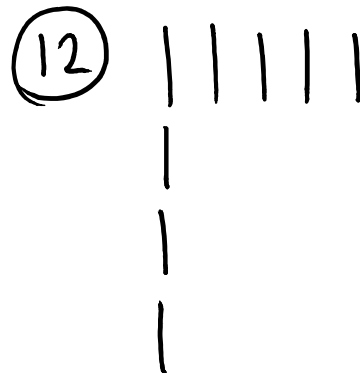
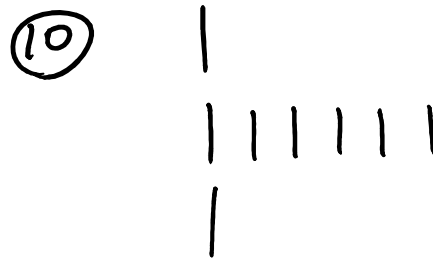
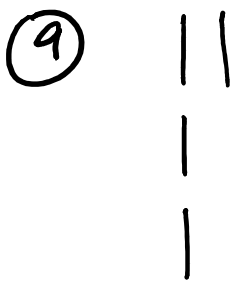
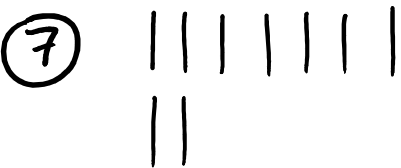
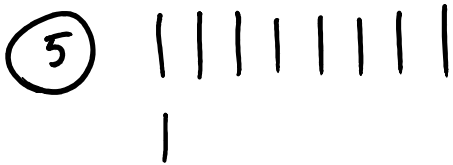
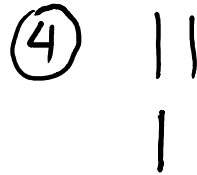
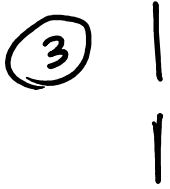


For each of the following examples, what is the best move to make if it is your turn? If you are going to lose for sure, write ☹️



How to use math to win at Nim

The sequence of numbers 1, 2, 4, 8, 16, 32, ... are called POWERS OF 2. We get each power of 2 by starting with 1 and then doubling it a certain number of times

$$1 = 1 \quad \text{we call this "2 to the power 0"}$$

$$2 = 1 \times 2 \quad \text{this is "2 to the power 1"}$$

$$4 = 1 \times 2 \times 2 \quad \text{this is "2 to the power 2"}$$

$$8 = 1 \times 2 \times 2 \times 2 \quad \text{"2 to the power 3"}$$

$$16 = 1 \times 2 \times 2 \times 2 \times 2 \quad \text{"2 to the power 4"}$$

Another way to think of it is that powers of two (apart from 1) are just some number of 2s multiplied together.

example: $16 = 2 \text{ to the power } 4 = 2 \times 2 \times 2 \times 2$

We can also have powers of other numbers,

example: $3 \text{ to the power } 4 = 3 \times 3 \times 3 \times 3 = 81$

A cool math fact: every whole number can be written as a sum of powers of two with no power of two repeated.

examples: $3 = 2 + 1$ $9 = 8 + 1$ $7 = 4 + 2 + 1$

Try it! How can you write the following numbers as a sum of powers of 2?

$$6 = \qquad 13 =$$

$$20 = \qquad 33 =$$

(hint: always start with the largest power of 2 possible)

How to win at Nim: write the number of objects in each row as a sum of powers of 2. On your turn, try to make it so that every power of 2 appears an even number of times. Keep doing this until you see a way to win.

example:

$$\begin{array}{l} | \quad \rightarrow 1 \\ ||| \quad \rightarrow 1 + 2 \\ |||| \quad \rightarrow 4 \\ ||||| \quad \rightarrow 4 + 2 + 1 \end{array}$$

Here 1 appears 3 times, so we remove 1 object from row 1, 2, or 4. After that