1) Isovector E1 example

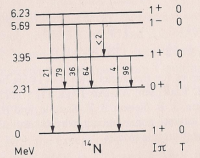
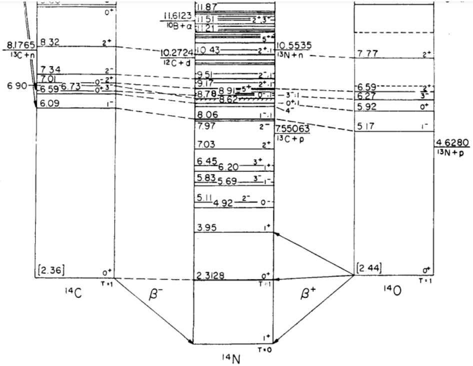


Figure 1-8 Dipole transitions in ^{14}N. The numbers on the arrows represent relative γ intensities as determined by S. Gorodetzky, R. M. Freeman, A. Gallmann, and F. Haas, *Phys. Rev.* **149**, 801 (1966).



Consider the relative rates of the E1 decays of the E=5.69 MeV $J^\pi = 1^-; T = 0$ state that are shown in the figure. Assume the spatial $\psi(r)$ of the ground state and first excited state is the same. **a)** Why is that OK?

These are likely deuteron-like states, with symmetric spatial wf's, and S;T having the antisymmetry.

Limitation: we saw an excited $1^+; 0\ ^3D_1$ L=2 configuration in ^{6}Li (see L12-13...v2.pdf p.19), which could be part of the 3.95 MeV state here, and mixing of that configuration with the ^{14}N g.s. (i.e. the D-state configuration contained in the deuteron g.s.) JB is explicitly ignoring. All papers on isospin mixing in ^{14}N use simple shell models in which this is surely included.

b) To which final states are M2 decays allowed from the E=5.69 MeV state?

M2 flips parity. M2 is allowed from 1^- to the $1^+; T = 0$ states. You can't vector-add 2 and 0 to get 1^-, so M2 is not allowed from 1^- to 0^+.

Ignoring that M2 possibility:

c) Assuming the known dependence on E_γ of the E1 rate, compute the E1 rate ratio from the 5.69 MeV state $\Gamma_{E=0}/\Gamma_{E=2.31}$, ignoring the isospin selection rule. (By summing over final and averaging over initial states, JB concludes from C-G coefficient squares that this E1 rate should also scale with $2J_f + 1$.)

scaling by E_γ^3 and $2J+1$, JB gets

X $\Gamma_{E=0}/\Gamma_{E=2.31}$ = $(3.95/(3.95-2.31))^3\sqrt{3}$ = 17.7 X

$\Gamma_{E=0}/\Gamma_{E=2.31}$ = $(5.69/(5.69-2.31))^3\sqrt{3}$ = 8.3

d) Compute the ratio of experimental to theoretical values of $\Gamma_{E=0}/\Gamma_{E=2.31}$.

Exp ratio is 36/64, so Exp/Theory(for good isospin) is X0.032X 0.068.

Noting this is small,

e) Still assuming identical $\psi(r)$ (no longer a good assumption), what admixture of the 1^-;T=1 state at 8.06 MeV would explain this E1 rate, to lowest order in perturbation theory? (The published literature needs a theory estimate of the space matrix element.)

Assuming that good isospin would produce zero E1 rate (there are higher-order corrections to the E1 operator JB is ignoriing) there is no allowed amplitude to interfere with, so the rate for same spatial wf will just go like an admixture α^2 = X0.032X 0.068 .

[extra: If one wants to estimate a Coulomb mixing matrix element, first order perturbation theory would be α= (matrix element)/(delta E), so $|\alpha|$= X 0.18X 0.26 $\Rightarrow$ 2.37 MeV * 0.26 = 0.62 MeV. That's unexpectedly large compared to all others. Likely the spatial wf of the 8.06 MeV state is different from the 3.95 MeV state. (We did predict 1^-;0 and 1^-;1 configurations with s,p symmetric occupation for A=6 that might be part of the ^{14}N configurations, but there is not much futher we can say from that approach.)]

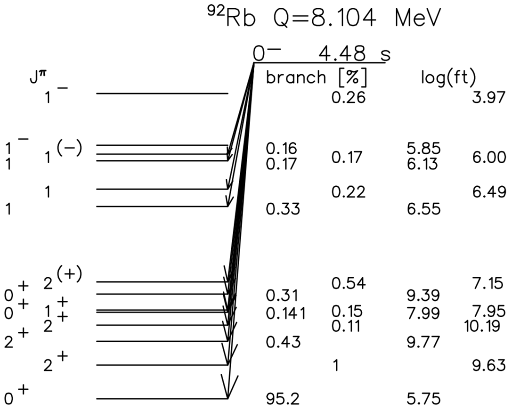
Using E1 transitions in N=Z nuclei has been a solid method historically to measure isospin breaking, but like many techniques it needs good nuclear wavefunctions. In this case, if there were lifetime measurements $\Rightarrow$ rate measurements, one could deduce reliable admixtures from how the rate was altered.

JB was surprised to see an isospin-allowed and an isospin-forbidden E1 with similar branches, so is glad to see working out the simple scalings is consistent with the E1 being isovector.

2) G-T and 1st-forbidden competing: Reactor ν's and ^{92}Rb decay

$^{92}_{37}$Rb^{55} $\sim$10% of reactor ν's 5-7 MeV

Levels from 2012 NDS compilation:



• Remember t = ln(2)/(partial decay rate) = (decay's $t_{1/2}$)/(branch fraction).

a) List one pair of possible orbitals for the odd proton and neutron of the ^{92}Rb $J^\pi = 0^-$ ground state. Note their relative π.

b) for 3 sets of states, comment on:
whether the β^- decay is allowed or forbidden, and on the size of the $log_{10}ft$ values:

- **i)** the J^π=0$^+$ ground state (branch 95.2%);
- **ii)** the highest $J^\pi = 1^-$ state;
- **iii)** the six other J=2 and J=0 states.

c) β^- decay to the highest $J^\pi = 1^-$ state (at 7.363 MeV excitation) has log(ft)=3.97, much faster than the 0^+ ground state with log(ft)=5.75. Given the maximum β kinetic energy, get 'Fermi integral' *f* (Wong Eq. 5-69) from Wong Fig. 5.7 for these two transitions (assuming $0^- \to 0^+$ is 'allowed'). Check these listed branch fractions and log(ft)'s for consistency.

d) Re: the other six J=1 states clustered together, state the compiler's reasoning and threshold log(ft) for determining allowed vs. forbidden and $\therefore$ parity

2a) Z=37, 3 short of closed Z=40 suggests f5/2 (parity minus)
N=55 closed N=50 + 5, suggests g7/2 (parity plus)
OK to add 5/2 and 7/2 and get 0. Parity is minus.
(Common in fission products to have 'valence' n and p in opposite-parity orbitals)

2b) i) 0- to 0+ changes nuclear parity so is '1st-forbidden.' log(ft) of 5.75 is one of the faster forbidden ones in the histogram Fig. 5-8 from Wong.
ii) log(ft) 3.97 suggests a fast Gamow-Teller. 0- to 1- keeps nuclear parity and changes J by 1, satisfies G-T selection rule.
iii) 0- to these 0+ and 2+ are all positive parity; all are 1st-forbidden. log(ft)'s are more than 10x slower than the strong 0- to 0+ g.s to g.s.

2c) Q=8.104 MeV is highest possible kinetic energy. Since Wong plots down to 0.1 MeV, less than mass of electron, he is clearly plotting maximum kinetic energy on x-axis(despite somewhat nonstandard E0 notation). 0- to 0+ has two operators, though it's commonly assumed one dominates for these higher-energy transitions, so it's ok to assume 'allowed.' Z=37 Q=8 MeV has log(f)=4.3 to 4.5 or so.
transition to the highest J=1- at 7.363 then has 8.104-7.363= 0.741 MeV, about f=-1.5. Branch should be lower by log(4.5-(-1.5)= log(6) = 10^6
from the momentum integral f alone. The log(ft) is 3.97 instead of 5.75, so that G-T (matrix element)^2 **is 10**(1.78) times bigger, and rate scales with (matrix element)**^{2}**. 6-1.78= 4.22 or** 10^4 **times smaller. The actual branch is 300x smaller, so something is not right here.**

2d) The compiler is saying a log(ft) of smaller than 6 is G-T, which is plausible but might not be perfect. One has to look very carefully at any state where one needs the answer.

Some qualitative considerations I did not go through:

Many of the 0- to 0+ transitions seem to have log(ft) at 6 or faster, faster than most 1st-forbidden transitions.

One doesn't get opposite-parity states, which are needed to get G-T transitions to bleed strength from the higher-energy nu's, until fairly high excitation. The g.s. to g.s. transition is often then the largest branch, making more energy come out in higher-energy ν's instead of γ's.

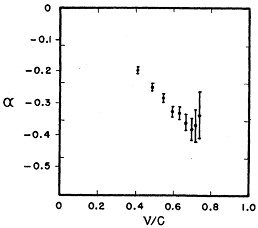
That highest 1^- state is likely part of the low-Ex tail of the giant dipole resonance, which has centroid energy (phenomenological from Berman+Fultz RevModPhys 47 713 (1975)) E_{GDR}=31.2$A^{-1/3}$ + 20.6 $A^{-1/6}$ = 16.6 MeV with full width half maximum $\sim$ 5 MeV.) This is one reason for the "pandemonium effect: strong E1 transitions at 5 MeV produce a forest of narrow lines with poor Germanium efficiency that makes them very hard to detect. (If both parent and progeny have same parity, the "Giant Gamow-Teller" resonance produces states with a similar role.) Total absorption spectrometers, 4 pi arrays of high-Z scintillator (with inherently poorer energy resolution)help by absorbing all the gamma energy and measuring beta feeding patterns averaged over many states.

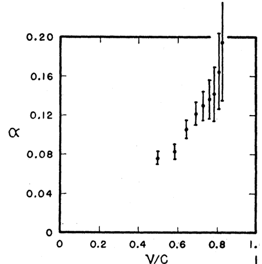
Pietro Spagnoletti from SFU presented details of 92Sr spectroscopy from 92Rb decay at WNPPC 2023, studying in detail this low-energy GDR tail. JB remains curious about this, though more curious about the ν spectrum measured by one of you.

3) β^- vs. β^+ asymmetry with respect to spin

E. Ambler, R. W. Hayward, D. D. Hoppes, R. P. Hudson, and C. S. Wu

Phys. Rev. 106, 1361 (1957)





Ambler measured the $2^+ \to 2^+$ β^+ decay of ^{58}Co to have A_β "roughly one third the magnitude and opposite sign" of ^{60}Co.

a) Why is the "opposite sign" interesting?

b) Explain "one third the magnitude" using the equation for pure G-T on p. 59 of L21_24_WeakInteractionsAndNuclei_v2.pdf
Which graph is ^{58}Co? Which graph is ^{60}Co?

c) The symbol α is the coefficient of $\cos[\theta]$ in the angular distribution
$W[\theta] = 1 + AP\frac{v}{c}\cos[\theta]$
extrapolate the data simply (by ruler) to $\frac{v}{c}$=1 and read off the measurement of AP.
Deduce the nuclear polarization P in each case.

(Should the polarizations be similar? Yes. The dilution refrigerator + B field selects the lowest-energy state, given by $\mu_{\rm nuclear} \cdot B_{\rm effective}[\rm nucleus]$. The effective B field is the same for any Co atom, and the μ's are measured to be within 10%.]
(The Fermi operator contribution to ^{58}Co decay has been measured separately (it changes γ-ray polarization...) and is small enough to ignore at the level of accuracy considered here.)

JB didn't explain features of the expression on p. 43 from JTW Phys Rev 106 517 (1957). (You should recognize the allowed decay expression $p_e E_e p_\nu E_\nu$. The Fermi function F(Z,E) is included in the later Coulomb corrections paper from Nucl Phys A quoted on p. 56.) This is the general decay distribution as a function of E_β and β and ν angle. All these correlations appear, including the a and A terms. If you integrate over ν angles (i.e. if you don't measure the ν), then the a, c, B, D terms vanish, and your experiment measures the β asymmetry A term. (bm/E is a normalization that is zero in the S.M.) If you average over the initial spin polarizations of the nucleus, the c, A, B, D terms vanish, and you're left with the $\beta - \nu$ correlation a term.

3) a) beta- and beta+ are lepton and antilepton and should have opposite helicity. (note though that the lambda on p. 54 depends on J and J', so one has to be careful). One could also say that CP is looking like it's conserved, since C is different and the P breaking is changing sign.

b) beta asymmetry wrt nuclear spin Abeta=A for 2+ to 2+ GT is +J/(J+1)=+1/3 for a positron emitter (compared to -1 for 5+ to 4+ for a beta minus decay).
So bottom graph must be from 58Co and top graph from 60Co.

c) AP is about 2/3 of the calculation in each case, so P is about 2/3 (the experimentalists note this in their paper– they have an independent measurement from the anisotropy of the gamma ray emission that agrees.)

4) ^{131}Cs decays by electron capture, J^{π}= $5/2^{+} \rightarrow 3/2^{+}$.
Assuming an angular distribution
$W(\theta) = 1 + A_{\nu} P cos(\theta)$
and polarization P=1, deduce A_{ν} assuming the SM left-handed ν helicity.

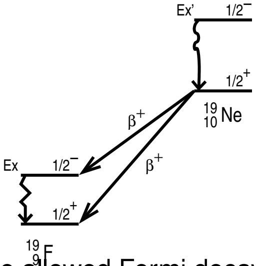
131Cs + electron $\rightarrow$ 131Something + neutrino

m initial = m final

Assume fully polarized up for 'derivation'

m Cesium + m electron = m (spin 3/2) + m(neutrino)
+5/2 + +- 1/2 = ($\leq$ +3/2) + m(nu)

want to find null of angular distribution, so look for projection to be not allowed by angular momentum conservation.
Consider nu going straight up. Then left-handed m(nu) always has projection -1/2. At least 2 on left-hand side, no more than 1 on right-hand side, so this is forbidden. Nu can't go straight up, cos(0)=1 in that case, so Anu= -1.
(If try nu going down, m(nu) = + 1/2, so it's possible to satify the spin projection conservation.)



5) Consider this simplified A=19 energy level diagram. All states shown have total isospin $T = 1/2$. Assume $E_\gamma = E_x$ (energy of recoils is negligible).

a) Identify the two pairs of isobaric analog states by J^π.

$1/2^-$ and $1/2^-$; $1/2^+$ and $1/2^+$

Consider the β^+ decay of the ^{19}Ne $1/2^+$ ground state to the two final states.

b) Calculate the absolute square of the nuclear matrix element for the allowed Fermi decay to the $1/2^+$ ground state (hint: use the coefficient of the isospin-raising operator we have considered, e.g. Wong Eq. 5-71.)

$T(T+1) - Tz(Tz \pm 1)$, raising Tz from -1/2 to +1/2, so $T(T+1) - Tz(Tz+1) = 1/2(3/2) - (-1/2)(1/2) = 1$

c) Compare the $1/2^+$ to $1/2^+$ Gamow-Teller rate to that of the neutron, assuming a single valence particle for $\langle \sigma^2 \rangle$ use deShalit and Feshbach table on p.18 of Lecture 21-24

They are both s1/2 neutron $\rightarrow$ s1/2 proton, l+1/2 to l+1/2 with l=0, so $\langle \sigma \rangle^2 = 3$

Yes, this is larger than the Fermi component. For the A_β coefficient, the contributions nearly cancel and A_β is near-zero in standard model, but non-SM effects don't cancel

d) Do you expect the β^+ branching ratio to the excited $1/2^-$ state to be larger or smaller than to the $1/2^+$ state? Give two reasons.

smaller to the $1/2^-$, changing nuclear wf parity means it's a 1st forbidden transition.

Also smaller because energy release is (somewhat) smaller.

6) a) What γ-ray multipolarity is allowed for a $1/2^-$ to $1/2^+$ transition?

E1

b) Assuming that multipolarity and good isospin symmetry (i.e. assume matrix element is the same), compute the ratio of γ decay rates in terms of Ex' and Ex. Compare to experiment, given Ex=110 keV, Ex'=275 keV, and $\Gamma_\gamma(^{19}\mathrm{Ne})/\Gamma_\gamma(^{19}\mathrm{F})$ = 13.9$\pm$ 0.7

ratio of E_γ^3 is 15.6, higher by 1.12 at 2.4 σ significance.

Estimate of the admixture of atomic P state into the 1st excited S state in atomic parity violation in 1st-order perturbation theory:

M.A. Bouchiat C. Bouchiat J. de Physique 35 899 (1974) https://hal.science/jpa-00208216v1/file/ajp-jphys_1974_35_12_899_0.pdf

Ch.4, I.B. Khriplovich, "Parity Nonconvservation in Atomic Phenomena" 1991 Gordon and Breach

The matrix element: $\langle s_{1/2}|H_W|p_{1/2}\rangle \propto Z^2 NR$, where

$R = \frac{4(\frac{a}{2Zr_0})^{2-2\gamma}}{(\Gamma(2\gamma+1))^2}$ is a relativistic correction for e^- probability and momentum near the nucleus

r_0= 1.2×10^{-13}cm $\times A^{-1/3}$ is the nuclear charge radius

$a = 1/m\alpha = 0.529177 \times 10^{-8}$cm is the Bohr radius of the e^-

$(2\gamma+1)$ is the argument of the Gamma function Γ

$\gamma = \sqrt{(j+1/2)^2 - Z^2\alpha^2}$ with (atomic!) j=1/2 for these s and p states, α=1/137

The weak coupling between e^- and p is proportional to (1/4 - $\sin^2(\theta_W)$) which is small, so atomic parity violation counts neutrons N. The e^- ψ and momentum at nucleus makes Z^2

7) Complete the table: (the last 2 columns are ratios to deuterium)

	Z	N	R	$\langle s\|H_W\|p\rangle/\langle s\|H_W\|p\rangle_d$	ΔE (GHz)	$(\langle s\|H_W\|p\rangle/\Delta E)/(\langle s\|H_W\|p\rangle/\Delta E)_d$
^{2}H	1	1	1.0006	$\equiv$1	1.058	$\equiv 1$
^{133}Cs	55	78	4.86	1.15×10^6	2.21×10^5	5.6
^{211}Fr	87	124	59.3	5.57×10^7	2.25×10^5	262
alkali?	119	179	5318	1.35×10^{10}	$\sim 2 \times 10^5$	7.1×10^4

the larger atomic fine-structure splitting of Cs compared to d almost cancels the larger matrix element and makes a similar admixture. Atomic hydrogen experiments have not worked, because atomic (unlike nuclear) systematic effects from imperfect external fields are also enhanced like $1/\Delta E$