Fall 2025 Physics Qualifying Exam
for Advancement to Candidacy
Part 1
August 29, 2025
9:00-11:15 PDT

If you are in the PhD in astronomy or PhD in medical physics programs, stop! This is the physics version of the exam. Please ask the proctor for the version appropriate for your program instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours and 15 minutes to complete 3 questions.

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Bohr radius of hydrogen atom	a_0	5.3×10^{-11} m
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.5110 MeV/c^2
mass of a neutron	m_n	1.67493×10^{-27} kg = 939.5654 MeV/c^2
mass of a proton	m_p	1.67262×10^{-27} kg = 938.2721 MeV/c^2
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of C		12
molecular weight of He		4
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

1. A particle starts in the ground state of a 1D quantum harmonic oscillator with characteristic frequency ω_0. The frequency of the oscillator is suddenly (instantly) changed to $\alpha\omega_0$, without changing the wavefunction of the particle. Then, the energy of the particle is measured. What are the possible outcomes of this measurement? Derive an expression for the probability of the system being in the ground state of the new Hamiltonian.

2. Provide an order-of-magnitude estimate for the power dissipated by a copper spherical shell with a radius of R and a wall thickness $d \ll R$, spinning at n revolutions per second in a uniform magnetic field of strength B oriented along the axis of rotation. The resistivity of copper is ρ.

3. A thin ring of electric charge with a total charge of $Q > 0$ is assembled in zero gravity, with the charge distributed uniformly in a ring of radius R that lies in a plane.

A. A test particle with charge $q < 0$ and mass m is placed in the centre of the ring, then displaced by distance $d \ll R$ from the centre in the direction perpendicular to the plane of the ring. The particle is then released. Calculate the frequency of oscillation of its motion around the ring's centre.

B. If the particle is now placed in the centre of the ring and then displaced radially by distance $d \ll R$ in the plane of the ring, calculate the force on the particle, including the direction of the force.

4. Consider a three-level system described by the Hamiltonian

$$\frac{H}{\hbar} = \Omega(|e\rangle\langle g_1| + |g_1\rangle\langle e| + |e\rangle\langle g_2| + |g_2\rangle\langle e|),$$

where $\Omega > 0$ is a real, positive coupling parameter, and the three levels are the normalized states $|g_1\rangle$, $|g_2\rangle$, and $|e\rangle$.

(a) Find all the eigenstates of this Hamiltonian and their energies.

(b) Assume an initial state $|\psi(0)\rangle = |g_1\rangle$ and calculate the state $|\psi(t)\rangle$ at time t. Find the minimum time $T > 0$ when $|\psi(T)\rangle$ is given by $|g_2\rangle$.

(c) We may generalize the Hamiltonian above to an n-level system ($n \geq 3$), described by

$$\frac{H}{\hbar} = \sum_{j=1}^{n-1} \left[\Omega|e\rangle\langle g_j| + \Omega|g_j\rangle\langle e|\right],$$

where the n levels are the normalized states $|g_j\rangle$ ($j = 1, \ldots, n-1$) and $|e\rangle$. How many eigenstates are there with zero energy?

(d) Give an explicit example of a normalized eigenstate with zero energy for $n = 10$.

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Part 2
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13:30-15:45 PDT

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5. A hydrogen fuel cell works based on the reaction

$$H_2\ (g) + 1/2\ O_2\ (g) \rightarrow H_2O\ (l),$$

where the molar entropies and enthalpies of formation of the constituents at 300 K and 1 atm are:

S_{H_2} (g):	131 J/K
S_{O_2} (g):	205 J/K
S_{H_2O} (l):	70 J/K
H_{H_2} (g):	0
H_{O_2} (g):	0
H_{H_2O} (l):	-286 kJ.

The reaction at the anode is

$$H_2 + 2OH^- \rightarrow 2H_2O + 2e^-$$

and the reaction at the cathode is

$$1/2\ O_2 + H_2O + 2e^- > 2OH^-.$$

A. Explain why (from first principles) $S_{H_2O}\ (l) < S_{H_2} < S_{O_2}$.

B. Calculate the open-circuit voltage of the fuel cell based on this information, assuming the reaction is carried out at atmospheric pressure.

C. What fraction of the energy from this reaction is released in the form of heat, assuming optimal efficiency?

6. Radial and vertical oscillations of a star in the Milky Way

In the Milky Way, stars do not follow perfect circular orbits in the disk but instead undergo small oscillations in both the radial (R) and vertical (z) directions about their guiding center radius. Consider a star located near the Sun, at a galactocentric radius $R_0 = 4.4 \times 10^{18}$ m. Assume the local gravitational potential is axisymmetric and can be approximated near R_0 as harmonic in R and z.

(a) **Radial (epicyclic) motion**

The epicyclic frequency κ characterizes radial oscillations about the guiding center. For a circular orbit in an axisymmetric potential $\Phi(R)$, the epicyclic frequency (defined as 2π divided by the oscillation period) is given by:

$$\kappa^2 = R\frac{d(\Omega^2)}{dR} + 4\Omega^2, \quad \text{where } \Omega(R) = \frac{v_c(R)}{R}.$$

Assume a flat rotation curve near the Sun: $v_c(R) = 220$ km/s (constant). Compute κ and the corresponding radial oscillation period T_R for a star near R_0.

(b) **Vertical oscillations and harmonic approximation**

Near the Galactic plane, stars oscillate vertically due to the restoring force of the local gravitational potential.

(i) Show that near $z = 0$, the vertical gravitational potential of a particle of mass m can be approximated as harmonic, i.e.:

$$\Phi(z) \approx \frac{1}{2}m\nu^2 z^2$$

by performing a Taylor expansion of the vertical potential around the midplane.

(ii) Assuming that $\nu \approx 1.4 \times 10^{-13}$ s^{-1}, compute the vertical oscillation period T_z for a star near the Sun.

(iii) If the star reaches a maximum vertical height of $z_{\max} = 10^{17}$ m, determine the maximum vertical speed $v_{z,\max}$.

7. Tiny Thermodynamics

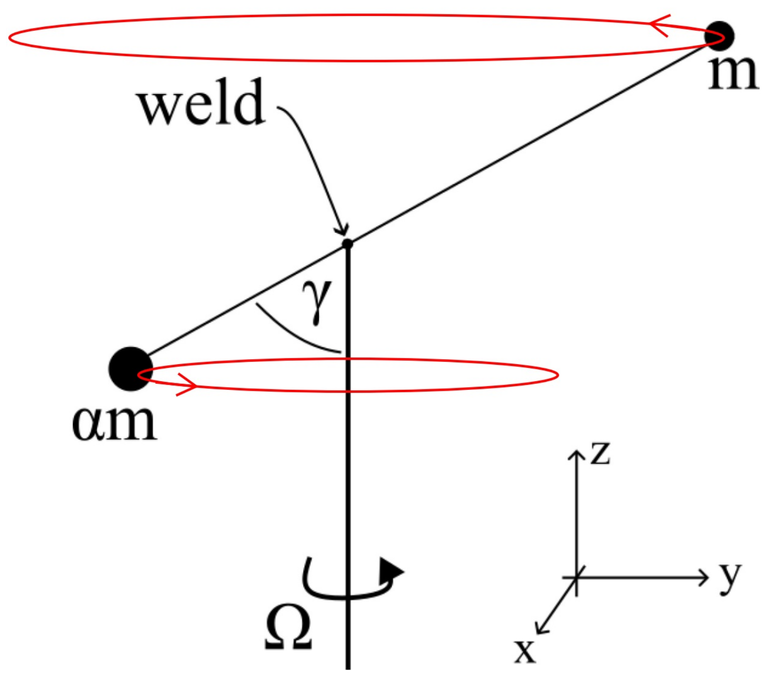
Suppose that you have two tiny pieces of metal. They are so small, in fact, that the internal energy in each piece is contained within five harmonic oscillators. The two pieces of metal are brought into thermal contact and reach equilibrium with ten units of energy ($\hbar\omega$ of the oscillators) in total. You can measure how much energy is in each piece of metal, but not which oscillators contain the energy.

A. How many macrostates are there?

B. How many total microstates are there?

C. Assuming that the system is ergodic (i.e. all microstates are accesible), what would the chance be of seeing all of the energy in one piece of metal (either one)?

8. Consider two point-like masses, m and αm, connected by a light, rigid rod of length d. The centre of mass of this 'baton' is welded at an angle γ to a vertical rod spinning with a constant angular frequency Ω. Ignore gravity.



A. At the instant when the baton lies in the $x = 0$ plane as shown, compute the rate of change of angular momentum, $d\vec{L}/dt$. Hint: draw a force diagram for the system.

B. At that instant, the weld breaks. Obtain the subsequent motion of the baton.