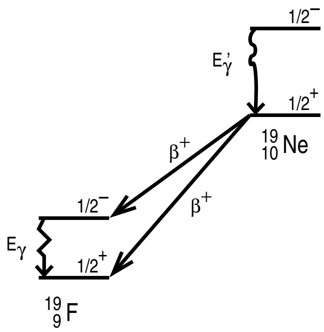

Consider this simplified A=19 energy level diagram.
All states shown have total isospin $T = 1/2$.

1 a) Identify the two pairs of isobaric analog states by J^π. **$1/2^-$ and $1/2^-$; $1/2^+$ and $1/2^+$**

Consider the β^+ decay of the ^{19}Ne $1/2^+$ ground state to the two final states.

b) Calculate the absolute square of the nuclear matrix element for the allowed Fermi decay to the $1/2^+$ ground state (hint: use the coefficient of the isospin-lowering operator we have considered, e.g. Wong Eq. 5.71)

$\boldsymbol{\sqrt{T_i(T_i+1) - T_{0i}(T_{0i}-1)} = \sqrt{1/2(3/2) - 1/2(-1/2)} = 1}$

c) Can Gamow-Teller transitions contribute? Why or why not?

(Nuclear π same, spin changes by +-1 or 0, no 0 to 0) To the $1/2^+$ to $1/2^+$, yes. To the $1/2^+$ to $1/2^-$, no

d) Do you expect the branching ratio to the excited $1/2^-$ state to be larger or smaller? Give two reasons.

Smaller. 1st forbidden non-unique will have usually much smaller matrix element i.e. larger fT, barring accidental GT-Fermi cancellation (tricky to do but not necessarily impossible). Lepton momentum integral $\sim Q^5$ so this is smaller as well.

e) What γ-ray multipolarity is allowed for a $1/2^-$ to $1/2^+$ transition? **E1**

f) The γ-ray decay rate scales with E^N: what is N? **N=2L+1 = 3**

g) Assuming good isospin symmetry, compute the ratio of γ-decay rates in terms of E'_γ and E_γ. Compare to experiment, given

E_γ=110 keV, E'_γ=275 keV, and $\Gamma_\gamma(^{19}\mathrm{Ne})/\Gamma_\gamma(^{19}\mathrm{F})$ = 13.9± 0.7

(19Ne decay rate)/ (19F decay rate) = $(E'_\gamma/E_\gamma)^3 = 15.6$, high by 10±5%

The E1 operator is purely isovector in the long-wavelength limit (see e.g. Bohr and Mottleson Eq. 1-63 and text following. The electric charge is 1/2 + t_3, the 1/2 multiplies the center of mass and does not drive transitions [Bohr and Mottleson are pretty smart to just assert that], the t_3 part is purely isovector).

For the isospin dependence, using Wong A-15 for Wigner-Eckart in terms of Wigner 3J's, with isospin operator of rank T_{kq} = T_{10}, these are symmetric under column permutations so the extra minus sign is easiest to see. With good isospin the other parts of the isobaric analog pairs are the

same, so the magnitude of the nuclear matrix element is the same for these γ-ray transitions in ^{19}F and ^{19}Ne systems, assuming good isospin symmetry.

Other multipolarities produce sums and differences of isoscalar and isovector parts for such isobaric analog transitions.

h) The electron capture (EC) branch is also allowed. If the maximum kinetic energy of the β^+ is 2.0 MeV, what is the maximum ν energy in EC capture to 2-digit accuracy? Name two things to take into account for a more exact computation.

an electron is captured and not created, so max $E_\nu = 2.0 + 2m_\beta = 3.0$MeV. Recoil kinetic energy is $\sim$ 100 eV. Binding energy of final atom electron is 0.7 keV for innermost atomic shell for ^{19}F.