Astronomy Comprehensive Exam, Fall 2021
Session 1

September 1, 2021

Note: if you are in the PhD in physics program, stop! This is the astronomy version of the exam. Please download the physics version instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer ***any three*** of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2.25 hours to complete 3 questions.

You are allowed to use two $8.5'' \times 11''$ formula sheets (each written on both sides), and a handheld, non-graphing calculator.

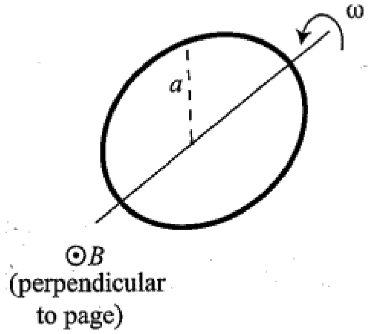
Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273.16°C
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	au	1.5×10^{11} m
luminosity of the sun	$L_\odot$	3.8×10^{26} W
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	$M_\odot$	2×10^{30} kg
molecular weight of H_2O		18
molecular weight of N_2		28
molecular weight of O_2		32
weight of Helium atom He		4
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
parsec	pc	3.086×10^{16} m
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the earth	$R_\oplus$	6.4×10^6 m
speed of the sun (galactocentric)	$v_\odot$	220 km s^{-1}
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Thomson cross section	σ_T	6.65×10^{-29} m^2

1. A conducting loop of radius a, resistance R, and moment of inertia I is rotating around an axis in the plane of the loop, initially at an angular frequency ω_0. A uniform static magnetic field B is applied perpendicular to the rotation axis (see figure).

   (a) Calculate the rate of kinetic energy dissipation, assuming it all goes into Joule heating of the loop resistance.

   (b) In the limit that the change in energy per cycle is small, derive the time dependence of the angular velocity ω. In particular, how long will it take for ω to fall to $1/e$ of its initial value? You may ignore any effects relating to self-inductance.



2. A flat plate with area A and negligible thickness sits inside an classical ideal gas with pressure P, temperature T, and molecular mass m. Calculate the rate at which molecules strike the plate. You may ignore edge effects. Do a full exact calculation, not just an order of magnitude estimate.

3. The rotation curves of spiral galaxies are flat as far as they can be measured, implying the existence of a dark matter halo. It is not known how far this dark matter halo extends. Imagine that the dark matter halo of the Milky Way extends half-way to the Andromeda Galaxy (which is itself 765 kpc away), so that the halos of the two galaxies just touch each other. We assume that the rotation curve remains flat to this point, so that the rotation speed at that radius is equal to that at our radius.

   (a) What is the total mass you infer for the Milky Way, in solar masses? What assumptions did you make?

   (b) The mean distance between massive galaxies in the universe is about 3 Mpc. Assuming that they all have the same mass that you just calculated above, and they represent all the mass of the universe, what is the mean density of the universe? What is the value of the corresponding cosmological density parameter Ω_m? Is your result consistent with what we know about Ω_m? If not, what could be wrong?

   (c) What volume of space at the density you calculated in part (b) would contain the mass of the Sun? Express your answer in terms of the side of a cube of this volume, in pc. How does your answer compare to the typical distance between stars in the solar neighbourhood?

4. Type Ia supernovae are believed to result from thermonuclear explosions of white dwarfs.

   (a) Calculate the nuclear energy released in a typical Type Ia supernova. The energy released from nuclear burning of carbon to the iron peak is about 0.001 m_pc^2 per baryon (proton or neutron), where m_p is the proton mass. You can assume the white dwarf is $1.3M_\odot$ of pure carbon and burns completely.

   (b) Calculate the gravitational binding energy of a $1.3M_\odot$ white dwarf. Assume that the radius of the white dwarf is 4000 km.

(c) Is there enough energy from nuclear burning to unbind the white dwarf? Can you think of any reasons why not all this energy might be available to disrupt the star?

(d) Estimate the typical velocity of the ejecta.

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1. Consider a particle of mass m trapped between two walls. We'll solve this problem in 1D, so the two walls effectively make an 1D infinite square well potential confining the particle. The distance between the walls is D.

   (a) What is the quantum mechanical energy of the particle in the n^{th} energy level?

   (b) Using your answer to Part A, what is the force on the walls when the particle is in the n^{th} level?

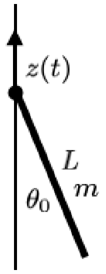
   (c) What is the average force on the walls due to a classical particle, whose kinetic energy matches your answer to Part A, bouncing back and forth between the walls? (Again assume this is a 1D problem.)

2. A comet is found in an orbit around the sun. When spotted, the comet is 10^{11} m from the sun, heading 20 degrees away from the direct path to the sun at a speed of 10^4 m s^{-1}. What is the distance of closest approach of the comet to the sun in its orbit, and what is the speed of the comet at that time?

3. A rod of length L, mass m and uniform cross-section and density, is attached at one end to a point about which it may pivot freely. It is taken to the International Space Station (a zero-gravity environment) and the pivot point is then moved in the z direction by a known function $z(t)$. The rod is initially at rest and makes an angle θ_0 with the z axis.

   (a) Find a differential equation for the angle $\theta(t)$ that the rod makes with the z axis, in terms of $z(t)$.

   (b) If the pivot point movies with constant acceleration, the pendulum may oscillate about an equilibrium position. For small oscillations, what is the frequency of oscillation?



4. Consider two black holes, of mass M_1 and M_2, which orbit each other, initially with negligible kinetic and gravitational potential energy. They eventually merge to produce a black hole of mass M_T. Some of the initial rest-mass energy of the black holes is converted to gravitational radiation which has total energy E. We also know from the work of Bekenstein and Hawking that the the entropy of a black hole is proportional to its surface area, which is proportional to the square of the mass.

   (a) Use conservation of energy and the second law of thermodynamics to derive Hawking's limit on the energy radiated in gravitational waves,

   $$E \leqslant M_1 + M_2 - \sqrt{M_1^2 + M_2^2},$$

   where we are using units in which the speed of light $c = 1$. (You can ignore the entropy of the gravitational radiation.)

   (b) If two black holes of equal mass merge, what is the upper limit on the energy in gravitational radiation, as a percentage of the initial mass-energy? Now suppose that a very large black hole accretes a small one, $M_2 \ll M_2$. What fraction of the initial mass-energy of the small black hole could be converted to gravitational radiation without violating Hawking's limit?

(c) Generalize Hawking's result to find a limit on the total energy radiated when N black holes merge to produce one large black hole.

(d) Thus show that if many stellar-mass black holes merge to produce a supermassive black hole, a large fraction of the total initial mass of the black holes could *in principle* be converted into gravitational radiation.