Spring 2024 Physics Qualifying Exam
for Advancement to Candidacy
Part 1
May 8, 2024
9:00-11:15 PDT

If you are in the PhD in astronomy or PhD in medical physics programs, stop! This is the physics version of the exam. Please ask the proctor for the version appropriate for your program instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours and 15 minutes to complete 3 questions.

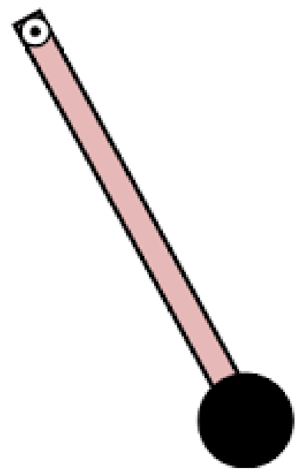
You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas (see back of page as well):

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Bohr radius of hydrogen atom	a_0	5.3×10^{-11} m
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.5110 MeV/c^2
mass of a neutron	m_n	1.67493×10^{-27} kg = 939.5654 MeV/c^2
mass of a proton	m_p	1.67262×10^{-27} kg = 938.2721 MeV/c^2
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

1. A superconducting microwave resonator is a small loop with an inductance of L and a capacitance of C. It can be modelled quantum mechanically. Its quantum state can be described by the charge Q on the capacitor and the magnetic flux ϕ through the loop. The operators for these quantities satisfy a uncertainty relation $[\hat{Q}, \hat{\phi}] = i\hbar$. Write down the Hamiltonian for this system, and determine its ground state energy.

2. Carbon monoxide (CO) has emission lines corresponding to transitions between a state with angular momentum J to a state with $J-1$. The transition with the lowest energy has a wavelength of 2.6 mm. Calculate the wavelength of the transition with the second-longest wavelength, and estimate the distance between the carbon atom (molecular weight = 12 amu) and oxygen atom (molecular weight = 16 amu).



3. Consider a pendulum which consists of a weight of mass M attached to a fixed point by a rigid rod of length ℓ. The only forces acting on the weight are gravity and the force due to the presence of the rigid rod. Assume that the rod is perfectly rigid, one end is attached to the fixed point, the other end is attached to the weight. Assume that, constrained by these attachments, the rod can take up any orientation, that it is free to move, that it has negligible mass and that its movement involves negligible friction.

This "spherical pendulum" has stable circular orbits where the height of the weight is a constant. Show that the stable circular orbits occur when and only when the height of the the weight is less than the height of the fixed point where the rod is attached.

If one considers a stable circular orbit and perturbs it slightly, stability implies that, for a sufficiently small perturbation, there is an oscillatory behaviour. Find the frequency of the oscillation as a function of the height of the stable orbit.

4. The interaction potential between atoms of mass M is given by a Lennard-Jones potential:

$$u(x) = \frac{A}{x^{12}} - \frac{B}{x^6}$$

Here A and B are positive constants. Consider a crystal formed from these atoms, with a cubic lattice structure. Calculate the density of the crystal. Give an expression for its specific heat capacity in the low temperature limit. Consider only the interactions of each atom with its six nearest neighbours. Calculate as well the approximate binding energy of the quantum mechanical ground state for the diatomic molecule formed from these atoms.

Spring 2024 Physics Qualifying Exam
for Advancement to Candidacy
Part 2
May 8, 2024
12:30-14:45 PDT

If you are in the PhD in astronomy or PhD in medical physics programs, stop! This is the physics version of the exam. Please ask the proctor for the version appropriate for your program instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours and 15 minutes to complete 3 questions.

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

5. A neutron star can be modelled as a degenerate Fermi gas of constant density made up of neutrons, held together by the gravitation attraction between the neutrons. Under this approximation, and assuming the neutrons are non-relativistic, derive the mass M of the neutron star with a radius of 10 km. Your answer may depend on the mass of a neutron m_n and fundamental constants.

6. Consider a non-relativistic electron near an infinite planar perfectly conducting surface. Recall that the boundary condition for the electric field at such a surface is that the transverse components must all vanish. Also recall that this boundary condition can be imposed by introducing an image charge at a mirror position on the opposite side of the surface. The question that we are interested in asks whether the electron can form a bound state "hydrogen atom" with its image charge. Can such a state exist? If so, estimate its ionization energy.

7.

A P-N junction device is shown below. The junction is centered at x=0 and the potential in the device is given by the function:

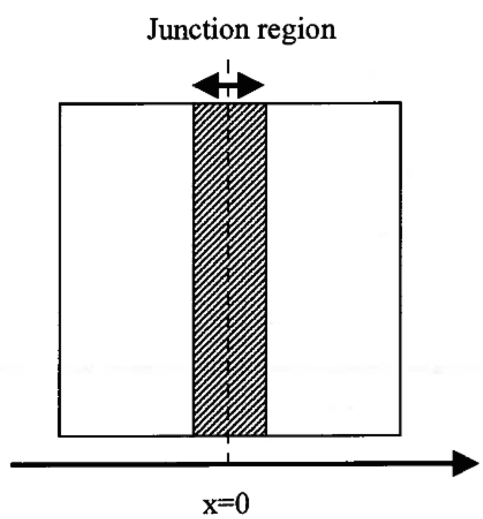
$$\varphi(x)=\left(1+\frac{x}{\sqrt{x^2+a^2}}\right)\cdot\frac{V_0}{2}$$

where a is a measure of the width of the junction. In the approximation that the width of the device is much greater than the width of the junction region, V_0 is the total potential difference across the device.

a) Find the electric field E and the charge density ρ as a function of x. Specify which set of units you are using (i.e., SI, Gaussian, etc.).

b) Plot the functions φ(x), E(x), and ρ(x) as functions of x. Label the axes so that the maxima or minima of the functions can be ascertained.

c) Find the net charge on the device.



8. Consider a diatomic gas, where each molecule may be modelled as two point-like atoms, each with mass 1×10^{-27} kg held at a distance of 2×10^{-10} m by a spring with $k = 2000$ N/m. Neglect any interactions between molecules, and assume that the molecules survive up to arbitrarily high temperature. Sketch the heat capacity of one mole of these molecules from 1 K up to 10,000 K, with your horizontal axis (temperature) shown in log scale, and your vertical axis in units of the ideal gas constant R.