September 2016 Physics & Astronomy Qualifying Exam
for Advancement to Candidacy
Day 1: September 1, 2016

Do not write your name on the exam. Instead, write your student number on each exam booklet. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading. If you use extra exam booklets, write your student number on the extra exam books as well. Write all answers in the blank exam booklet(s), not on this printout!

Today's portion of the exam has 8 questions. Answer *any five* of the eight. Do not submit answers to more than 5 questions—if you do, only the first 5 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 4 hours to complete 5 questions.

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

| | | |
|---|---|---|
| absolute zero | 0 K | -273°C |
| air pressure at sea level | 1 atm | $10^5$ N/m$^2$ |
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| Avogadro's constant | $N_A$ | $6.02 \times 10^{23}$ |
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| charge of an electron | $e$ | $1.6 \times 10^{-19}$ C |
| distance from earth to sun | 1 AU | $1.5 \times 10^{11}$ m |
| Laplacian in spherical coordinates | $\nabla^2 f =$ | $\frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$ |
| mass of an electron | $m_e$ | 0.511 MeV/c$^2$ |
| mass of the sun | $M_{sun}$ | $2 \times 10^{30}$ kg |
| mass of a proton | $m_p$ | 938 MeV/c$^2$ |
| mass of a neutron | $m_n$ | 940 MeV/c$^2$ |
| Newton's gravitational constant | $G$ | $6.7 \times 10^{-11}$ N m$^2$ kg$^{-2}$ |
| nuclear magneton | $\mu_N$ | $5 \times 10^{-27}$ J/T |
| permittivity of free space | $\epsilon_0$ | $8.9 \times 10^{-12}$ C$^2$ N$^{-1}$/m$^2$ |
| permeability of free space | $\mu_0$ | $4\pi \times 10^{-7}$ N/A$^2$ |
| Planck's constant | $h$ | $6.6 \times 10^{-34}$ J·s |
| radius of the Earth | $R_{earth}$ | $6.4 \times 10^6$ m |
| speed of light | $c$ | $3.0 \times 10^8$ m/s |
| Stefan-Boltzmann constant | $\sigma$ | $5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ |
| Stirling's approximation | $N!$ | $e^{-N}N^N\sqrt{2\pi N}$ |

1. DNA is a long flexible molecule. The DNA from one cell of your body, if stretched out in a line, would extend 2 meters in length but a DNA double helix is just 2 nm across. The persistence length of a DNA molecule is $\sim 50$ nm, meaning that we can model a random tangle of DNA as a 3D random walk with a step size of 50 nm.

A. Provide an estimate of the diameter of a ball of DNA strand, 2 m in length, using the random walk model. How does this compare to the cell nucleus diameter of 10 $\mu$m?

B. In reality the DNA is neatly coiled in chromosomes. In the approximation that the entropy of this state is much lower than that of the "balled up" state in part A, do an order of magnitude estimate of the change in entropy when the DNA changes from the "balled up" state to the chromosomal state, and estimate the minimum amount of energy the cell must spend to accomplish this.

2. C3PO told Han Solo, "Sir, the possibilty of successfully navigating an asteroid field is approximately 3,720 to 1." In this problem you will check C3PO's math.

Suppose that the asteroid belt was formed by an Earth-sized planet that was broken into 100 m wide chunks, perhaps by the Death Star. Our solar system's asteroid belt has an inner radius of 2.1 AU, an outer radius of 3.3 AU, and a height of 2 AU.

Suppose Han Solo flies the Millenium Falcon from Earth to Jupiter at close to light speed, passing through the asteroid belt in the process. Calculate the probability that the Falcon (diameter = 30 m) will collide with an asteroid, assuming that it takes no evasive action. Neglect gravitational interactions.

3. A 1D *relativistic* harmonic oscillator obeys Hooke's law (F=-kx). If $A$ is the maximum amplitude of oscillation and $m$ is the mass, calculate:

A. The total energy of the oscillator

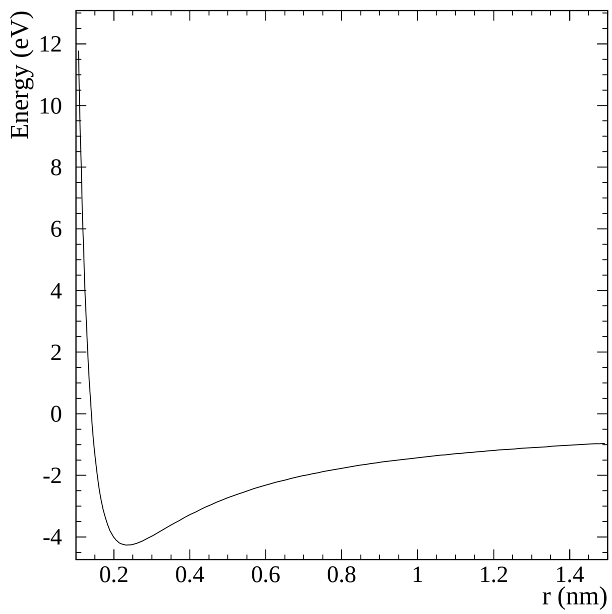
B. The maximum speed of the mass

C. The magnitude of the acceleration of the mass when $x = A/2$

Hint: the relativistic Lagrangian in this problem is

$$L = mc^2(1 - \sqrt{1-\beta^2}) - \frac{1}{2}kx^2$$

where $\beta \equiv v/c$.




4. A NaCl molecule has an ionic bond between a $Na^+$ and a $Cl^-$ ion. The potential for this system as a function of $r$, the distance between the ions, may be described by:

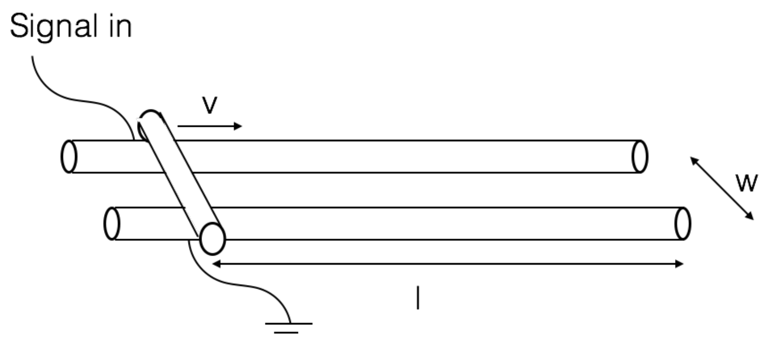
$$V(r) = -\frac{e^2}{4\pi\epsilon_0 r} + \frac{A}{r^k}$$

where $A$ and $k$ are constants that characterize the repulsive potential between the ions at small $r$.

Measurements show that this potential has a minimum of $-4.26$ eV at $r = 0.236$ nm. The atomic masses of Na and Cl are 22 and 37. Using this information and the above potential, estimate the energy gap between two lowest vibrational modes of the molecule.

5. Mars has a mass of $M = 6.4 \times 10^{23}$ kg and a radius of 3400 km. Suppose that Mars were given an Earthlike atmosphere: pressure = 100 kPa, temperature = 10°C, made of 78% nitrogen (atomic mass 28) and 22% oxygen (atomic mass 32).

Is Mars's gravity strong enough to retain this atmosphere? Do an order of magnitude estimation of how long it would take for the atmosphere to escape into space. Neglect the effects of external factors such as solar wind or meteors.

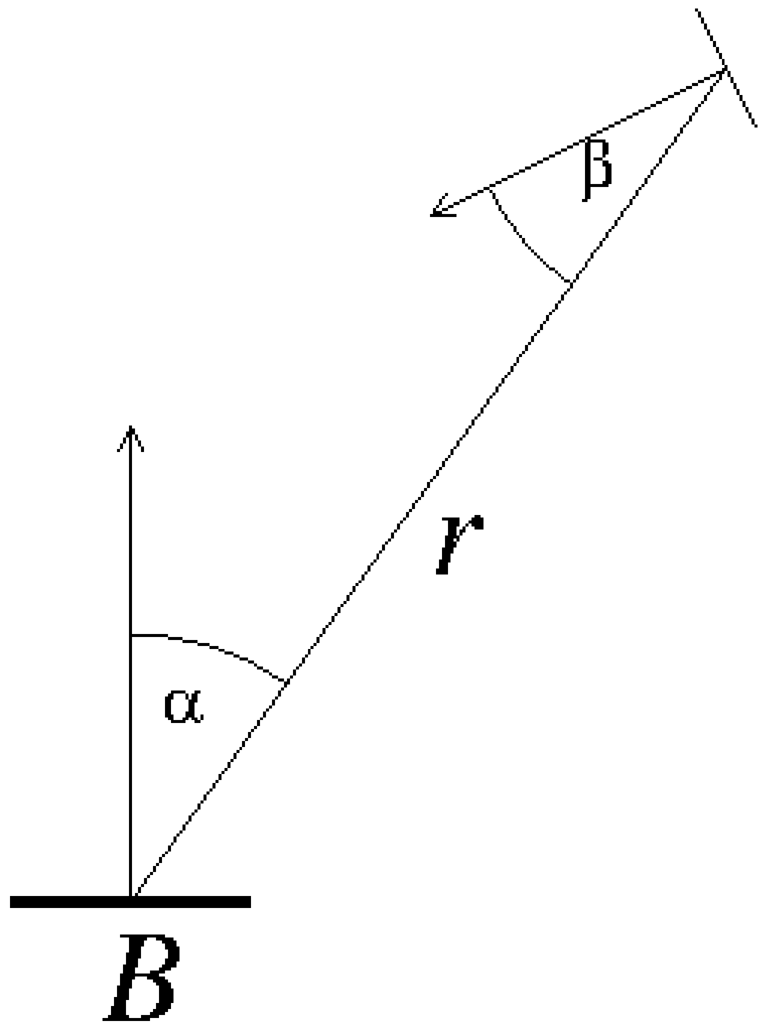



6. A rail gun is an instrument that uses magnetic forces to accelerate a conducting object that spans two conducting rails. The schematic above shows two long rails, with a shorter rod that spans the distance $w = 4$ cm between them and extends an extra 1 cm on either side. The rods slide without rolling along the rails; assume here that the points of contact between rod and rails have zero resistance and zero friction. Driving a current from “Signal in” to ground creates a magnetic field that interacts with the rod and causes it to accelerate down the rails, in the direction marked “v”. Assume that the rod starts 10 cm from the connection point. You should make appropriate simplifying assumptions about the $\vec{B}$ field from the rails.

The rods and rails are each 1 cm diameter. They are made of copper (density $= 9000$ kg/m$^3$, resistivity $= 2 \times 10^{-8}$ $\Omega$·m). From the starting point (at rest, 10 cm from where the signal and ground are connected), to the ends of the rails, is a distance $l = 1.9$ m. Calculate the velocity of the rod when it flies off the rails, assuming it is powered by an ideal current source that supplies $10^4$ A.

7. A bowling ball is thrown with an initial horizontal translational velocity of $v_0 = 7$ m/s but a rotational speed of 0. It initially skids until it rolls without slipping. Given a coefficient of friction $\mu = 0.05$, mass $M = 7$ kg, and ball radius $R = 11$ cm, at what time $t$ in seconds after being released does the ball stop slipping?

8. Consider a disk of radius $R$ emitting radiation, and a detector B. Every point on the surface of the disk emits radiation isotropically into $2\pi$ steradians. The disk is located at distance $r$ from detector B and makes an angle $\alpha$ from the perpendicular to the detector. The surface of the emitting disk makes an angle $\beta$ with respect to the line of sight to the detector.




Assuming $R \ll r$, calculate the flux measured at detector B as a function of $\alpha, \beta, R, r$ and the intensity $I$ of the emitted radiation. (Dimensionally, intensity is power per area, W/m$^2$.)

September 2016 Physics Qualifying Exam
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Day 2: September 2, 2016

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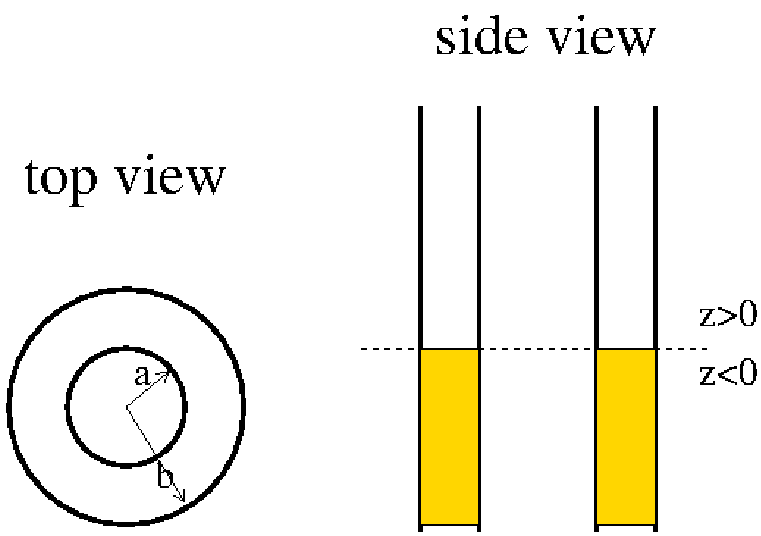
9. Provide an order of magnitude estimate, from first principles, for the elastic modulus (Young's modulus) of solid hydrogen. Reminder: the elastic modulus is defined as the ratio of the force per area along an axis to the fractional change in length along that axis:

$$E = \frac{Force/Area}{\Delta L/L}$$




10. A dielectric fluid ($\epsilon > \epsilon_0$) fills the space between two infinitely long concentric, perfectly conducting cylinders for $z < 0$. The region $z > 0$ between the cylinders is open to the atmosphere ($\epsilon = \epsilon_0$). The cylinders have radii $a$ and $b$ and are sufficiently long that edge effects can be neglected. The inner cylinder is charged to potential $\Phi = V > 0$, while the outer cylinder is grounded at $\Phi = 0$.

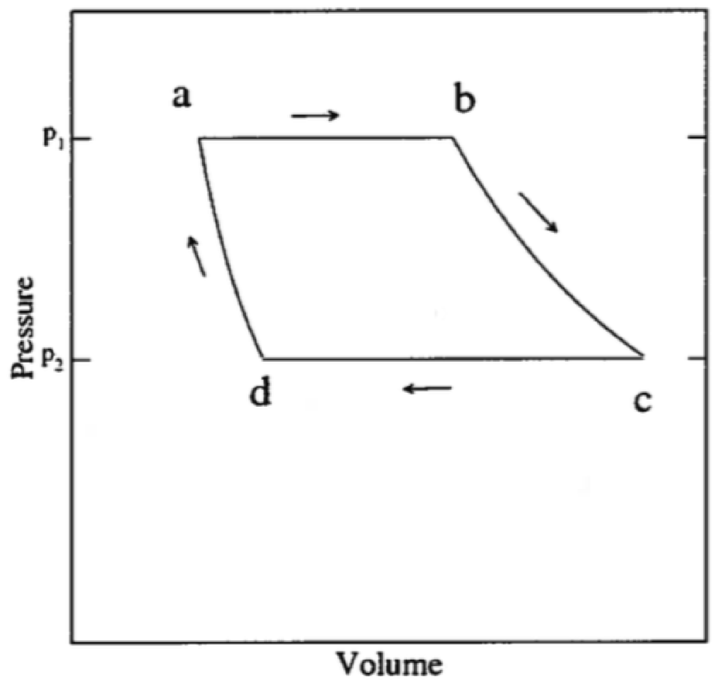
A. Solve for the electric field as a function of the cylindrical radius coordinate $\rho$ for $z > 0$ and for $z < 0$.

B. Find the charge per unit length at the inner surface of the dielectric. Do this both for free charge on the conducting cylinder, and bound charge in the dielectric, for both $z < 0$ and $z > 0$.

11. A charged pion (mass = 140 MeV/c$^2$) travels with velocity $v$ in the $+z$ direction in the lab frame. It decays in flight to a muon (mass = 106 MeV/c$^2$) and a neutrino (mass = 0). In the pion's rest frame, this decay is isotropic, with the muon equally likely to go in all directions, and the neutrino going in the opposite direction from the muon.

A. Calculate the energy of the muon in the pion's rest frame.

B. The average velocity vector of the muon in the pion's rest frame is $(0, 0, 0)$, by isotropy. What is the average velocity vector of the muon as measured in the lab frame? Hint: it is *not* simply (0,0,v)!




12. An ideal gas of N molecules, each with $g$ degrees of freedom, is carried around the reversible closed cycle shown above. Two of the segments are adiabatic, and two are isobaric (constant pressure).

A. What is the value of $g$ for a monatomic gas? What is $g$ for a gas of diatomic molecules that can be treated as rigid rotators?

B. Show that when an ideal gas undergoes an adiabatic process, $PV^\gamma =$ constant, where $\gamma$ is related to $g$. Determine the relation between $\gamma$ and $g$.

C. Of the states labelled a, b, c, and d in the diagram, which has the highest temperature? Which has the lowest?

D. For each segment of the loop, state whether the entropy of the gas is increasing, decreasing, or staying constant. Explain how you can tell.

E. For each segment of the loop, state whether the gas is doing positive work, negative work, or no work.

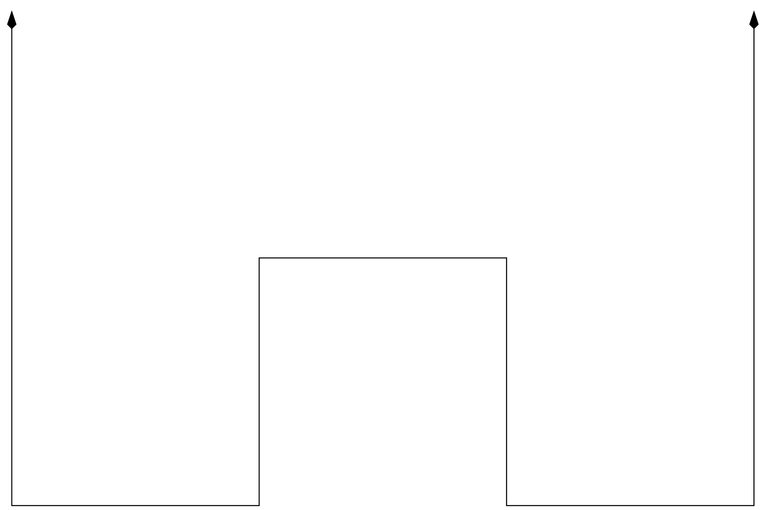
F. When cycled in this way, does the system act as a heat engine or a refrigerator? Explain.

13. A plate of cross-sectional area $A$ and small thickness $h$ is made of metal with electrical resistivity $\rho$. The plate is suddenly placed in an external electric field $E_0$ normal to its surface. Find the rate of heat production by Joule heating in the plate, in watts, as a function of time, and calculate the total energy lost in the Joule heating.

14. A quantum mechanical particle of mass $m$ and charge $Q$ is confined to move in a circle of radius $R$ in the $xy$ plane. There is no potential.

A. Calculate the energy eigenstates and the degeneracy of each energy level.

B. Now a weak electric field $\vec{E} = F\hat{x}$ is applied. Calculate the electric dipole moment of the system in its ground state, to lowest non-zero order in $F$.



15.

A particle is initially trapped in the ground state of the left side of a double well potential with an infinite energy barrier separating left and right. Now the barrier is suddenly lowered to a finite but still high value (see the figure). Because of tunnelling, the particle appears, for the first time, completely on the right side of the double well potential at $t = 9$ s. Calculate the probability of finding the particle on the right side at $t = 12$ s.

16. Prove that for both ideal non-interacting, non-relativistic fermion and boson gases at arbitrary temperatures in three spatial dimensions, the pressure is always equal to 2/3 of the energy density.