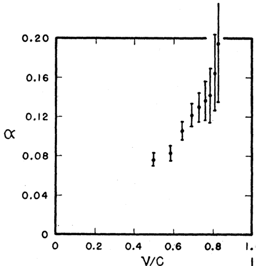




Ambler measured the $2^+ \to 2^+$ β^+ decay of ^{58}Co to have A_β "roughly one third the magnitude and opposite sign" of ^{60}Co.

a) Why is the "opposite sign" interesting?

b) Explain "one third the magnitude" using the equation for pure G-T on p. 63 of L19-22_WeakInteractionsAndNuclei_2025_v3.pdf
Which graph is ^{58}Co? Which graph is ^{60}Co?

c) The figures' "α" is the coefficient of $\cos[\theta]$:
$W[\theta] = 1 + AP\frac{v}{c}\cos[\theta]$
Extrapolate the data simply (by ruler) to $\frac{v}{c}$=1 and read off the measurement of AP.
What is the approximate nuclear polarization P in each case?
(The Fermi operator contribution to ^{58}Co decay has been measured separately (it changes γ-ray polarization...) and is small enough to ignore at the level of accuracy considered here.)

JB didn't explain features of the expression on p. 49 from JTW Phys Rev 106 517 (1957):

- **This is the general decay distribution as a function of E_β and β and ν angle. All these correlations appear, including the a and A terms. If you integrate over ν angles (i.e. if you don't measure the ν), then the a, c, B, D terms vanish, and your experiment measures the β asymmetry A term. (bm/E is a normalization that is zero in the S.M.) If you average over the initial spin polarizations of the nucleus, the c, A, B, D terms vanish, and you're left with the $\beta - \nu$ correlation a term.**

- **You should recognize the allowed decay expression $p_e E_e p_\nu E_\nu$. The Fermi function F(Z,E) is included in the later Coulomb corrections paper from Nucl Phys A quoted on p. 63.**

3) a) beta- and beta+ are lepton and antilepton and should have opposite helicity. (note though that the quantity λ on p. 63 depends on J and J', so one has to be careful). One could also say that CP is looking like it's conserved, since C is different and the P breaking is changing sign.

b) beta asymmetry wrt nuclear spin Abeta=A for 2+ to 2+ GT is +J/(J+1)=+1/3 for a positron emitter (compared to -1 for 5+ to 4+ for a beta minus decay).
So bottom graph must be from 58Co and top graph from 60Co.

c) AP is about 2/3 of the calculation in each case, so P is about 2/3 (the experimentalists note this in their paper– they have an independent measurement from the anisotropy of the gamma ray emission that agrees.)

2) ^{131}Cs decays by electron capture, J^{π}= $5/2^{+} \rightarrow 3/2^{+}$.
Assuming an angular distribution
$W(\theta) = 1 + A_{\nu} P cos(\theta)$
and polarization P=1, deduce A_{ν} assuming the SM left-handed ν helicity
Hint: assume initial nucleus is fully polarized up, and conserve total spin project *m* as in the ^{60}Co derivation. Can the ν go straight up?
The result and proof in this extreme case are independent of the captured electron's polarization, and you do not need to formally average over it.
(Electron capture EC decay is not in the lookup table.)

131Cs + electron $\rightarrow$ 131Something + neutrino

m initial = m final

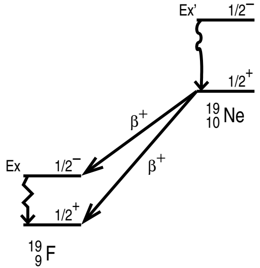
Assume fully polarized up for 'derivation'

m Cesium + m electron = m (spin 3/2) + m(neutrino)
+5/2 + +- 1/2 = ($\leq$ +3/2) + m(nu)

want to find null of angular distribution, so look for projection to be not allowed by angular momentum conservation.
Consider nu going straight up. Then left-handed m(nu) always has projection -1/2. At least 2 on left-hand side, no more than 1 on right-hand side, so this is forbidden. Nu can't go straight up, cos(0)=1 in that case, so Anu= -1.
(If try nu going down, m(nu) = + 1/2, so it's possible to satify the spin projection conservation. So this simple but crude method can need two guesses to get the clean answer.)

I skipped a TRINAT slide on what happens if you measure both beta and nu directions, and how it can sometimes reflect both lepton helicities. We are still hoping it turns into a competitive experiment. This will not be on the final, it's just if you ever want more info on this topic.

3) Consider this simplified A=19 energy level diagram.
All states shown have total isospin $T = 1/2$.
Assume $E_\gamma = E_x$ (energy of recoils is negligible).



a) Identify the two pairs of isobaric analog states by J^π.

$1/2^-$ and $1/2^-$; $1/2^+$ and $1/2^+$

Consider the β^+ decay of the ^{19}Ne $1/2^+$ ground state to the two final states.

b) Calculate the absolute square of the nuclear matrix element for the allowed Fermi decay to the $1/2^+$ ground state (hint: use the coefficient of the isospin-raising operator we have considered, e.g. Wong Eq. 5-71 or on page 37 of the L19-22 notes.)

$T(T+1) - Tz(Tz \pm 1)$**, raising Tz from -1/2 to +1/2, so** $T(T+1) - Tz(Tz+1) = 1/2(3/2) - (-1/2)(1/2) = 1$

c) Compare the $1/2^+$ to $1/2^+$ Gamow-Teller rate to that of the neutron, assuming a single valence particle for $\langle \sigma^2 \rangle$ for each use deShalit and Feshbach table on p.17 of L19-22

They are both s1/2 neutron $\rightarrow$ s1/2 proton, l+1/2 to l+1/2 with l=0, so $\langle \sigma \rangle^2 = 3$

Yes, this is larger than the Fermi component. For the A_β coefficient, the contributions nearly cancel and A_β is near-zero in standard model, but non-SM effects don't cancel

d) Do you expect the β^+ branching ratio to the excited $1/2^-$ state to be larger or smaller than to the $1/2^+$ state? Give two reasons.

smaller to the $1/2^-$, changing nuclear wf parity means it's a 1st forbidden transition.

Also smaller because energy release is (somewhat) smaller.

Consider the same A=19 system.

4) a) What γ-ray multipolarity is allowed for a $1/2^-$ to $1/2^+$ transition?

E1

b) Compute the ratio of γ decay rates in terms of Ex' and Ex. Compare to experiment, given Ex=110 keV, Ex'=275 keV, and $\Gamma_\gamma(^{19}\mathrm{Ne})/\Gamma_\gamma(^{19}\mathrm{F})$ = 13.9$\pm$ 0.7

Hint: This multipole is purely isovector in the long-wavelength limit (see p. 3 of L19-22 i.e. Bohr and Mottleson Eq. 1-63 and text following). So the nuclear matrix element is the same for these γ-ray transitions in ^{19}F and ^{19}Ne systems, assuming good isospin symmetry.

ratio of E_γ^3 is 15.6, higher by 1.12 at 2.4 σ significance.