Autumn 2020 Physics & Astronomy Qualifying Exam
for Advancement to Candidacy
September 3, 2020
12:30-14:45 PDT

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer *any three* of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded.

You have 2.25 hours to complete 3 questions. **This is a closed book exam. During the exam, you shall not use Google or other online/off-line resources, other than your own formula sheet. At the end of the exam, please scan and upload your papers to the corresponding Assignment on Canvas. Your paper should be uploaded by 15:00 PDT. In case of emergency you can email your work to feizhou@phas.ubc.ca.**

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
Laplacian in spherical coordinates	$\nabla^2 f =$	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
molecular weight of N_2		28
molecular weight of O_2		32
weight of Helium atom He		4
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^{6} m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^{8} m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

Session One: 12:30 –14:45pm, Sept 3, 2020

1. Just as motion of charged particles can result in radiation, motion of mass can lead to gravitational radiation. The power L emitted in the form of gravitational radiation by a mass distribution depends on the third time derivative of the quadrupole moment of its mass distribution:

$$L = A \left\langle \sum_{j,k} \left(\frac{d^3 Q_{jk}}{dt^3} \right)^2 \right\rangle .$$

Here the sum is over elements of the quadrupole moment, the brackets indicate an average over many periods of rotation, and A is a proportionality constant that can be written in terms of fundamental constants.

A. Use dimensional analysis to produce an order of magnitude estimate of the value of the constant A.

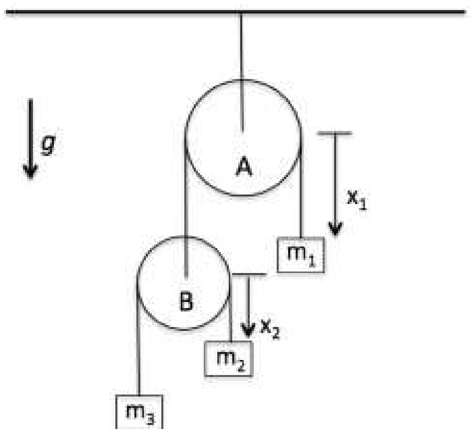
B. Consider a rotating neutron star with radius 10 km and rotational frequency of 100 Hz. If the neutron star is perfectly spherical then its quadrupole moment is constant and no radiation occurs. Suppose however that there is a bulge on its equator with a height of 10 cm, covering an area of 100 m $\times$ 100 m on the surface. Assuming that the material in the bulge has nuclear density, estimate the bulge's mass, and then do an order of magnitude estimate of the power radiated as gravitational waves by this star as it rotates.

---

2. An atom is trapped in the ground state of a 3D harmonic potential $V_c(\mathbf{r}) = \frac{1}{2} m\omega^2 r^2$ where ω is the trap frequency and m is the mass. In modern laboratories, such confining potentials can be easily induced by lasers.

(a) Now an additional potential is adiabatically turned on so that effectively $V_a(\mathbf{r}) = \infty$ for the half space of $z > 0$. How much work has been done on the *atom* during this adiabatic switching on?

(b) After keeping $V_{tot} = V_c + V_a$ for a while, one decides to suddenly switch off all the potentials to release the particle into free space. How much work has been done on the *atom* after the potentials are off?



3. Consider the rope and pulley system that is depicted in the figure. Assume that the pulleys and ropes are ideal, that is, in particular, they are weightless and frictionless and the ropes are infinitely flexible and do not stretch. Assume that at some time the system is released from rest, in the configuration shown. Find the relationship between the masses m_1, m_2, and m_3 so that, when the system is released, the mass m_1 starts moving downward. Does its acceleration increase, decrease or stay constant with time?

---

4. In answering the questions below, make reasonable estimates for any needed parameters/quantities not already provided. Consider an idealized Sun and Earth as blackbodies in otherwise empty space. The Sun has a surface temperature $T_S = 6000K$, and heat transfer processes on the Earth are effective enough to keep the Earth's surface temperature uniform. The radius of the Earth is $R_E = 6.4 \times 10^6$ m, the radius of the Sun is $R_S = 7.0 \times 10^8$ m, and the Earth-Sun distance is $d = 1.5 \times 10^{11}$ m. The mass of the Sun is $M_S = 2.0 \times 10^{30}$ kg.

(a) Find the temperature of the Earth.

(b) Find the radiation force on the Earth.

(c) Compare these results with those for an interplanetary granule in the form of a spherical, perfectly black body with a radius $R = 0.1$ cm, moving in a circular orbit around the Sun at a radius equal to the Earth-Sun distance d.

(d) For what size particle would the radiation force calculated in part (c) be equal to the gravitational force from the Sun at a distance d?

Astronomy Comprehensive Exam, Autumn 2020
Session 2

September 3, 2020 (15:45-18:00 PDT)

Note: if you are in the PhD in physics program, stop! This is the astronomy version of the exam. Please download the physics version instead.

Do not write your name on your exam papers. Instead, write your student number on each page. This will allow us to grade the exams anonymously. We'll match your name with your student number after we finish grading.

This portion of the exam has 4 questions. Answer any three of the four. Do not submit answers to more than 3 questions—if you do, only the first 3 of the questions you attempt will be graded.

You have 2.25 hours to complete 3 questions. **This is a closed book exam. During the exam, you shall not use Google or other online/off-line resources, other than your own formula sheet. At the end of the exam, please scan and upload your papers to the corresponding Assignment on Canvas. Your paper should be uploaded by 18:15 PDT. In case of emergency you can email your work to hickson@phas.ubc.ca.**

You are allowed to use one $8.5'' \times 11''$ formula sheet (both sides), and a handheld, non-graphing calculator.

Here is a possibly useful table of physical constants and formulas:

absolute zero	0 K	-273°C
atomic mass unit	1 amu	1.66×10^{-27} kg
Avogadro's constant	N_A	6.02×10^{23}
Boltzmann's constant	k_B	1.38×10^{-23} J/K
charge of an electron	e	1.6×10^{-19} C
distance from earth to sun	1 AU	1.5×10^{11} m
mass of an electron	m_e	0.511 MeV/c^2
mass of hydrogen atom	m_H	1.674×10^{-27} kg
mass of a neutron	m_n	1.675×10^{-27} kg
mass of a proton	m_p	1.673×10^{-27} kg
mass of the sun	M_{sun}	2×10^{30} kg
molecular weight of H_2O		18
molecular weight of N_2		28
molecular weight of O_2		32
weight of Helium atom He		4
Newton's gravitational constant	G	6.7×10^{-11} N m^2 kg^{-2}
nuclear magneton	μ_N	5×10^{-27} J/T
permittivity of free space	ϵ_0	8.9×10^{-12} C^2 N^{-1}/m^2
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Planck's constant	h	6.6×10^{-34} J·s
radius of the Earth	R_{earth}	6.4×10^{6} m
radius of a neutron	$R_{neutron}$	3×10^{-16} m
speed of light	c	3.0×10^{8} m/s
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Stirling's approximation	$N!$	$e^{-N}N^N\sqrt{2\pi N}$

1. A uniform spherical planet sees an instantaneous arriving solar energy flux $dE/dt = K/r^2$ where K is the proportional to the 'solar constant' ($\sim$ 1300 W m^{-2} for Earth), and r is the heliocentric distance. For a circular orbit, the total absorbed energy per m^{-2} is just $E = KP$ where P is the orbital period.

   Consider a hypothetical case in which the Earth is given an orbital eccentricity and different semi-major axis. Near perihelion there is more flux, but more time is spent near aphelion, and it is not obvious which effect dominates, making it unclear whether the planet receives a higher or lower energy input rate (again, time averaged over the orbit - this number is a key input for climate forcing).

   Determine the explicit dependence of the energy input rate (time averaged over the orbit) on the semi-major axis a and eccentricity e of the elliptical orbit. Recall that the rates of change of the true and eccentric anomaly are $df/dt = h/r^2$ and $dX/dt = an/r$, respectively, where $h = \sqrt{GMa(1-e^2)}$ is the specific angular momentum and n is the orbital mean motion. Conclude whether or not a planet on an eccentric orbit gets more or less flux per orbit than the circular case with the same semi-major axis a.

2. X-ray photons are produced in a cloud of radius R at the uniform rate Γ (photons per unit volume per unit time). the cloud is a distance d away. Assume that the cloud is optically thin. A detector at Earth has an angular acceptance beam of half-angle $\Delta\theta$ and an effective area A.

   (a) If the cloud is fully resolved by the detector, what is the observed intensity of the radiation as a function of position?

   (b) If the cloud is fully unresolved, what is the average intensity when the source is in the detector?

3. The neutron star that powers the Crab Pulsar can be assumed to have a mass of $1.4\mathrm{M}_\odot$ and a radius of 10 km with constant internal density and an effective temperature of 10^6 K. The frequency of the Crab Pulsar is 30 Hz and its period increases by 38 ns each day. Compare the power from the surface emission to the power lost as the neutron star spins down. The total power of the Crab Nebulae is about 75,000 times that of the Sun. What is the likely source of this power?

4. The cosmological scale factor $a(t)$ satisfies the equation

$$\frac{\dot{a}}{H_0} = \left\{ \frac{\Omega_r(t_0)}{a^2} + \frac{\Omega_m(t_0)}{a} + \Omega_\Lambda(t_0)a^2 + [1 - \Omega_0(t_0)] \right\}^{1/2}, \tag{1}$$

   where t_0 refers to the present moment.

   (a) To what does each of the four terms within the square brackets refer?

   (b) Obtain expressions for $a(t)$ for the four different cases that only one term is non-zero.