1) β^- vs. β^+ asymmetry with respect to spin

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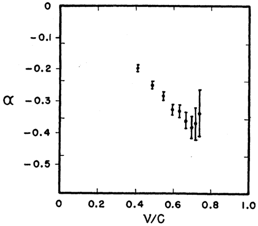
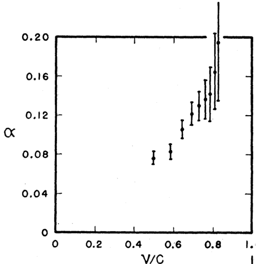




Ambler measured the $2^+ \rightarrow 2^+$ β^+ decay of ^{58}Co to have A_β "roughly one third the magnitude and opposite sign" of ^{60}Co.

a) Why is the "opposite sign" interesting?

b) Explain "one third the magnitude" using the equation for pure G-T on p. 63 of L19-22_WeakInteractionsAndNuclei_2025_v3.pdf
Which graph is ^{58}Co? Which graph is ^{60}Co?

c) The figures' "α" is the coefficient of $\cos[\theta]$:
$W[\theta] = 1 + AP\frac{v}{c}\cos[\theta]$
Extrapolate the data simply (by ruler) to $\frac{v}{c}$=1 and read off the measurement of AP.
What is the approximate nuclear polarization P in each case?
(The Fermi operator contribution to ^{58}Co decay has been measured separately (it changes γ-ray polarization...) and is small enough to ignore at the level of accuracy considered here.)

2) ^{131}Cs decays by electron capture, J^π= $5/2^+ \rightarrow 3/2^+$.

Assuming an angular distribution

$W(\theta) = 1 + A_\nu P cos(\theta)$

and polarization P=1, deduce A_ν assuming the SM left-handed ν helicity

Hint: assume initial nucleus is fully polarized up, and conserve total spin project *m* as in the ^{60}Co derivation. Can the ν go straight up?

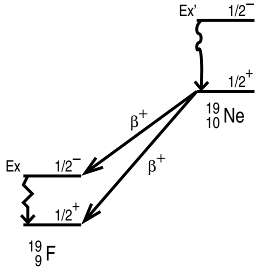
The result and proof in this extreme case are independent of the captured electron's polarization, and you do not need to formally average over it.

(Electron capture EC decay is not in the lookup table.)

3) Consider this simplified A=19 energy level diagram.
All states shown have total isospin $T = 1/2$.
Assume $E_\gamma = E_x$ (energy of recoils is negligible).

a) Identify the two pairs of isobaric analog states by J^π.

Consider the β^+ decay of the ^{19}Ne $1/2^+$ ground state to the two final states.



b) Calculate the absolute square of the nuclear matrix element for the allowed Fermi decay to the $1/2^+$ ground state (hint: use the coefficient of the isospin-raising operator we have considered, e.g. Wong Eq. 5-71 or on page 37 of the L19-22 notes.)
c) Compare the $1/2^+$ to $1/2^+$ Gamow-Teller rate to that of the neutron, assuming a single valence particle for $\langle\sigma^2\rangle$ for each use deShalit and Feshbach table on p.17 of L19-22
d) Do you expect the β^+ branching ratio to the excited $1/2^-$ state to be larger or smaller than to the $1/2^+$ state? Give two reasons.

Consider the same A=19 system.

4) a) What γ-ray multipolarity is allowed for a $1/2^-$ to $1/2^+$ transition?

b) Compute the ratio of γ decay rates in terms of Ex' and Ex. Compare to experiment, given Ex=110 keV, Ex'=275 keV, and $\Gamma_\gamma(^{19}\mathrm{Ne})/\Gamma_\gamma(^{19}\mathrm{F})$ = 13.9$\pm$ 0.7

Hint: This multipole is purely isovector in the long-wavelength limit (see p. 3 of L19-22 i.e. Bohr and Mottleson Eq. 1-63 and text following). So the nuclear matrix element is the same for these γ-ray transitions in ^{19}F and ^{19}Ne systems, assuming good isospin symmetry.