Autumn 2022 Physics Qualifying Exam
for Advancement to Candidacy
Part 1

August 26, 2022
11:30-13:45 PDT

If you are in the PhD in astronomy or PhD in medical physics programs, stop! This is the physics version of the exam. Please ask the proctor for the version appropriate for your program instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2 hours and 15 minutes to complete 3 questions.

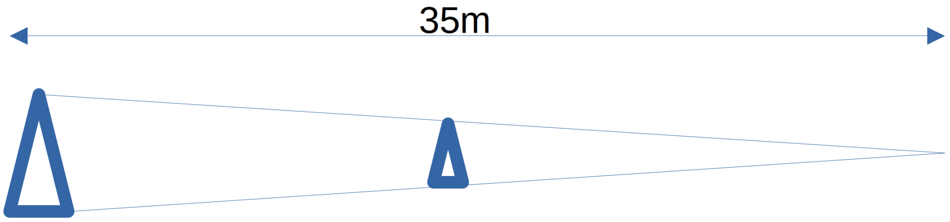
You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.661×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2\psi =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

1. A wind turbine has three blades, 35 m long, with a mass of 5200 kg each. The blades have a hollow cross section and taper linearly in width (see diagram).

When the wind is blowing steadily, the blades rotate with a period of 5 s. The wind suddenly stops, and the angular rotation is observed to decrease exponentially with a time constant of 3 s. Assuming that the turbine converts mechanical energy to electric energy with 80% efficiency, estimate the power output of the turbine during the period when the wind was blowing.



2. A closed sphere contains an ideal gas, which has a pressure of 10^5 Pa at a temperature of 300 K. At what temperature will the radiation pressure pushing outward on the walls of the sphere be equal to the pressure from the gas? Assume that the gas is in thermal equlibrium with the walls, and that the walls (inside and outside) emit as blackbodies.

3. A bead of mass m moves along a circular wire of radius R in zero gravity.

A. Treating the problem quantum mechanically, solve for the energy eigenstates of the system.

B. If the system is heated to some temperature T, calculate the expectation value of the magnitude (absolute value) of the system's angular momentum, $\langle |L| \rangle$, in the high temperature limit.

4. A 1 mm diameter ring is made from superconducting wire that itself is 0.1 mm diameter. The density of the wire is 10000 kg/m^3, and the ring carries a circulating current of 1 A.

Two 1 m×1 m metal plates are located at ±0.005 m on the z axis, parallel to the xy plane, and a voltage V is applied between them.

The ring is initally oriented with its normal vector $\hat{n} = \hat{z}$ parallel to the z axis, and is fired at velocity $\vec{v} = v\hat{x}$ along the x axis so it passes between the capacitor plates, entering at $\{x = -0.5\text{ m}, y = 0, z = 0\}$.

When $V = 0$, the ring leaves the capacitor in the same orientation as it began, and still at $y = 0, z = 0$.

When V is small but not 0, what are the y and z coordinates of the ring when it leaves the capacitor, and what is the $\hat{n}$? Because V is small, you should express your answer to first order in V. Your answer may be approximate, but you should state any approximations you make. You may ignore the influence of gravity entirely. *Hint: what is the B field in the frame of the ring?*

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Part 2
August 26, 2022
15:00-17:15 PDT

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5. A perfectly reflective mirror of mass M hangs from massless wires with length L. A laser with power P is directed at normal incidence onto one side of the mirror. The laser light is linearly polarized. A linear polarizer is placed between the laser and the mirror with its axis of polarization perpendicular to the beam, and rotates with angular frequency ω about the beam direction. Calculate the amplitude of the resulting oscillatory motion of the mirror.

6. A jigsaw puzzle with 1000 pieces has an area of 70 cm × 50 cm when assembled. The pieces of the puzzle are dumped out onto a table with dimensions of 1 m × 2 m, and spread out face-up in a single layer with no pieces overlapping. You then assemble the jigsaw puzzle on the table. Do an order of magnitude estimate of the change in entropy of the puzzle after it's assembled. Clearly explain what assumptions you make.

7. The surface of a perfectly conducting sphere of radius a carries surface charge density $\sigma(\theta) = \sigma_0 \cos\theta$ where σ_0 is a constant and θ is the polar angle (with respect to the z-axis). Now a small dipole with dipole moment $p\hat{z}$ is placed at distance R (much bigger than a) from the centre of the sphere in the $\theta = \pi/2$ plane, i.e. the plane of the equator. Find the force (direction and magnitude) between the charged sphere and the small dipole. How does the force change if the distance is increased from R to $2R$?

8. *Emancipation from a distance:* Consider a one-dimensional quantum mechanical system with an attractive short-ranged potential which we approximate by a delta function so that the initial potential energy is

$$U_0(x) = -\frac{\hbar^2\kappa}{m}\delta(x)$$

We assume that we set this system up so that one particle occupies the one and only bound state of this potential. This bound state has energy $E_0 = -\frac{\hbar^2\kappa^2}{2m}$. Consider the stationary state of this system where the particle occupies the bound state.

Then we modify the problem by placing an impenetrable wall at position $x = -L$ so that the resulting potential energy function is well approximated by

$$U(x) = \begin{cases} -\frac{\hbar^2\kappa}{m}\delta(x) & -L < x < \infty \\ \infty & -\infty < x < -L \end{cases}$$

with L initially very large. Indeed, if L is very large, the system should be undisturbed by the presence of the wall.

Now, let us imagine that we adiabatically decrease the distance L. We expect that the binding energy of the bound state begins to depend on L. Is there a value of L at which the binding energy goes to zero and the particle is liberated? If so, what is that value?