August 2008 Physics & Astronomy Qualifying Exam
for Advancement to Candidacy
Day 1: August 27, 2008

Do not write your name on the exam. Instead, sign up on the sign-up sheet, and note the serial number next to your name. Write that number on your exam in place of your name. This will allow us to grade the exams anonymously. We'll match your exam with your name using the sign-up sheet after we finish grading. If you use extra exam booklets, write your serial number on the extra exam books as well.

Today's portion of the exam has 8 questions. Answer *any five* of the eight. Do not submit answers to more than 5 questions—if you do, only the first 5 of the questions you attempt will be graded. If you attempt a question and then decide you don't want to it count, clearly cross it out and write "don't grade".

You have 4 hours to complete 5 questions.

You are allowed to use one $8'' \times 11''$ formula sheet (both sides), and a hand-held, non-graphing calculator.

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1. A very diffuse cloud of interstellar gas consists of molecules that decay with a lifetime of 1 s from an excited state to the molecular ground state, emitting a photon with wavelength λ=1000 nm in the process. The gas molecules have a temperature of 300,000 K, and a molecular weight of 26 amu. Estimate the width in nm of the resulting spectral line observed from this cloud.

2. Surface tension is the amount of energy per unit area needed to increase the surface area of a volume of liquid by a small amount. Water is poured onto a surface such as teflon that has zero adhesion to water. The water forms a thin puddle on the surface with thickness h. If the surface tension of the water is γ, derive an expression for h in terms of γ and any other relevant quantities. You may assume the diameter of the puddle is large compared to its thickness, but small compared to the width of the teflon surface it rests upon.

3. A binary asteroid was discovered in August 2001, consisting of two components both of apparent magnitude $m = 24.0$ separated by 4.0 arcseconds on the sky. The binary's center of mass orbits the Sun on an orbit with semimajor axis a =44.0 AU. Trackingfrom 2001-2008 has shown that the orbit between the asteroids is essentially circular, seen edge on, and the discovery occurred at maximum separation. The orbital period is 30 years.The Sun's apparent magnitude is $m = -27$. Stating any assumptions you make, provide estimates of the physical radii of each of the two components based first upon their observed brightness, and secondly upon their dynamical motion. (Hints: magnitude $m \equiv -2.5 \log_{10}$ Flux. 1 degree=3600 arcseconds. 1 AU= 1.5×10^{11} m.)

4. Optical tweezers have been used to control and manipulate atoms. For simplicity, we model a very small quantum tweezer as a structure having quantum levels with energies $E_n = n\epsilon$, where $n = 0, 1, 2...N$, and $N \gg 1$.

A) Assume that the atoms are *distinguishable* and that there are ten atoms in the tweezer. Calculate the partition function for this quantum tweezer as a function of temperature. At temperature T, what is the average occupation number of atoms in the ground state (i.e. $n = 0$ level) following this assumption?

B) Fermionic atoms are indistinguishable and obey the Pauli exclusion principle. (For Parts B-D of this problem you should assume that the atoms are all spin polarized—in other words, assume the fermions are all identical spin states.) Find out in which limit the results in part A are approximately correct for fermionic atoms.

C) Use the Pauli exclusion principle and derive the exact partition function for a quantum tweezer weakly connected to an atom reservoir (so that atoms can be exchanged) with the chemical potential set to μ.

D) By varying the chemical potential, one can control the number of atoms in the tweezer. For what value of μ can you be certain to load exactly 3 atoms into the tweezer in the low T limit?

5. A solar panel generates electricity by absorbing solar radiation. In this problem, you are asked to estimate the area of a solar panel needed to power an air conditioner in a house.

A) The Sun is a black body and its surface temperature is about 6000K. What is the radiation intensity (radiation power per unit area in units of W/m^2) on the surface of the Sun?

B) The earth is heated to its surface temperature mainly by the solar radiation. Estimate the solar radiation intensity on the earth using what you know about the earth and the Sun. There is more than one way to obtain the results. (Hint: For instance, assume the total incoming solar radiation intercepted by the earth is balanced by the total outgoing radiation of the earth, neglecting the greenhouse effects of the atmosphere.)

C) Assume that a solar panel is under direct sunshine for 12 hours a day in summers in southern California. To power an air conditioner 24 hours a day with 1kW power, how big should the solar panel be?

6. Photons in a very thin isolated optical fibre of length L propagate along the direction of the fibre with the free-space speed of light and live in an effectively one-dimensional space. Suppose that the fiber is not hooked up to any light source, and that it is cooled to a very low temperature $T = 200K$.

A) Estimate the typical wavelengths of photons in the fibre. What is the average energy density of photons in the fibre? Leave your answer as a constant times a definite integral over a dimensionless quantity (you don't have to and probably can't do the integral).

B) Comment on how your results in Part A depend on the temperature T. Can you explain why your results here are similar to or different from those of photons in a free space?

C) Now this isolated fibre is compressed adiabatically to half of its original length. Find the photon energy density and typical wavelengths of photons at the final stage of compression.

7. This problem will test your ability to do order-of-magnitude estimation.

A) The Great Library of Alexandria is estimated to have held around 1 million scrolls. A typical scroll was 0.3 m $\times$ 9 m when unrolled. How many CD-ROMs would it take to hold an electronic copy of the entire library? State clearly any assumptions you make

B) In his dying breath, Julius Caesar reportedly said, “Et tu, Brute?” How many air molecules from Caesar’s dying breath were contained in your last breath?

8. Consider a charged polymer consisting of N links and $N+1$ beads *in one dimension* (1-d) so links point only *right* or *left*. Each bead has position x_i and charge q_i, and the whole polymer is in solution at temperature T in an electric field of strength ϵ. The links have length d, and the 0^{th} bead is *tethered* at the origin so that $x_0 = 0$.

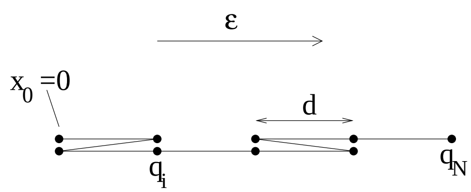

Figure 1:

A) Let all the charges be zero ($q_i = 0$) except for the N^{th} bead, $q_N = q$. Find the thermal average distance $\langle X_N \rangle$ of the N^{th} bead from the origin, as a function of the above variables.

B) Now suppose that every bead has a charge $q_i = q$. Again find the thermal average distance $\langle x_N \rangle$ from the origin of the N^{th} bead, as a function of the above variables. *Neglect* the interaction of the beads with each other (but not with the field!) and assume $N \gg 1$.

August 2008 Physics & Astronomy Qualifying Exam
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Day 2: August 28, 2008

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9. A satellite designer wants to design a weather satellite. The satellite will be placed in geosynchronous orbit and must be capable of resolving cloud features 100 m wide. The satellite will contain a spherical mirror with a camera mounted in its focal plane.

A) What is the minimum needed diameter of the mirror?

B) If the focal plane of the mirror must be 60 cm away from the mirror, what should the mirror's radius of curvature be?

C) Suppose a thin sheet of glass with index of refraction n and thickness t is placed between the mirror and the camera. Show that the focal plane shifts in position by a distance $t(n-1)/n$.

10. ^{57}Fe decays by emitting a single gamma-ray, liberating 14.4 keV of energy in the process. The natural linewidth of this transition is measured to be 10^{-8} eV (full width at half-maximum).

A) What is the lifetime for this transition?

B) Demonstrate that the gamma-ray emitted from an isolated Fe nucleus at rest cannot be absorbed by another isolated Fe nucleus that is also at rest (hint: consider recoil effects).

C) An Fe nucleus decaying in solid iron can undergo recoilless gamma-ray emission if the recoil momentum of the decay is transferred to the entire lattice as a whole. (This is the well-known Mössbauer effect.) This gamma-ray can then be reabsorbed by another crystal in a recoil-free absorption. In an experiment to measure the gravitational redshift predicted by general relativity, a ^{57}Fe crystal was placed at the top of a 100 m tower, and an absorbing crystal was placed at the bottom of the tower. With what velocity should the bottom crystal be moved in order to allow for recoil-free absorption of gamma-rays from the upper crystal? Assume g=9.8 m/s^2. Note that the gravitational redshift of a photon in the Earth's gravitational field is given by $\Delta\lambda/\lambda = GM/c^2r$, where G is Newton's constant, M is the mass of the Earth, and r is the distance of the photon from the centre of the Earth.

11. The nuclear shell model treats the nucleons in a nucleus as independent particles moving in a common 3D harmonic oscillator well, with a small spin-orbit coupling. The simplest shell model Hamiltonian sets the spin-orbit coupling to 0, and is written as:

$$H = \frac{p^2}{2m} + \frac{1}{2}mr^2\omega^2$$

where m is the nucleon mass ($mc^2 = 938$ MeV) and ω is a constant.

A) List the energy levels and orbital angular momentum ℓ values for all of the protons in ^{16}O (Z=8).

B) Suppose the RMS width of an ^{16}O nucleus in its ground state is $\sqrt{(x-\bar{x})^2} =$ 3.1 fm. To reproduce this radius, what value of ω should one use for the above shell model Hamiltonian? (Hint: use the virial theorem, which for the oscillator potential V states that $\langle V \rangle = \langle E \rangle / 2$).

Suppose that we now introduce a small spin-orbit coupling that couples the orbital angular momentum of each nucleon to its spin:

$$H = \frac{p^2}{2m} + \frac{1}{2m}r^2\omega^2 - B(\vec{\ell} \cdot \vec{s})$$

where B is a positive constant.

C) What is the total angular momentum j of the proton with the highest energy in the ^{16}O nucleus?

D) Suppose $B = 0.1$ MeV. How much energy is needed to excite ^{16}O from its ground state to its first excited state? Use the value of ω derived in part B.

12. A proton moving in the $-x$ direction encounters a region of space with a magnetic field that randomizes the direction of the particle. The magnetic field is at rest. After scattering off the magnetic fields the proton is ejected from the region and is moving in the $+x$ direction.

A) If the original energy of the proton is 10^{12} eV and the mean strength of the magnetic field is 1 mT, what is the final energy of the proton after scattering?

B) Now suppose that a 10^{12} eV proton moving in the $-x$ direction encounters an identical region of magnetic field that happens to be moving in the $+x$ direction at 10,000 km/s. After scattering the proton again is moving in the $+x$ direction. What is the proton's final energy?

C) Given that the size of the region with the magnetic field is 10^7 km across, provide an order of magnitude estimate of the highest energy proton that could be scattered backwards off this "magnetic mirror".

13.

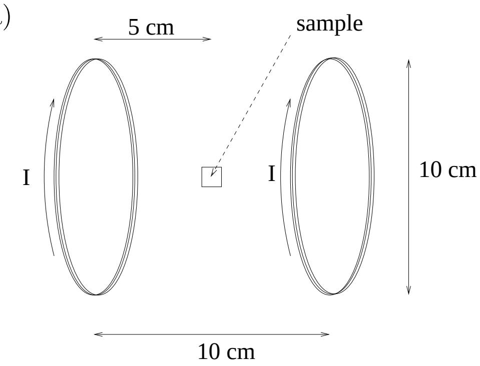

Figure 2:

Nuclear magnetic resonance (NMR) is a technique in which a nucleus can be resonantly excited between two different angular momentum states that are split in energy by an applied magnetic field. It can be used to measure the abundance of ^{17}O in water. ^{17}O is a spin-1/2 nucleus with a gyromagnetic ratio of $\gamma = -36.3$ MHz/T. (Note that the magnetic moment $\vec{\mu} = \gamma\vec{J}$, where $\vec{J}$ is the spin angular momentum.) A very small sample of water is placed inside a magnetic field produced by two coils of wire each carrying identical currents I, as shown. Each coil has 400 loops of wire and a diameter of 10 cm. The loops are placed 10 cm apart with the sample centered between them. A 1 MHz continuous RF signal is applied to the sample, and the magnetic field strength is increased from zero until a resonance from the ^{17}O is seen.

A) What is the magnetic field strength at which the resonance occurs?

B) How much current flows in the two magnetic field coils at this point?

C) Can NMR be used to measure the abundance of ^{16}O? Why or why not?

14. An observation is made of a distant galaxy as follows: First, a small area of sky is observed which contains the object of interest. Then, an equal area of sky, in nearly the same direction but not containing the galaxy (or any other object), is observed for an equal length of time. The signals from these two observations are then subtracted to determine the flux from the source alone.

A) If n_1 photons are detected in the first observation and n_2 in the second, estimate the signal-to-noise ratio that will be obtained for the flux.

B) The quantum efficiency of a photon detector is the efficiency with which it detects individual photons. Due to instrumental noise in the system, a noisy detector with quantum efficiency Q may have a signal-to-noise ratio equivalent to that of a noiseless detector with quantum efficiency Q'. Suppose that a detector with quantum efficiency Q adds noise which has a root-mean-squared (RMS) value equal to the signal produced by r photons. Derive an expression for the effective quantum efficiency Q'. Show that in the limit when the detector noise can be neglected that $Q' = Q$.

C) When photomultipliers are used to count photons, there is a brief period of time, immediately after a photon is detected, when the photomultiplier, amplifier, counter, etc, is unable to respond to any other photons. This is called the dead time of the system. Therefore, the measured count rate will be smaller than the true rate (the rate that would be measured if there was no dead time), because some photons will arrive during the dead time and will be missed. Because the dead time is usually a constant, independent of the count rate, a larger fraction of photons will be missed if the count rate is high. Derive an expression relating the observed count rate R, the true rate R_0, and the dead time t_d.

15. Interference of matter waves has been studied using ultra-cold atoms. The phase of a matter wave for free-falling cold atoms at time t and height y is given as

$$\phi(y,t) = \phi(y_0,t) + Et - \int_{y_0}^{y} dy'\ k(y')$$

where E is the total energy of individual coherent atoms and $\hbar k(y)$ is the semi-classical momentum of falling atoms at height y. A recent experiment observed coherent matter waves continuously emitted from two vertical micro-traps (you can neglect the size of the traps) with height $-\lambda/2$ and $\lambda/2$. The atoms are emitted at rest from the traps.

A) For a given height y (which is below the two micro-traps), derive the conditions of constructive and destructive interference.

B) At a given t, describe quantitatively the spatial interference pattern. Is the interference pattern uniform in the y-direction?

C) Summarize your results in A and B and describe the spatial-temporal interference using qualitative plots.

16. A rocket in empty space (no gravitational field) must accelerate a 100 kg payload from rest to a velocity of 6000 m/s. Assume that the rocket's exhaust is ejected with a velocity of 1500 m/s, as measured in the rest frame of the rocket. An empty rocket stage with mass M (without payload) can carry 10 times its weight in fuel.

A) By calculating the maximum velocity of the rocket as a function of its empty mass M, show that no single-stage rocket could ever accelerate this payload to 6000 m/s.

B) Consider a two-stage rocket. In this rocket, the first stage has mass M_1 and carries $10M_1$ kg of fuel. After it burns and becomes empty, the stage is detached. The second stage, with mass M_2, containing $10M_2$ kg of its own fuel plus the payload, is then ignited. Show that a two-stage rocket can reach the desired velocity, and find the optimial values of M_1 and M_2 that minimize the sum of the two rockets' weight.