Spring 2023 Physics Qualifying Exam
for Advancement to Candidacy
Part 1
August 25, 2023
9:00-11:15 PDT

If you are in the PhD in astronomy or PhD in medical physics programs, stop! This is the physics version of the exam. Please ask the proctor for the version appropriate for your program instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours and 15 minutes to complete 3 questions.

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^{6} m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^{8} m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

1.

A. Find the temperature dependence of the energy density associated with photons at temperature T in 3D space, assuming equilibrium. (You need the temperature dependence and the dependence on the speed of light; the dimensionless prefactor doesn't play much role below.)

B. For photons, the entropy density can be directly related to the energy density from part A. Using dimensional analysis, can you find the temperature and speed dependence of the entropy as well?

C. In complex quantum systems, photon-like excitations can emerge, which propagate with a tunable "speed of light" v. If this "speed of light" adiabatically changes from v to $2v$, what is the change of the emergent photon temperature in this case? (You may assume that the excitations have a thermal spectrum.)

2. Order of magnitude estimation:

A. Estimate the total energy stored in ocean waves on the surface of the earth at any moment in time.

B. Estimate how close (in orders of magnitude) a modern-day computer processor is to fundamental thermodynamic limits on energy efficiency.

3. A spring of uniform stiffness and composition has a mass of m and an unstretched length of L. Its spring constant is measured to have some value k. The spring is allowed to hang under its own weight. Let f denote the fraction of the spring's mass that is below some location x, measured from the lower end of the spring. Calculate $x(f)$. What is the total length of the spring when hanging under its own weight: in other words, what is $x(1)$?

4. A particle moves in a potential well given by

$$V(x) = (Ax^2 - B)\exp[-\alpha x^2],$$

where A, B, and α are positive constants. At time $t = 0$ the particle is at $x = 0$ and has velocity v_0 in the $+x$ direction. For the following, treat the problem relativistically.

A. For what range of v_0 will the particle escape to infinity?

B. If the particle escapes to infinity, what velocity will it have in the limit that $t \to \infty$?

C. Suppose that a laser beam with power P, measured in the lab frame, is applied in the $-x$ direction. What power does the particle measure in its rest frame at $t = 0$?

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Part 2
August 25, 2023
12:30-14:45 PDT

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Here is a possibly useful table of physical constants and formulas (see back of page as well):

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atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Bohr radius of hydrogen atom	a_0	5.3×10^{-11} m
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.5110 MeV/c^2
mass of a neutron	m_n	1.67493×10^{-27} kg = 939.5654 MeV/c^2
mass of a proton	m_p	1.67262×10^{-27} kg = 938.2721 MeV/c^2
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
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Table of Spherical Harmonics

ℓ = 0

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

ℓ = 1

$$\begin{aligned}
Y_1^{-1}(\theta,\varphi) &= \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta &&= \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x-iy)}{r}\\
Y_1^{0}(\theta,\varphi) &= \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\cos\theta &&= \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\frac{z}{r}\\
Y_1^{1}(\theta,\varphi) &= -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta &&= -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x+iy)}{r}
\end{aligned}$$

ℓ = 2

$$\begin{aligned}
Y_2^{-2}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot e^{-2i\varphi}\cdot\sin^2\theta &&= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x-iy)^2}{r^2}\\
Y_2^{-1}(\theta,\varphi) &= \frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta\cdot\cos\theta &&= \frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x-iy)\cdot z}{r^2}\\
Y_2^{0}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{5}{\pi}}\cdot(3\cos^2\theta-1) &&= \frac{1}{4}\sqrt{\frac{5}{\pi}}\cdot\frac{(3z^2-r^2)}{r^2}\\
Y_2^{1}(\theta,\varphi) &= -\frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta\cdot\cos\theta &&= -\frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x+iy)\cdot z}{r^2}\\
Y_2^{2}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot e^{2i\varphi}\cdot\sin^2\theta &&= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x+iy)^2}{r^2}
\end{aligned}$$

5. A neutral tritium atom (Z=1, A=3) has a mass of 2809.4321 MeV/c^2, while a neutral helium-3 atom (Z=2, A=3) has a mass of 2809.4135 MeV/c^2. Use the measured masses to estimate the distance between the protons in tritium.

6. Many copies of a system of two spin-1/2 particles are prepared in an identical spin state given by:

$$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle$$

An experimenter performs many measurements of these identical systems, and finds the following results:

- If only the first spin is measured, the result is spin up with 30% probability.
- If only the second spin is measured, the result is spin up with 40% probability.
- If both spins are measured, the result is both spins up with 10% probability.

Derive the upper and lower limits on the probability that a measurement of the total spin of this state will yield zero.

7.
Consider a particle of mass m which at a particular instant of time, say $t = 0$, has the wave-function

$$\psi(x, y, z, t = 0) \ \propto \sqrt{\frac{2}{\pi a^3}} \left(\frac{x}{r} + \frac{z}{r}\right) e^{-r/a} + \sqrt{\frac{80}{3\pi a^3}} \frac{z^2}{r^2} e^{-2r/a}$$

where (x, y, z) are Cartesian coordinates of space and $r = \sqrt{x^2 + y^2 + z^2}$. If, at time zero, we simultaneously measure $\vec{L}^2$ and L_x, what is the probability that the result of the measurement is $\vec{L}^2 = 2\hbar^2$ and $L_x = 0$.

8. To reduce interactions between charges confined in an XY-plane at $z = 0$, we can sandwich the $z = 0$ plane with dielectric materials. Assume that the dielectric constant in the upper ($z > 0$) region is ϵ_1, and that it is ϵ_2 for $z < 0$. Find the reduced force between a charge Q at $(0, 0, 0)$ and a dipole moment, $p\hat{x}$, located at either point $(r, 0, 0)$ or $(0, r, 0)$, both in the $z = 0$ plane. Express your results for both point locations in terms of p, r, Q and the dielectric constants given above. *Hint: the electric field produced by the charge at the origin will be in the radial direction everywhere.*