Astronomy Comprehensive Exam, Spring 2024

Session 1

August 30, 2024

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This portion of the exam has 4 questions. Answer ***any three*** of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours 15 minutes to complete 3 questions.

You are allowed to use two $8.5'' \times 11''$ formula sheets (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Bohr radius of hydrogen atom	a_0	5.3×10^{-11} m
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to moon	d_{moon}	3.8×10^{8} m
distance from earth to sun	1 AU	1.5×10^{11} m
electron volt	1 eV	1.6×10^{-19} J
mass of an electron	m_e	0.5110 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg = 939.5654 MeV/c^2
mass of a proton	m_p	1.673×10^{-27} kg = 938.2721 MeV/c^2
mass of the moon	M_{moon}	7×10^{22} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the earth	R_{earth}	6.4×10^{6} m
radius of the sun	R_{sun}	7×10^{8} m
speed of light	c	3.0×10^{8} m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N} N^N \sqrt{2\pi N}$

1. (a) Suppose that galactic supernovae obey Poisson statistics. The mean number of supernovae per century is 3. What is the most likely date for the next supernova? What is the probability distribution for the length of the interval between now and the next galactic supernova?

   (b) The Planck collaboration obtained the following measurements and constraints (among others) on cosmological parameters:

$$H_0 = 67.27 \pm 0.60 \text{ km/s/Mpc},$$
$$\Omega_m = 0.3166 \pm 0.0084,$$
$$\Omega_m (H_0)^3 = 96330 \pm 290 \text{ (km/s/Mpc)}^3.$$

   Are the errors in H_0 and Ω_m correlated? Use the error propagation equation to determine the covariance between these two parameters, and the correlation coefficient.

2. Astronomers learn a great deal about processes in the universe by studying the ratios of various elemental isotopes. Here we concentrate on the isotopic ratios that can be measured in meteoritical samples.

   (a) The very important concept of *cosmochronology* involves measuring the isotopic abundances (relative to some reference isotope of one of the elements in question) in a set of different minerals found in a meteorite. In this case there is a radioactive decay of some parent isotope P to a daughter isotope D with some known decay constant timescale λ (at which time $1/e$ of the parent remains). Explain, using a sketched graph, what information can be obtained about the meteorite and/or the early Solar System with measurements of the isotopic ratios. If you wish to use the example of a specific isotopic system, you may. This part of the question is thus concentrated on how the technique works.

   (b) Still in the topic of cosmochronology, regardless of your explanation in part (a) of *how* the technique works, name four specific results that cosmochronology has told us about the history of objects in the Solar System.

   (c) A related, but not identical, topic is that of *extinct radionuclides*. Explain both (1) generically how this technique works, and (2) two qualitative results we know about either the history of the solar system or processes in the galaxy (or planet formation) that scientists have learned.

3. There is a coin at the bottom of a two-metre deep pool. You are standing at the edge of the pool and your line of sight makes a thirty degree angle with the surface of the water where you see the coin. How deep does the coin appear? Now you turn your head sideways so your eyes are arranged vertically one above the other; how deep does it appear now? (Take the index of refraction of water to be 1.33).

4. (a) An exoplanet is on a close, planar, circular orbit around a solar-type star. Tides between the star and planet are causing the planet's orbit to decay (semi-major axis decreases). The period decay timescale is given by $\tau_P = P/\dot{P}$. Let $\tau_P = -1$ Myr. What is the corresponding semi-major axis decay timescale $\tau_a = a/\dot{a}$?

   (b) Assume the planet is tidally locked and remains that way as the planet's orbit decays. Further assume the structure of the star and planet remain constant and that the total angular momentum of the star and planet is conserved. How does the star's rotation rate change as a function of $\dot{P}$ and P (and any other appropriate constants)?

(c) Describe the condition necessary for tides to cause the planet's orbit to decay. Consider the cases where the spin and orbital angular momentum vectors are parallel and anti-parallel. Do the star and planet spin up, spin down, or do something else? Do not worry about inclined cases. Only consider basic tidal interactions.

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5. An object of mass M is embedded in homogeneous expanding flat universe that has matter density ρ_m and dark energy density ρ_Λ, with $\rho_m < 2\rho_\Lambda$. Close to the mass, matter decouples from the expansion and falls towards the mass. Further away, the expansion prevents the matter from ever reaching the mass. Find the radius that separates these two regimes and estimate the mass of the matter that accretes onto the object.

6. The neutron star that powers the Crab Pulsar can be assumed to have a mass of 1.4 $M_\odot$ and a radius of 10 km with constant internal density and an effective temperature of 10^6 K. The frequency of the Crab Pulsar is 30 Hz and its period increases by 38 ns each day. Compare the power from the surface emission to the power lost as the neutron star spins down. The total power of the Crab Nebulae is about 75,000 times that of the Sun. What is the likely source of this power?

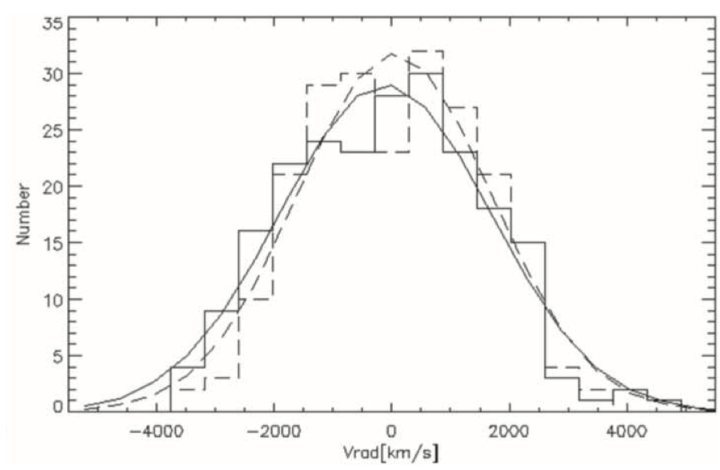
7. Consider a star of initial radius 5×10^8 cm and mass 1.4 $M_\odot$, with a uniform temperature of 10^{10}K, that explodes as a supernova. Assuming constant density, total ionization, $Z/A = 0.5$, homologous expansion ($v \propto r$), and opacity due to electron scattering, calculate the radius the expanding sphere would have when it first became optically thin. (use $\kappa_e = 0.2$ cm^{2}g^{-1} throughout) If the expansion were adiabatic as well as homologous what would be the temperature of the radiation at this point (assume that radiation entropy is separately conserved).

8. (a) Using the virial theorem, show that the mass of a galaxy cluster, $M_{\rm vir}$, is related to the line-of-sight velocity dispersion of galaxies σ_v (assumed to be isotropic) as:

$$M_{\rm vir} = \frac{3 r_{\rm vir} \sigma_v^2}{G} \quad (0.1)$$

where $r_{\rm vir}$ refers to the virial radius and G is the gravitational constant.

(b) A galaxy cluster has a virial radius of 2 Mpc. The following plot shows its radial velocity distribution. Estimate its virial mass in solar mass unit.



(c) State one limitation in using the virial theorem to accurately weigh a galaxy cluster.

(d) Describe another method to weigh a galaxy cluster.