1) Isovector E1 example

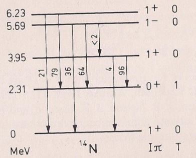
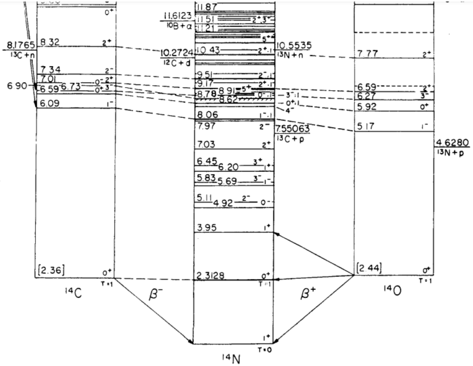


Figure 1-8 Dipole transitions in ^{14}N. The numbers on the arrows represent relative γ intensities as determined by S. Gorodetzky, R. M. Freeman, A. Gallmann, and F. Haas, *Phys. Rev.* **149**, 801 (1966).



Consider the relative rates of the E1 decays of the E=5.69 MeV $J^\pi = 1^-$; $T = 0$ state that are shown in the figure. Assume the spatial $\psi(r)$ of the ground state and first excited state is the same. **a)** Why is that OK?

b) To which final states are M2 decays allowed from the E=5.69 MeV state?

Ignoring that M2 possibility:

c) Assuming the known dependence on E_γ of the E1 rate, compute the E1 rate ratio from the 5.69 MeV state $\Gamma_{E=0}/\Gamma_{E=2.31}$, ignoring the isospin selection rule. (By summing over final and averaging over initial states, JB concludes from C-G coefficient squares that this E1 rate should also scale with $2J_f + 1$.)

d) Compute the ratio of experimental to theoretical values of $\Gamma_{E=0}/\Gamma_{E=2.31}$.

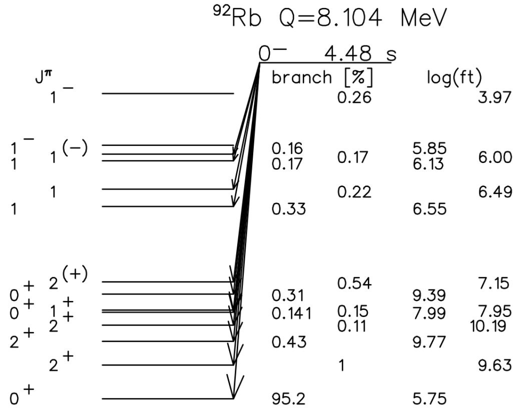
Noting this is small,

e) Still assuming identical $\psi(r)$ (no longer a good assumption), what admixture of the 1^-;T=1 state at 8.06 MeV would explain this E1 rate, to lowest order in perturbation theory? (The published literature needs a theory estimate of the space matrix element.)

2) G-T and 1st-forbidden competing: Reactor ν's and ^{92}Rb decay

$^{92}_{37}$Rb^{55} $\sim$10% of reactor ν's 5-7 MeV

Levels from 2012 NDS compilation:



• Remember t = ln(2)/(partial decay rate) = (decay's $t_{1/2}$)/(branch fraction).

a) List one pair of possible orbitals for the odd proton and neutron of the ^{92}Rb $J^\pi = 0^-$ ground state. Note their relative π.

b) for 3 sets of states, comment on:
whether the β^- decay is allowed or forbidden, and on the size of the $log_{10}ft$ values:

- **i)** the J^π=0$^+$ ground state (branch 95.2%);
- **ii)** the highest $J^\pi = 1^-$ state;
- **iii)** the six other J=2 and J=0 states.

c) β^- decay to the highest $J^\pi = 1^-$ state (at 7.363 MeV excitation) has log(ft)=3.97, much faster than the 0$^+$ ground state with log(ft)=5.75. Given the maximum β kinetic energy, get 'Fermi integral' *f* (Wong Eq. 5-69) from Wong Fig. 5.7 for these two transitions (assuming $0^- \to 0^+$ is 'allowed'). Check these listed branch fractions and log(ft)'s for consistency.

d) Re: the other six J=1 states clustered together, state the compiler's reasoning and threshold log(ft) for determining allowed vs. forbidden and $\therefore$ parity

3. Simple Fermi function:

One simplified expression for the Fermi function mentioned in lecture (p. 32 L19-22...) produces a simple analytic form for the β energy spectrum.

a) **Setting m_β=0,** integrate that spectrum to find a quantity *dN* proportional to the phase space integral *f*:

$dN = \int_0^{E0} F_{PR}(E, Z) pE(E0 - E)^2 dE$

where E and p are the relativistic energy and momentum of the β, $E0$ is the maximum total energy possible for the beta, and

$F_{PR}(E, Z)$ is Primakoff and Rosen's Fermi function:

$F_{PR}(E, Z) = a|\frac{1}{(1-e^{\pm a})}|\frac{E}{p}$

where $\pm$ is for $\beta^\pm$ decay, and $a = 2\pi Z\alpha$ is a constant that is often not small.

This is based on the "nonrelativistic" version in the lecture notes, simplified without rigor to make the integrations easily analytic.

(Hint: The answer is proportional to $E0^N$ where N is an integer.

If you just square the parentheses there are only 3 terms to integrate and then sum.

b) Considering this total decay rate proportional to $E0^N$ (and ignoring the dependence on Z), how much does this β decay phase space change between $E0$ = 1 and 5 MeV?

(Note how powerful the *ft* concept is, producing an intrinsic decay strength for β decay constant to parts per thousand despite the enormous variation in *f*.)

4) Low-mass unknown boson exchange (keep setting c=1 here)
An unknown boson with relatively low mass wrt the $W^{\pm}$ will have a lowest-order correction from the propagator (p. 5 of lecture notes):

$$\frac{g_x^2}{M_x^2}\left(1+\frac{q^2}{M_x^2}\right)$$

where g_x is the vertex coupling constant of the boson (assumed same for quarks and leptons), and q is some momentum transfer. Set $q = p_\beta$ below.
Two neutron β decay experiments (Beck PRL 2024) indicate a correction of 0.01 of the weak interaction. This could be explained by an interaction with

$$\frac{g_x^2}{M_x^2} \approx 0.01\,\frac{g_W^2}{M_W^2}$$

Suppose $M_x \approx 0.001 M_W \approx 80$ MeV

a) How big is $\frac{g_x^2}{g_W^2}$?

b) Momentum spectra for the β decay of the neutron, which have average momentum $p_\beta \approx 0.8 MeV$, are distorted by about how much?

c) Similarly, spectra from the decay of $^{38\mathrm{m}}$K, with average $p_\beta \approx 3.2 MeV$, are distorted by how much?

(In present experiments, c) might be barely observable, while b) is not.)