Astronomy Comprehensive Exam, Spring 2024
Session 1

May 8, 2024

Note: if you are in the PhD in physics program, stop! This is the astronomy version of the exam. Please ask the proctor for the version appropriate for your program instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer ***any three*** of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours 15 minutes to complete 3 questions.

You are allowed to use two $8.5'' \times 11''$ formula sheets (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Bohr radius of hydrogen atom	a_0	5.3×10^{-11} m
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.5110 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg = 939.5654 MeV/c^2
mass of a proton	m_p	1.673×10^{-27} kg = 938.2721 MeV/c^2
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

1. A superconducting microwave resonator is a small loop with an inductance of L and a capacitance of C. It can be modelled quantum mechanically. Its quantum state can be described by the charge Q on the capacitor and the magnetic flux Φ through the loop. The operators for these quantities satisfy an uncertainty relation $[\hat{Q}, \hat{\Phi}] = i\hbar$. Write down the Hamiltonian for this system, and determine its ground state energy.

2. Carbon monoxide (CO) has emission lines corresponding to transitions between a state with angular momentum J to a state with $J - 1$. The transition with the lowest energy has a wavelength of 2.6 mm. Calculate the wavelength of the transition with the second-longest wavelength, and estimate the distance between the carbon atom (molecular weight = 12 amu) and oxygen atom (molecular weight = 16 amu).

3. Consider a pendulum which consists of a weight of mass m attached to a fixed point by a rigid rod of length l. The only forces acting on the weight are gravity and the force due to the presence of the rigid rod. Assume that the rod is perfectly rigid, one end is attached to the fixed point, the other end is attached to the weight. Assume that, constrained by these attachments, the rod can take up any orientation in three dimensions, that it is free to move, that it has negligible mass and that its movement involves negligible friction.

   This "spherical pendulum" has stable circular orbits where the height of the weight is a constant. Show that the stable circular orbits occur when and only when the height of the the weight is less than the height of the fixed point where the rod is attached. If one considers a stable circular orbit and perturbs it slightly, stability implies that, for a sufficiently small perturbation, there is an oscillatory behaviour. Find the frequency of the oscillation as a function of the height of the stable orbit.

4. The interaction potential between atoms of mass M is given by a Lennard-Jones potential:

$$u(x) = \frac{A}{x^{12}} - \frac{B}{x^6}$$

   Here A and B are positive constants and x is the separation between the two atoms. Consider a crystal formed from these atoms, with a cubic lattice structure. Calculate the density of the crystal. Give an expression for its specific heat capacity in the low temperature limit. Consider only the interactions of each atom with its six nearest neighbours. Calculate as well the approximate binding energy of the quantum mechanical ground state for the diatomic molecule formed from these atoms.

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Session 2

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You have 2 hours 15 minutes to complete 3 questions.

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Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	$-273.16°$C
astronomical unit	au	$149,597,870.7$ km
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
luminosity of the sun	$L_\odot$	3.8×10^{26} W
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the earth	$M_\oplus$	5.97219×10^{24} kg
mass of the sun	$M_\odot$	2×10^{30} kg
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
parsec	pc	3.086×10^{16} m
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the earth	$R_\oplus$	$6,371.0$ km
radius of Jupiter	$R_{♃}$	$69,911$ km
radius of the sun	$R_\odot$	$696,342$ km
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Thomson cross section	σ_T	6.65×10^{-29} m^2

1. Consider a planetesimal that contains N atoms. At what value of N does gravity begin to dominate over other forces in determining the structure of the object. What is the mass where gravity just begins to dominate? Is this mass reasonable? Suppose that the object consists mainly of water ice (density $= 917$ kg m^{-3}, compressive strength $= 4$ MPa, heat of fusion $= 333$ J g^{-1}). At approximately what minimum radius would you expect the object to be roughly spherical. Please give your assumptions in your analysis.

2. Two stars, having masses m_1 and m_2 orbit each other in a binary system that has semimajor axis a.

   (a) Assuming circular orbits, determine the kinetic and potential energies of the system and show that the (scalar) virial theorem is satisfied. Would you expect the virial theorem to hold? Explain.

   (b) Suppose that star #1 explodes, leaving behind a relic that has a fraction f of its original mass. Show that the system will be disrupted if

$$f < \frac{1}{2 + m_2/m_1}$$

3. Consider a comet that falls onto a neutron star which has mass $M = 2M_\odot$ and radius $R = 10$ km. Take the comet to be spherical with a diameter of 10 km and composed entirely of water ice.

   (a) Ignoring relativistic effects, find the speed with which the comet hits the neutron star, and estimate the energy released in the resulting explosion. (Ignore any energy produced by nuclear reactions.)

   (b) Suppose that the energy released all goes in to heating the constituent atoms of the comet, which then dissociate and ionize, forming an optically-thick plasma. What would be the temperature the plasma, and where in the electromagnetic spectrum would the resulting black body emission peak (eg. radio, IR, optical, UV, X-ray, gamma ray)?

   (c) Provide a plausible estimate for the duration of this event and thus estimate the luminosity of the explosion, in units of the solar luminosity. Assuming that the neutron star is within the Milky Way, do you think that we might be able to detect this event?

4. Current observations indicate that, on large scales, the universe is flat, with dark energy contributing about 70% of the mass/energy density of the universe at the present time. Yet, estimates of the masses of clusters of galaxies ignore the gravitational repulsion produced by dark energy. Typically one estimates the mass within a radius r_{200} centred on the cluster, within which the mean matter density is 200 times the critical density, by relating the mass to the radial velocity dispersion σ of galaxies in the cluster using the virial theorem. Does this result in an overestimate or an underestimate of the mass? Estimate the fractional error in the mass of a low-redshift galaxy cluster that results from ignoring the repulsion of dark energy. (Treat dark energy as a homogeneous fluid having a pressure $p = -\varepsilon$, where ε is the dark energy density.)