1) Isovector E1 example

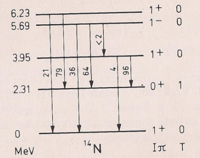
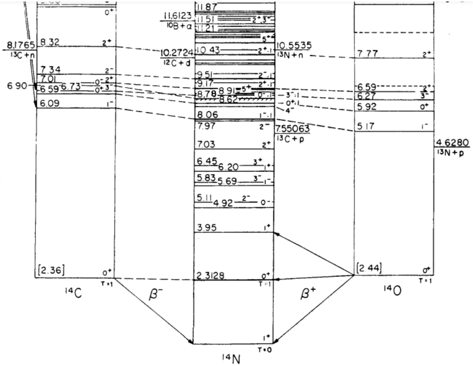


Figure 1-8 Dipole transitions in ^{14}N. The numbers on the arrows represent relative γ intensities as determined by S. Gorodetzky, R. M. Freeman, A. Gallmann, and F. Haas, *Phys. Rev.* **149**, 801 (1966).



Consider the relative rates of the E1 decays of the E=5.69 MeV $J^\pi = 1^-$; $T = 0$ state that are shown in the figure. Assume the spatial $\psi(r)$ of the ground state and first excited state is the same. **a)** Why is that OK?

These are likely deuteron-like states, with symmetric spatial wf's, and S;T having the antisymmetry.

Limitation: we saw an excited $1^+; 0\ {}^3D_1$ L=2 configuration in ^{6}Li (see L_13-14...v2.pdf p.21), which could be part of the 3.95 MeV state here, just like the D-state configuration contained in the deuteron g.s. All papers on isospin mixing in ^{14}N use simple shell models in which this is surely included.

b) To which final states are M2 decays allowed from the E=5.69 MeV state?

M2 flips parity. M2 is allowed from 1^- to the 1^+; $T = 0$ states. You can't vector-add 2 and 0 to get 1^-, so M2 is not allowed from 1^- to 0^+.

Ignoring that M2 possibility:

c) Assuming the known dependence on E_γ of the E1 rate, compute the E1 rate ratio from the 5.69 MeV state $\Gamma_{E=0}/\Gamma_{E=2.31}$, ignoring the isospin selection rule. (By summing over final and averaging over initial states, JB concludes from C-G coefficient squares that this E1 rate should also scale with $2J_f + 1$.)

scaling by E_γ^3 and $2J+1$, JB gets

$\Gamma_{E=0}/\Gamma_{E=2.31}$ = $(5.69/(5.69-2.31))^3\sqrt{3}$ = 8.3

d) Compute the ratio of experimental to theoretical values of $\Gamma_{E=0}/\Gamma_{E=2.31}$.

Exp ratio is 36/64, so Exp/Theory(for good isospin) is X0.032X 0.068.

Noting this is small,

e) Still assuming identical $\psi(r)$ (no longer a good assumption), what admixture of the 1^-;T=1 state at 8.06 MeV would explain this E1 rate, to lowest order in perturbation theory? (The published literature needs a theory estimate of the space matrix element.)

Assuming that good isospin would produce zero E1 rate (there are higher-order corrections to the E1 operator JB is ignoriing) there is no allowed amplitude to interfere with, so the rate for same spatial wf will just go like an admixture α^2 = X0.032X 0.068 .

[extra: If one wants to estimate a Coulomb mixing matrix element, first order perturbation theory would be α= (matrix element)/(delta E), so $|\alpha|$= X 0.18X 0.26 $\Rightarrow$ 2.37 MeV * 0.26 = 0.62 MeV. That's unexpectedly large compared to all others. Likely the spatial wf of the 8.06 MeV state is different from the 3.95 MeV state. (We did predict 1^-;0 and 1^-;1 configurations with s,p symmetric occupation for A=6 that might be part of the ^{14}N configurations, but there is not much futher we can say from that approach.)]

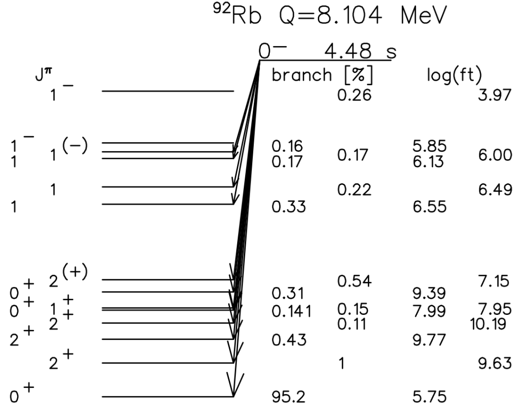
Using E1 transitions in N=Z nuclei has been a solid method historically to measure isospin breaking, but like many techniques it needs good nuclear wavefunctions. In this case, if there were lifetime measurements $\Rightarrow$ rate measurements, one could deduce reliable admixtures from how the rate was altered.

JB was surprised to see an isospin-allowed and an isospin-forbidden E1 with similar branches, so is glad to see working out the simple scalings is consistent with the E1 being isovector.

2) G-T and 1st-forbidden competing: Reactor ν's and ^{92}Rb decay

$^{92}_{37}\mathrm{Rb}^{55}$ $\sim$10% of reactor ν's 5-7 MeV

Levels from 2012 NDS compilation:



• Remember t = ln(2)/(partial decay rate) = (decay's $t_{1/2}$)/(branch fraction).

a) List one pair of possible orbitals for the odd proton and neutron of the ^{92}Rb $J^\pi = 0^-$ ground state. Note their relative π.

b) for 3 sets of states, comment on:
whether the β^- decay is allowed or forbidden, and on the size of the $log_{10} ft$ values:

- **i)** the J^π=0^+ ground state (branch 95.2%);
- **ii)** the highest $J^\pi = 1^-$ state;
- **iii)** the six other J=2 and J=0 states.

c) β^- decay to the highest $J^\pi = 1^-$ state (at 7.363 MeV excitation) has log(ft)=3.97, much faster than the 0^+ ground state with log(ft)=5.75. Given the maximum β kinetic energy, get 'Fermi integral' *f* (Wong Eq. 5-69) from Wong Fig. 5.7 for these two transitions (assuming $0^- \to 0^+$ is 'allowed'). Check these listed branch fractions and log(ft)'s for consistency.

d) Re: the other six J=1 states clustered together, state the compiler's reasoning and threshold log(ft) for determining allowed vs. forbidden and $\therefore$ parity

2a) Z=37, 3 short of closed Z=40 suggests f5/2 (parity minus)
N=55 closed N=50 + 5, suggests g7/2 (parity plus)
OK to add 5/2 and 7/2 and get 0. Parity is minus.
(Common in fission products to have 'valence' n and p in opposite-parity orbitals)

2b) i) 0- to 0+ changes nuclear parity so is '1st-forbidden.' log(ft) of 5.75 is one of the faster forbidden ones in the histogram Fig. 5-8 from Wong.
ii) log(ft) 3.97 suggests a fast Gamow-Teller. 0- to 1- keeps nuclear parity and changes J by 1, satisfies G-T selection rule.
iii) 0- to these 0+ and 2+ are all positive parity; all are 1st-forbidden. log(ft)'s are more than 10x slower than the strong 0- to 0+ g.s to g.s.

2c) Q=8.104 MeV is highest possible kinetic energy. Since Wong plots down to 0.1 MeV, less than mass of electron, he is clearly plotting maximum kinetic energy on x-axis(despite somewhat nonstandard E0 notation). 0- to 0+ has two operators, though it's commonly assumed one dominates for these higher-energy transitions, so it's ok to assume 'allowed.' Z=37 Q=8 MeV has log(f)=4.3 to 4.5 or so.
transition to the highest J=1- at 7.363 then has 8.104-7.363= 0.741 MeV, about f=-1.5. Branch should be lower by log(4.5-(-1.5)= log(6) = 10^6 **from the momentum integral f alone. The log(ft) is 3.97 instead of 5.75, so that G-T (matrix element)**^{2} **is 10**(1.78) times bigger, and rate scales with (matrix element)**^{2}**. 6-1.78= 4.22 or** 10^4 **times smaller. The actual branch is 300x smaller, so something is not right here.**

2d) The compiler is saying a log(ft) of smaller than 6 is G-T, which is plausible but might not be perfect. One has to look very carefully at any state where one needs the answer.

Some qualitative considerations I did not go through:

Many of the 0- to 0+ transitions seem to have log(ft) at 6 or faster, faster than most 1st-forbidden transitions.

One doesn't get opposite-parity states, which are needed to get G-T transitions to bleed strength from the higher-energy nu's, until fairly high excitation. The g.s. to g.s. transition is often then the largest branch, making more energy come out in higher-energy ν's instead of γ's.

That highest 1^- state is likely part of the low-Ex tail of the giant dipole resonance, which has centroid energy (phenomenological from Berman+Fultz RevModPhys 47 713 (1975)) E_{GDR}=31.2$A^{-1/3}$ + 20.6 $A^{-1/6}$ = 16.6 MeV with full width half maximum $\sim$ 5 MeV.) This is one reason for the "pandemonium effect: strong E1 transitions at 5 MeV produce a forest of narrow lines with poor Germanium efficiency that makes them very hard to detect. (If both parent and progeny have same parity, the "Giant Gamow-Teller" resonance produces states with a similar role.) Total absorption spectrometers, 4 pi arrays of high-Z scintillator (with inherently poorer energy resolution)help by absorbing all the gamma energy and measuring beta feeding patterns averaged over many states.

Pietro Spagnoletti from SFU presented details of 92Sr spectroscopy from 92Rb decay at WNPPC 2023, studying in detail this low-energy GDR tail. JB remains curious about this, though more curious about the ν spectrum measured by one of you.

3. Simple Fermi function:

One simplified expression for the Fermi function mentioned in lecture (p. 32 L19-22...) produces a simple analytic form for the β energy spectrum.

a) **Setting m_β=0,** integrate that spectrum to find a quantity dN proportional to the phase space integral f:

$$dN = \int_0^{E0} F_{PR}(E,Z)pE(E0-E)^2 dE$$

where E and p are the relativistic energy and momentum of the β, $E0$ is the maximum total energy possible for the beta, and

$F_{PR}(E,Z)$ is Primakoff and Rosen's Fermi function:

$$F_{PR}(E,Z) = a\left|\frac{1}{(1-e^{\pm a})}\right|\frac{E}{p}$$

where $\pm$ is for $\beta^\pm$ decay, and $a = 2\pi Z\alpha$ is a constant that is often not small.

This is based on the "nonrelativistic" version in the lecture notes, simplified without rigor to make the integrations easily analytic.

(Hint: The answer is proportional to $E0^N$ where N is an integer.

If you just square the parentheses there are only 3 terms to integrate and then sum.

b) Considering this total decay rate proportional to $E0^N$ (and ignoring the dependence on Z), how much does this β decay phase space change between $E0$ = 1 and 5 MeV? $5^5 = 3125$

(Note how powerful the ft concept is, producing an intrinsic decay strength for β decay constant to parts per thousand despite the enormous variation in f.)

4) Low-mass unknown boson exchange (keep setting c=1 here)

An unknown boson with relatively low mass wrt the $W^{\pm}$ will have a lowest-order correction from the propagator (p. 5 of lecture notes):

$$\frac{g_x^2}{M_x^2}(1+\frac{q^2}{M_x^2})$$

where g_x is the vertex coupling constant of the boson (assumed same for quarks and leptons), and q is some momentum transfer. Set $q = p_\beta$ below.

Two neutron β decay experiments (Beck PRL 2024) indicate a correction of 0.01 of the weak interaction. This could be explained by an interaction with

$$\frac{g_x^2}{M_x^2} \approx 0.01\frac{g_W^2}{M_W^2}$$

Suppose $M_x \approx 0.001 M_W \approx 80$ MeV

a) How big is $\frac{g_x^2}{g_W^2}$?

b) Momentum spectra for the β decay of the neutron, which have average momentum $p_\beta \approx 0.8 MeV$, are distorted by about how much?

$(1+\frac{p_beta^2}{M_x^2}) \approx 1 + 0.0001$

c) Similarly, spectra from the decay of $^{38\mathrm{m}}$K, with average $p_\beta \approx 3.2 MeV$, are distorted by how much?

(In present experiments, c) might be barely observable, while b) is not.)