Fall 2024 Physics Qualifying Exam
for Advancement to Candidacy
Part 1
August 30, 2024
9:00-11:15 PDT

If you are in the PhD in astronomy or PhD in medical physics programs, stop! This is the physics version of the exam. Please ask the proctor for the version appropriate for your program instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

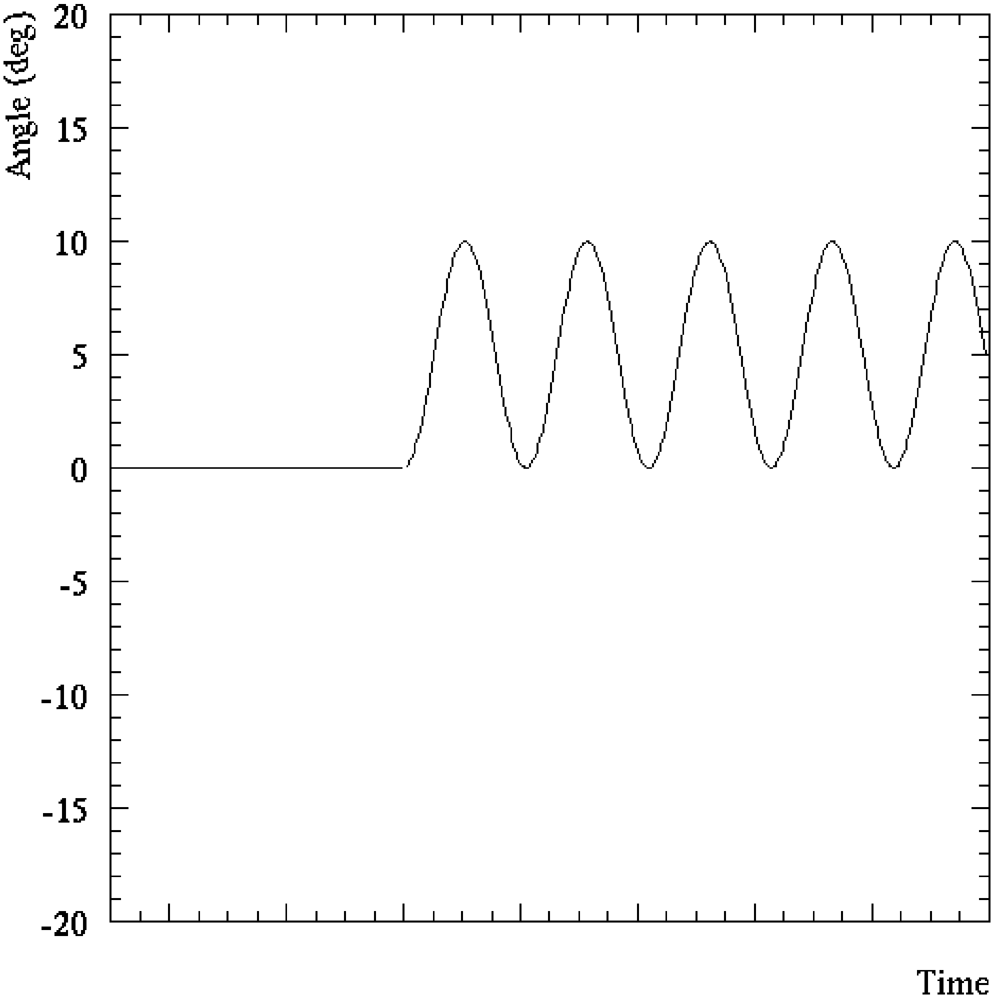
This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours and 15 minutes to complete 3 questions.

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas (see back of page as well):

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Bohr radius of hydrogen atom	a_0	5.3×10^{-11} m
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.5110 MeV/c^2
mass of a neutron	m_n	1.67493×10^{-27} kg = 939.5654 MeV/c^2
mass of a proton	m_p	1.67262×10^{-27} kg = 938.2721 MeV/c^2
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$



1 A traveler brings a small pendulum from her physics lab to a well isolated room in a moving train. The pendulum was tuned to a natural frequency of $f = 10$ Hz in her lab. At $t = 0$, the pendulum which was sitting still vertically suddenly starts moving backwards (with respect to the motion of the train) and keeps oscillating with an amplitude of 5 degrees for 20 s. All observed oscillations are centred around a position that is 5 degrees (backward) away from the vertical direction.

A. Describe the motion of the train before and after $t = 0$ as quantitatively as you can, up to $t = 20$ s.

B. What is your best estimate of the oscillation frequency during that 20 s time window?

2. A 10 kg weight hangs vertically from a 1 m long string. A muon with a total energy of 100 GeV passes horizontally through the weight and loses 100 MeV of energy in the process. Calculate the resulting amplitude of oscillation of the pendulum, in radians.

3. Consider a cold system of N identical spin-1/2 fermions, each with a charge of e, trapped in a one-dimensional harmonic well and subject to a constant electric field E so that their dynamics are described by the Hamiltonian

$$H = \sum_{i=1}^{N} \left(\frac{p_i^2}{2m} + \frac{k}{2} x_i^2 - eEx_i \right).$$

Assume that all of the particles are somehow kept in their spin-up state[1]. Then the system has the lowest N levels filled and the Fermi energy is defined as the value of the energy that is equidistant between the highest filled and the lowest unfilled energy level. Find the Fermi energy. Once you find it, you might note that it has the interesting property that it scales $\sim N$, unlike that of a usual Fermi gas where the Fermi energy is an intensive quantity. In the usual cold ($T \to 0$) Fermi gas, the chemical potential and the Fermi energy are equal. Are the chemical potential and the Fermi energy equal in this system? Justify your answer. Also, compute the location of the center of mass of the system.

4. A quantum crystal can have spatially extended states with a quadratic dispersion

$$\epsilon_k = \frac{\hbar^2 k^2}{2m},$$

where $\hbar k$ is the momentum associated with a particle. In addition, particles in this system can also occupy highly degenerate states that are fully localized and spatially well-separated, all with the same negative eigenvalue $-\epsilon_b$ (with $\epsilon_b > 0$). The number density of such localized states (i.e. the number of localized states per unit volume) is n_b. As the temperature is increased, more particles start occupying the extended states with positive quadratic dispersion. Estimate the temperature at which about 50% of the particles still remain in the localized states of energy $-\epsilon_b$. Hint: assume that the particle density is much lower than n_b, so that it can be treated as a dilute gas. Furthermore, assume that

$$\epsilon_b \gg \epsilon_0 = \frac{\hbar^2 n_b^{2/3}}{2m}$$

to simplify your calculations.

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[1] If it helps, you can instead imagine these particles as spinless particles that happen to also obey the Pauli exclusion principle.

Fall 2024 Physics Qualifying Exam
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Part 2
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14:00-16:15 PDT

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mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
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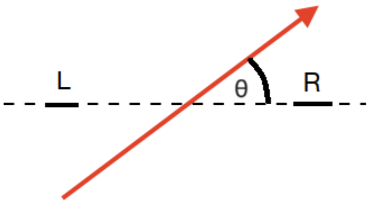

5. In a mini-field detector based on temporal interference effects, an electron can tunnel between two sites, L and R, separated by distance D (see above) along the direction of a dashed line, with a known amplitude "t". So, a particle initialized in site L will tunnel between L and R sites, resulting in oscillatory behaviours in the probabilities of being in site L or site R.

Robert has a very precise clock to track these temporal oscillatory behaviours and he wants to use it as a detector of very weak static electric fields around his lab. He knows that the field is uniform with a known magnitude E but he doesn't know its direction with respect to his device (as indicated above: the direction of electric field represented by an arrowed line has a unknown angle θ with respect to the dashed line, with $0 \leq \theta < 2\pi$). How will the oscillation frequency Ω in this case depend on the angle if Robert initializes the particle in site L ?

6. Calcite has an anisotropic index of refraction, varying from 1.49 for light polarized along one optical axis to 1.66 for light polarized along the perpendicular optical axis. A linearly polarized laser beam with a power of one watt and a wavelength of 500 nm is normally incident onto a sheet of calcite, whose optical axes lie in the plane of the sheet. The beam's plane of polarization makes a 45° angle with either optical axis. Calculate the minimum thickness for the calcite sheet that will convert the incident light to be circularly polarized. Then calculate the torque imparted by the laser to a sheet of this thickness. Assume that 100% of the incident flux is transmitted through the sheet.

7. Make an order-of-magnitude estimate of the efficiency of converting solar energy to heat by growing pine trees and burning them. That is, estimate the ratio of the amount of heat released by burning a pine tree to the amount of solar energy incident onto the tree as it grows. Justify your approximations.

8. A circular loop with radius R_0 and electric current I_0 (in the clockwise direction when viewed from the top) is placed in the xy-plane centered at $(0, 0, 0)$. Slightly above the centre of the loop along the z-axis at $(0, 0, z), z \ll R_0$, there is another tiny current loop of mass m and radius r_0 ($\ll R_0$) and current i_0 (also in the xy-plane, in the clockwise direction when viewed from the top). Find the force acting on the small current loop (magnitude and direction) in terms of the parameters given above. Describe the motion of the tiny current loop when released from $(0, 0, z)$. Neglect gravity. You may also assume that the motion is small enough that inductive currents may be ignored (i.e. the currents are constant).