Astronomy Comprehensive Exam, Spring 2021
Session 2

April 29, 2021

Note: if you are in the PhD in physics program, stop! This is the asronomy version of the exam. Please download the physics version instead.

Do not write your name on your exam papers. Instead, write your student nnumber on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2.25 hours to complete 3 questions.

You are allowed to use two $8.5'' \times 11''$ formula sheets (each written on both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
molecular weight of N_2		28
molecular weight of O_2		32
weight of Helium atom He		4
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N} N^N \sqrt{2\pi N}$

1. Two rockets travel in open space along the x axis, far from any gravitational fields, with identical speeds and constant acceleration a. At $t = 0$, the first rocket is at $x = 0$ and the second rocket is ahead of it at $x = L$, as measured in the rest frame of the rockets. (You may assume that $aL \ll c^2$.)

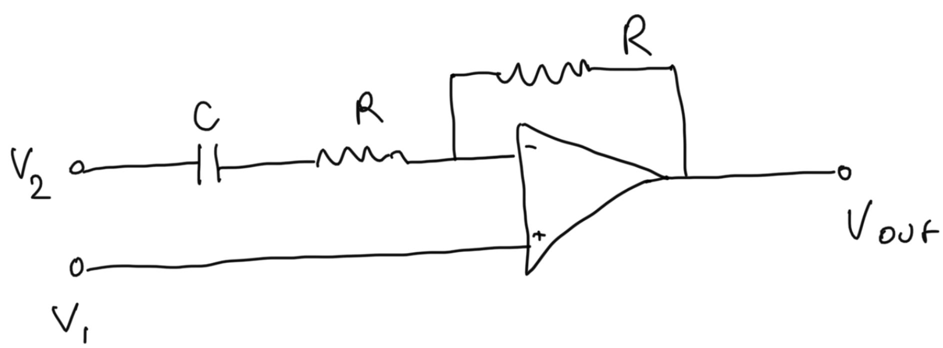
   (a) The first rocket fires a laser at the rocket in front of it. The laser's wavelength, measured in the first rocket's rest frame, is λ. What is the wavelength of the laser light that the second rocket detects, as measured in its rest frame?

   (b) Consider a coordinate system in which the rockets are moving at speed v at $t' = 0$, where the coordinate systems in the moving frame and the rockets' rest frame coincide at $t = t' = 0, x = x' = 0$. What is the distance between the two rockets as measured in the second (primed) coordinate system?

   (c) Einstein's equivalence principle says that a constant acceleration cannot be distinguished from a constant gravitational field. Apply this principle to your result from part A to calculate the gravitational redshift in wavelength of a 500 nm laser fired from the bottom of a 100 m tall tower as measured by a sensor at its summit.

2. Derive the relation between the scale height of stars in a thin galactic disk and their vertical velocity dispersion. You can assume that the potential is that of an infinitely thin disk, $\Phi(z) = 2\pi G\Sigma|z|$, where Φ is the gravitational potential, Σ is the mass surface density of the disk and z is the height above the disk. It will help to know that the Jeans equation in one dimension takes the form
$$\rho F_z = \frac{d}{dz}\rho\sigma^2.$$
where F_z is the vertical force and ρ is the mass density.

3. A standard operational amplifier (op-amp) is connected up with the following circuit. Recall that the voltage output of an op-amp is a huge factor (you can assume it to be infinity) times the voltage difference between the input terminals. The inputs of an op-amp present a very large (essentially infinite) impedance so that no current flows through them into the op-amp. Assume the op-amp is properly powered, with large positive and negative supply voltages. V_2 is held at +1 V with respect to ground, and a sine signal is applied to V_1 with respect to ground: $V_1 = 0.001V\sin(\omega t)$. What is V_{out} in the limit that $\omega \ll 1/RC$? What is it in the limit that $\omega \gg 1/RC$?



4. Three identical point masses m are constrained to move on a circle, as shown in the figure below. The masses are connected with identical springs each with spring constant k, that

obey Hooke's Law. There is no friction, gravity or motion outside the circle.

(a) Find all the natural or characteristic frequencies of oscillation for this system.

(b) Solve for the normal modes, labelling each with a Roman numeral (I, II, III, ...) then sketch or describe in words the motion for each mode, and list what its frequency will be.

(c) Suppose mass #1 is displaced slightly to the left (by dx) at $t = 0$, but none of the masses are moving and the other two are not displaced. At $t = 0$, mass #1 is released. Decompose the initial displacements at $t = 0$ into the normal modes you found in Part B, then solve for the positions of each mass for $t > 0$.