

Astronomy Comprehensive Exam, Summer 2026

Session 1
Sep 2nd, 2026

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Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 problems. Answer *any three* of the four. Do not submit answers to more than 3 problems—if you do, only the first 3 of the problems you attempt will be graded. If you attempt a problem and then decide you don't want it to count, clearly cross it out and write "don't grade".

You have 2 hours 15 minutes to complete 3 problems.

You are allowed to use two 8.5" × 11" formula sheets (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

atomic mass unit	$1 \text{ amu} = 1.661 \times 10^{-27} \text{ kg}$
Avogadro's constant	$N_A = 6.02 \times 10^{23}$
Bohr radius of hydrogen atom	$a_0 = 5.3 \times 10^{-11} \text{ m}$
Boltzmann's constant	$k_B = 1.38 \times 10^{-23} \text{ J/K}$
charge of an electron	$e = 1.6 \times 10^{-19} \text{ C}$
Thomson cross-section	$\sigma_T = 6.6524 \times 10^{-29} \text{ m}^2$
distance from earth to moon	$d_{\text{moon}} = 3.8 \times 10^8 \text{ m}$
distance from earth to sun	$1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$
electron volt	$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
mass of an electron	$m_e = 0.5110 \text{ MeV}/c^2$
mass of hydrogen atom	$m_H = 1.674 \times 10^{-27} \text{ kg}$
mass of a neutron	$m_n = 1.675 \times 10^{-27} \text{ kg} = 939.5654 \text{ MeV}/c^2$
mass of a proton	$m_p = 1.673 \times 10^{-27} \text{ kg} = 938.2721 \text{ MeV}/c^2$
mass of the moon	$M_{\text{moon}} = 7 \times 10^{22} \text{ kg}$
mass of the sun	$M_{\odot} = 2 \times 10^{30} \text{ kg}$
molecular weight of H ₂ O	18
Newton's gravitational constant	$G = 6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
permittivity of free space	$\epsilon_0 = 8.9 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$
Planck's constant	$h = 6.6 \times 10^{-34} \text{ J}\cdot\text{s}$
radius of the earth	$R_{\text{earth}} = 6.4 \times 10^6 \text{ m}$
radius of the sun	$R_{\odot} = 7 \times 10^8 \text{ m}$
speed of light	$c = 3.0 \times 10^8 \text{ m/s}$
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Stirling's approximation	$N! \approx e^{-N} N^N \sqrt{2\pi N}$

Problem 1: Why does an accelerated charge radiate?

A point charge q moves non-relativistically along a prescribed trajectory confined to a region of characteristic size d . Consider the electromagnetic field at an observation point

$$\mathbf{x} = r\hat{\mathbf{n}},$$

in the far-field limit $r \gg d$. In this limit, the electric field contains the retarded contribution

$$\mathbf{E}_{\text{rad}}(\mathbf{x}, t) = \frac{q}{4\pi\epsilon_0 c^2 r} \hat{\mathbf{n}} \times [\hat{\mathbf{n}} \times \mathbf{a}(t - r/c)],$$

where $\mathbf{a}$ is the acceleration of the charge evaluated at the retarded time. The associated magnetic field is

$$\mathbf{B}_{\text{rad}} = \frac{1}{c} \hat{\mathbf{n}} \times \mathbf{E}_{\text{rad}}.$$

1. From these expressions, derive the total radiated power in the non-relativistic limit (also known as the Larmor formula).
2. Show that the $1/r$ dependence of $\mathbf{E}_{\text{rad}}$ leads to a finite power radiated to infinity, whereas fields falling faster than $1/r$ do not.
3. The radiation field vanishes for a charge moving with constant velocity. Why is this required by special relativity?

Problem 2: Mass of an X-ray cluster from hydrostatic equilibrium

A galaxy cluster contains hot X-ray emitting gas with density $\rho(r)$ and temperature $T(r)$. Assume spherical symmetry, hydrostatic equilibrium, and an ideal gas equation of state.

1. Derive the expression

$$M(< r) = -\frac{k_B T(r) r}{G \mu m_p} \left[\frac{d \ln \rho}{d \ln r} + \frac{d \ln T}{d \ln r} \right],$$

where $M(< r)$ is the total gravitating mass inside radius r and μm_p is the mean particle mass.

2. Apply the result to an isothermal gas with $\rho(r) \propto r^{-\alpha}$, and show that, in that case, the circular velocity is independent of the radius r . If there are non-thermal sources of pressure support, how biased is the mass derived by this method (low or high)?

Problem 3: Orbital decay of the Swift gamma-ray telescope

The Swift gamma-ray telescope is currently in a decaying orbit due to atmospheric drag. Presently it is in a circular orbit at an elevation of 400 km, where the air density is $\sim 3 \times 10^{12} \text{ kg/m}^3$. Swift has a mass of 800 kg and is approximately $5.5 \text{ m} \times 5.5 \text{ m}$ in size.

Do an order of magnitude estimate of the rate at which the telescope is losing altitude.

Problem 4: Dynamics of small bodies in the Solar System

A small body of negligible mass orbits the Sun with semi-major axis a and eccentricity e . It is perturbed by a planet on a nearly circular orbit of semi-major axis a_p around the Sun. Assume that the planet's mass is much smaller than the solar mass.

1. Using Kepler's third law, derive the orbital period of the small body as a function of a .
2. A small body is said to be in a mean-motion resonance with the planet if the two orbital periods are in a ratio of small integers. Derive the semi-major axis of a resonance for which

$$\frac{P}{P_p} = \frac{q}{p},$$

where P is the orbital period of the small body and P_p is the orbital period of the planet.

3. Apply your result to find the location of the 3 : 1 resonance with Jupiter, in terms of Jupiter's semi-major axis a_J .
4. The perihelion and aphelion distances of the small body are

$$r_{\text{peri}} = a(1 - e), \quad r_{\text{aph}} = a(1 + e).$$

Explain how an increase in eccentricity can turn an asteroid-belt object into a Mars-crossing or Earth-crossing object.

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Problem 5: Reionization and the CMB anisotropies

After recombination, the universe is mostly neutral. Suppose, however, that at a redshift z_{ion} the intergalactic medium becomes fully ionized and remains ionized until today. CMB photons then have a probability of being rescattered by free electrons.

Assume a flat matter-dominated universe for this estimate, so that

$$H(z) \simeq H_0 \Omega_m^{1/2} (1+z)^{3/2},$$

and assume that the free-electron number density after reionization is

$$n_e(z) \simeq X \frac{\Omega_b \rho_{\text{crit}}}{m_p} (1+z)^3,$$

where X is the hydrogen mass fraction. The Thomson cross section is σ_T .

1. From $d\tau = n_e(z) \sigma_T c dt$, derive an expression for the optical depth $\tau(z_{\text{ion}})$.
2. Show that, in the limit $z_{\text{ion}} \gg 1$,

$$\tau(z_{\text{ion}}) \propto (1+z_{\text{ion}})^{3/2}.$$

3. How are the primary temperature anisotropy amplitude $(\Delta T/T)_{\text{primary}}$ and temperature power spectrum C_ℓ suppressed with τ ?
4. Give a physical explanation for why reionization damps small-scale temperature anisotropies but generates new polarization on large angular scales.

Problem 6: Centrifugal support in the Crab pulsar

The Crab pulsar has a rotation period $P \simeq 33$ ms. Model it as a neutron star of mass $M \simeq 1.4M_\odot$ and radius $R \simeq 10$ km.

Estimate the ratio of centrifugal acceleration to gravitational acceleration at the equator,

$$\frac{a_{\text{cent}}}{g}.$$

Then estimate the rotation period at which the star would be close to centrifugal break-up, and compare this value with the observed period of the Crab pulsar. State clearly the approximations used.

Problem 7: Transiting exoplanet: radius, orbit, and density

A star of mass $M_\star$ and radius $R_\star$ is observed to host a transiting planet. The transit light curve shows a periodic dimming with period P , fractional loss of light δ , total transit duration t_{tr} , and negligible ingress/egress time. Assume that the planet is much smaller than the star, that the orbit is circular, and that the transit is central.

1. Derive an expression for the planet radius R_p in terms of δ and $R_\star$.
2. Use Kepler's third law to derive the orbital semi-major axis a in terms of P and $M_\star$, assuming $M_p \ll M_\star$.

3. Show that the transit duration is approximately

$$t_{\text{tr}} \simeq \frac{P}{\pi} \frac{R_{\star}}{a}.$$

State clearly the assumptions entering this result.

4. Suppose that radial-velocity observations give a stellar velocity semi-amplitude K . For an edge-on circular orbit, derive an expression for the planet mass M_p in terms of K , P , and $M_{\star}$.
5. Combine the transit and radial-velocity information to obtain the mean density of the planet.
6. Explain how the result would change qualitatively if the orbit were not central or if the orbit were eccentric.

Problem 8: Planetary Astronomy

Scientists using the *Kepler* spacecraft discovered Kepler-69 c, a small planet in the habitable zone of the star Kepler-69. The star has equilibrium temperature $T_{\text{eff}} = 5637 \pm 107$ K, radius $R_{\star} = 0.943 \pm 0.117 R_{\odot}$, and mass $M_{\star} = 0.813 \pm 0.054 M_{\odot}$. The system is estimated to be 2,390 light-years (LY) from Earth.

- a) The host star is a G dwarf (Sun-like star). What is the apparent visual magnitude of Kepler-69, assuming it has the same luminosity as the Sun? The Sun's absolute visual magnitude is 4.83, and $1 \text{ pc} = 3.26 \text{ LY}$.
- b) *Kepler* time-series photometry data indicate a brightness drop of 340 ± 15 ppm in Kepler-69 during a transit. Assuming the planet is opaque, estimate the radius of Kepler-69 c in units of the Earth radius. Recall that $R_{\odot} = 109 R_{\oplus}$.
- c) The photometry data indicate an orbital period of 242.5 days for Kepler-69 c. Assuming a circular orbit, estimate the semi-major axis of the orbit, in AU.
- d) For certain assumptions, a Solar System object in thermal equilibrium has a temperature $T = 280 \text{ K} (1 - A)^{1/4} / a^{1/2}$, where A is the planet's albedo and a its semi-major axis, in AU. Assuming this formula applies and that Kepler-69 c has an Earth-like albedo ($A = 0.39$), what is the equilibrium temperature of Kepler-69 c? Explain any assumptions you believe are inherent in this T estimate, and why the surface temperature of the planet might be different.
- e) Assuming the T derived in (d), what is the wavelength of peak emission from Kepler-69 c, in μm ? What is the corresponding frequency?