Spring 2026 Physics Qualifying Exam
for Advancement to Candidacy
Part 1
May 6, 2026
9:00-11:15 PDT

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Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours and 15 minutes to complete 3 questions.

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Bohr radius of hydrogen atom	a_0	5.3×10^{-11} m
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
dipole radiation formula		$P_{rad} = \mu_0\omega^4\lvert\vec{\boldsymbol{p}}\rvert^2/12\pi c$
mass of an electron	m_e	0.5110 MeV/c^2
mass of a neutron	m_n	1.67493×10^{-27} kg = 939.5654 MeV/c^2
mass of a proton	m_p	1.67262×10^{-27} kg = 938.2721 MeV/c^2
mass of the sun	M_{sun}	2×10^{30} kg
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

Table of Spherical Harmonics

ℓ = 0

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

ℓ = 1

$$\begin{aligned}
Y_1^{-1}(\theta,\varphi) &= \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta &&= \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x-iy)}{r}\\
Y_1^{0}(\theta,\varphi) &= \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\cos\theta &&= \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\frac{z}{r}\\
Y_1^{1}(\theta,\varphi) &= -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta &&= -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x+iy)}{r}
\end{aligned}$$

ℓ = 2

$$\begin{aligned}
Y_2^{-2}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot e^{-2i\varphi}\cdot\sin^2\theta &&= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x-iy)^2}{r^2}\\
Y_2^{-1}(\theta,\varphi) &= \frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta\cdot\cos\theta &&= \frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x-iy)\cdot z}{r^2}\\
Y_2^{0}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{5}{\pi}}\cdot(3\cos^2\theta-1) &&= \frac{1}{4}\sqrt{\frac{5}{\pi}}\cdot\frac{(3z^2-r^2)}{r^2}\\
Y_2^{1}(\theta,\varphi) &= -\frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta\cdot\cos\theta &&= -\frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x+iy)\cdot z}{r^2}\\
Y_2^{2}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot e^{2i\varphi}\cdot\sin^2\theta &&= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x+iy)^2}{r^2}
\end{aligned}$$

Useful mathematical identities

$$\int_0^\infty \frac{dx\ x^n}{e^x-1} = \Gamma(n+1)\zeta(n+1)$$

$$\int_0^\infty \frac{dx\ x^n}{e^x+1} = \left(1-\frac{1}{2^n}\right)\Gamma(n+1)\zeta(n+1)$$

where $\Gamma(n)$ is the gamma function (and equals $(n-1)!$ for integer n), and $\zeta(n)$ is the Riemann zeta function.

$$\frac{1}{e^x+1} = \frac{1}{e^x-1} - \frac{2}{e^{2x}-1}$$

1. A neutron star (radius 10 km, mass 4×10^{30} kg) slowly accretes helium (atomic mass: 4.0026 amu) from a companion star. The helium forms a thin layer on the surface of the neutron star. Eventually the pressure and/or temperature become large enough that the entire mass of helium in the layer rapidly undergoes thermonuclear fusion to form carbon-12 (atomic mass: 12.0000 amu). This process produces an X-ray burst.

A satellite observes such a burst from a neutron star 15,000 light-years from Earth. At the start of the burst, the flux of energy as measured at the satellite from the object quickly increases to 8×10^{-10} J/(m$^2\cdot$ s), and then exponentially decays to zero with a time constant of 4 seconds.

Estimate the total mass of helium that is burned in this burst, and the pressure at the bottom of the helium layer just before ignition. Assuming that the helium layer forms a degenerate gas supported by electron degeneracy pressure, do an order of magnitude estimation of the thickness of the layer.

2. A pendulum consists of a point mass with mass M and net charge Q hanging at the end of a string with length L. The pendulum is displaced by a small angle θ_0 from vertical and released at $t = 0$. Ignoring any air resistance or friction, derive an expression for the amplitude as a function of time. At what time will the amplitude of the oscillation decay to $\theta_0/2$?

3. Demonstrate that the average energy of a neutrino at temperature T is $7/6$ times higher than the average energy of a photon at the same temperature. Recall that neutrinos are spin-1/2 and have a single polarization state, while photons are spin-1 with two polarization states.

4. Consider the Morse potential, used as an interatomic interaction model for the potential energy of a diatomic molecule:

$$V(r) = D\Big(1 - e^{-a(r-r_e)}\Big)^2$$

where r is the distance between the two atoms, D and a are positive constants, and r_e is the molecular bond length (equilibrium distance).

The Lagrangian for a diatomic molecule with reduced mass μ is, in polar coordinates,

$$L = \frac{1}{2}\,\mu\Big(\dot{r}^{\,2} + r^2\,\dot{\theta}^{\,2}\Big) - V(r)\,.$$

Write your answers in terms of μ, r_e, D and a.

A. Assume the molecule is not rotating. Compute the frequency of small oscillations around the equilibrium separation $r = r_e$.

B. Assume the molecule is rotating about its centre of mass. Compute the angular frequency $\dot{\theta}$ at which the molecular bond is stretched to $r = 2r_e$.

C. In the situation described in Part B, what is the frequency of small oscillations in r around the new equilibrium separation $r = 2r_e$?

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mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of C		12
molecular weight of He		4
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
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5. In a strong magnetic field, for example near a neutron star, hydrogen atoms will show Zeeman splitting in their energy levels. For strong enough magnetic field, the interaction term in the Hamiltonian takes the form

$$H_{int} = -\vec{\mu} \cdot \vec{B}$$

where

$$\vec{\mu} = \mu_B (g_L \vec{L} + g_S \vec{S})/\hbar$$

where μ_B is the Bohr magneton, $\vec{B}$ is the magnetic field, $\vec{L}$ and $\vec{S}$ are the electron's orbital and spin angular momenta, $g_L = 1$, and $g_S = 2$.

We will ignore fine splitting of the energy levels due to spin-orbit or relativistic effects, as these will be small compared to the Zeeman splitting for large enough B.

A. Sketch the energy levels of all of the $n = 1$ and $n = 2$ energy levels for a hydrogen atom in this magnetic field. Label each with appropriate quantum numbers.

B. Keeping in mind the allowed electric dipole transition selection rules ($\Delta\ell = \pm 1, \Delta m_\ell = 0, \pm 1, \Delta m_s = 0$), calculate the transition energies for all allowed transitions between the $n = 2$ states and the $n = 1$ states, as a function of B. The transition energy for $B = 0$ is $E_0 = 10.2$ eV.

C. If the observer views the neutron star along a line of sight parallel to the external magnetic field at the atom, which of these transitions will be observed?

6. As an electron with charge e approaches the 2D surface of liquid helium (assuming that the surface remains a plane), the weak polarizability of He leads to an attractive potential due to the appearance of a small image charge of $-0.007e$. Once the electron is extremely close to the He, repulsion effects lead to a hard-wall potential that pushes the electron away. Assuming that the surface of the He is otherwise unperturbed, the interaction of the electron with the He is entirely along z (the normal to the surface); it is free to move along x and y. Modelling the repulsion at the surface as an infinite barrier at the surface of the He, $z = 0$, calculate the ground state wavefunction along the z axis for an electron bound to the surface of He. Calculate the average electron distance from the surface in the ground state.

7. Charged pions (mass = 140 MeV/c^2) decay into a muon (mass = 106 MeV/c^2) and a neutrino (essentially massless): $\pi \to \mu + \nu$.

A. When pions decay at rest, what is the energy in MeV of the resulting neutrino?

B. Pions at rest decay isotropically, since the pion has no spin. Consider a collimated beam of pions moving at a speed of $0.999c$. Such a beam can be used to make a beam of neutrinos. For this beam, what is the opening angle of the neutrino beam observed in the lab frame? Here "opening angle" is defined as the angle relative to the beam centre that contains half of the neutrinos.

C. If the mean lifetime of a pion at rest is 26 ns, what is the median distance that the pions travel in the lab frame before decaying, for the beam described in Part B?

D. Derive the distribution of neutrino energies of this beam as measured in the lab frame.

8. Picture the electron as a uniformly charged spherical shell, with radius R and surface charge density $\sigma = e/4\pi R^2$, spinning at angular velocity ω. The resulting magnetic field is

$$\vec{B} = \frac{2\mu_0 R\omega\sigma}{3}\left(\cos\theta\ \hat{\mathbf{r}} - \sin\theta\ \hat{\boldsymbol{\theta}}\right) \qquad \text{for } r \le R$$

and

$$\vec{B} = \frac{\mu_0 R^4\omega\sigma}{3r^3}\left(2\cos\theta\ \hat{\mathbf{r}} + \sin\theta\ \hat{\boldsymbol{\theta}}\right) \qquad \text{for } r > R.$$

A. Calculate the total energy contained in the electromagnetic fields.

B. Calculate the total angular momentum contained in the fields.

C. According to the Einstein formula ($E = mc^2$), the energy in the fields should contribute to the mass of the electron. Lorentz and others speculated that the entire mass of the electron might be accounted for in this way: $U_{em} = mc^2$. Suppose, moreover, that the electron's spin angular momentum is entirely attributable to the electromagnetic fields: $L_{em} = \hbar/2$. On these two assumptions, determine the radius and angular velocity of the electron. What is their product, ωR? Does this classical model make sense?