Astronomy Comprehensive Exam, Spring 2021
Session 1

April 29, 2021

Note: if you are in the PhD in physics program, stop! This is the astronomy version of the exam. Please download the physics version instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 2.25 hours to complete 3 questions.

You are allowed to use two $8.5'' \times 11''$ formula sheets (each written on both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
molecular weight of N_2		28
molecular weight of O_2		32
weight of Helium atom He		4
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^6 m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^8 m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N} N^N \sqrt{2\pi N}$

1. Einsteinium-253 (atomic mass 253.085 amu) decays to berkelium-249 (atomic mass 249.075 amu) and helium-4 (atomic mass 4.003 amu), with a half-life of 20.5 days. Its density is 8.8 g/cm^3. Consider a sphere of this isotope with a mass of 1 mg. Assuming that all of the heat produced by radioactive decay is radiated from the surface as black body radiation, calculate the surface temperature of the sphere. If the thermal conductivity of the material is 10 W/(m·K), calculate the temperature in the center of the sphere. (You may assume that all of the decay products stop inside the sphere, and that none escape.)

2. Consider a protoplanetary disc. Let R be the cylindrical distance from the star to a given point in the disc midplane and z the vertical distance from the midplane. At any given R, the vertical potential gradient is well described by $\Phi_z(R) = \Omega(R)^2 z^2/2$, where $\Omega(R)$ is the local angular orbital speed for the gas at R.

   Assume that the gas pressure is polytropic, where $P = K\rho^\gamma$, for gas density ρ and adiabatic index γ. The polytropic constant is given by K. Further let the midplane density at any given R be ρ_0.

   (a) With these assumptions in mind, what is the vertical density profile for the disc at any given R?

   (b) Such polytropic discs have a well-defined surface, which we will call H_s. What is H_s for any $\gamma > 1$? If you did not complete part (a), describe how you would approach the problem (partial credit).

   (c) Rewrite the the density profile from part (a) using the adiabatic sound speed in the disc midplane at the given R. If you did not complete part (a), write down the adiabatic sound speed for the midplane.

3. Assuming that the Sun is made of pure, fully ionized hydrogen gas and that only electron scattering plays a role in opacity: $\kappa = 0.04$ m^2 kg^{-1}, estimate the central pressure and temperature of the Sun and the luminosity that emerges. We are looking for orders of magnitude, so don't worry about π's and 2's.

4. Consider a flat radiation-only universe. This describes the early evolution of our own universe, $\Omega_{r0} = 1$ and $\Omega_{m0} = \Omega_{\Lambda 0} = \Omega_{k0} = 0$. Find the evolution of the scale factor with time and the comoving size of the past light cone (the comoving radius of the region at time t that can be seen by an observer at time t_0, where $t_0 > t$).