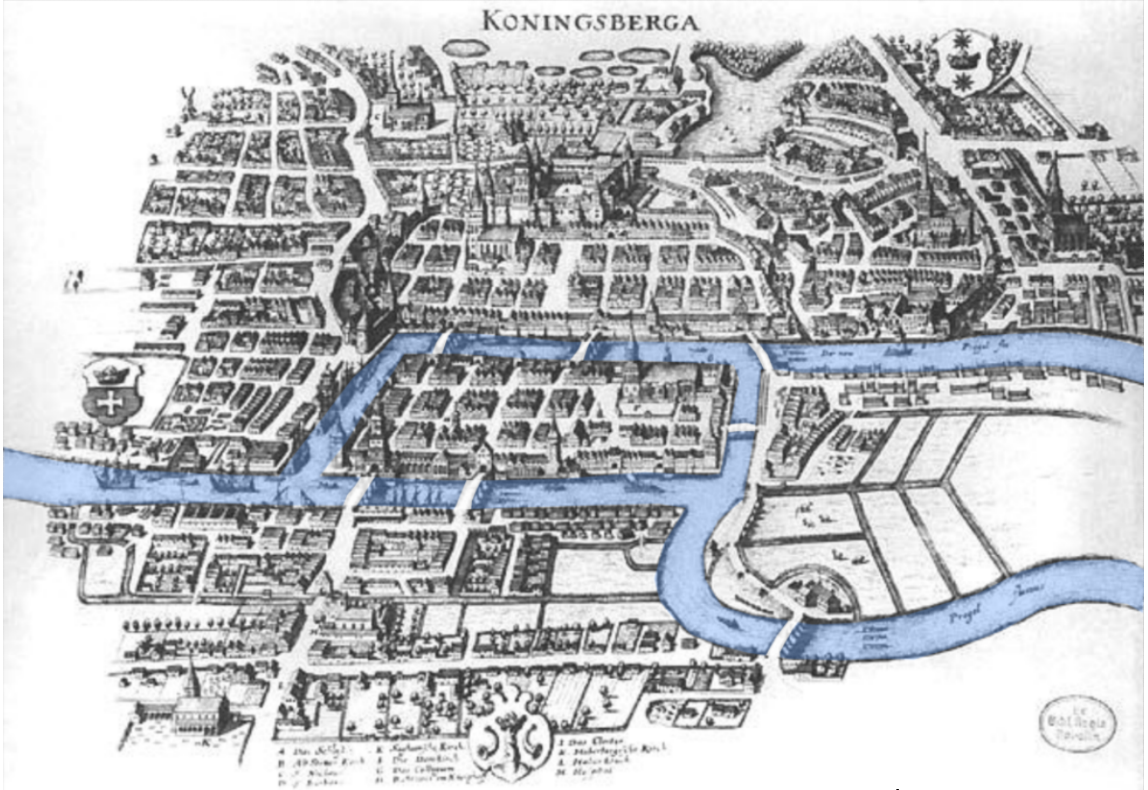
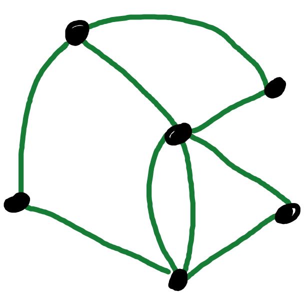
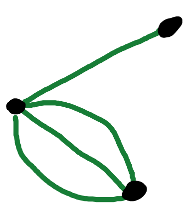
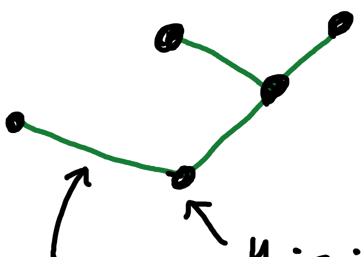


Can you cross every bridge in this city without crossing any one twice? This problem was given to a famous mathematician named Leonhard Euler. Today we'll understand how he solved it with a tool called GRAPH THEORY.





These pictures are called GRAPHs. Can you think of a way to draw a graph to represent how the various parts of the city are connected by brides?

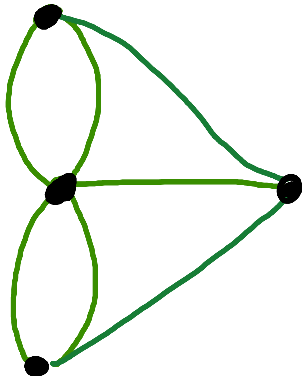


This is an EDGE

this is a VERTEX

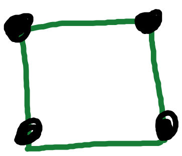
the plural of vertex is "VERTICES"

We can relate Euler's problem to a problem about graphs:



Can you start on some vertex and trace all the edges with your pencil without lifting it or going over any edge twice? If you can, the path you took is called an EULER PATH

Which of these graphs has an Euler path?

1


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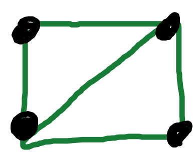

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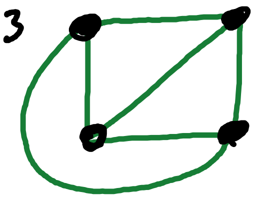

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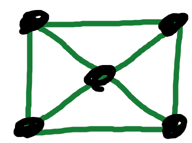

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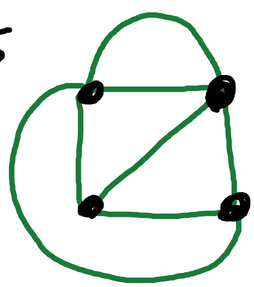

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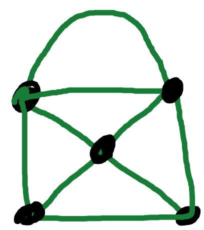

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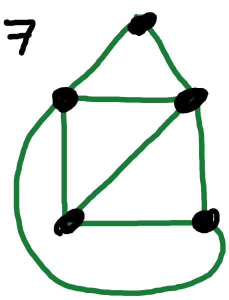

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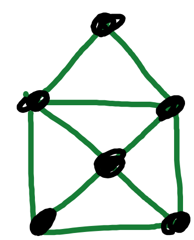
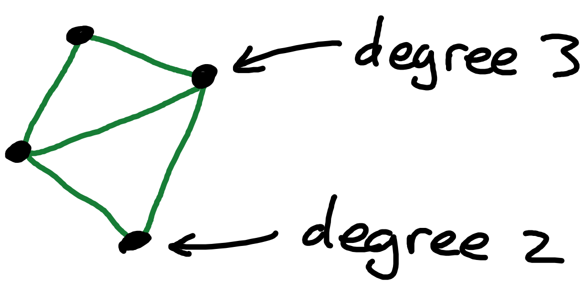
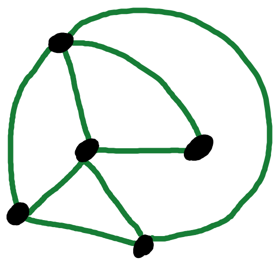


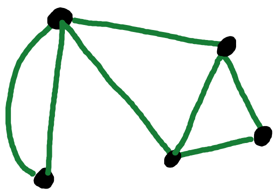
Euler figured out a way to tell just by looking at the graph. The secret is to count how many edges go into each vertex. This is called the DEGREE of the vertex.





For this graph, write a number beside each vertex to show its degree.

If it's possible to draw an Euler path starting from some vertex, circle that vertex.



Do the same thing for the second diagram.

What are all the degrees for the vertices that are not circled?

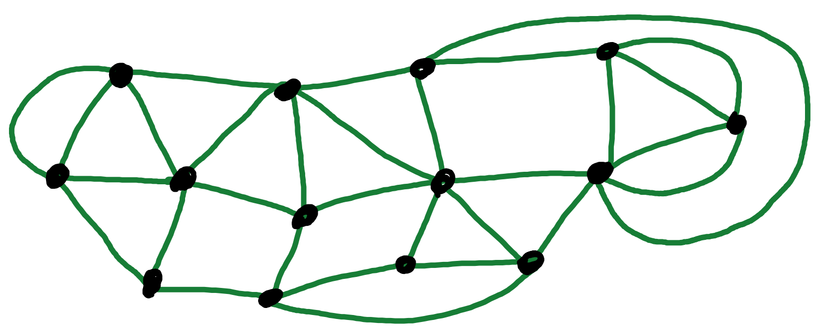
What are all the degrees for the vertices that are circled?

Do you notice anything here? Can you explain it?

See if you can use your observations to figure out how to tell if there is an Euler path just by looking at the degrees of the vertices.

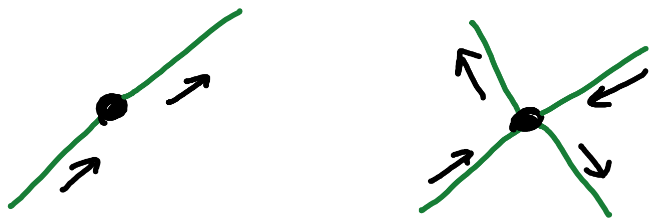
Use your method to decide if the original bridge problem is solvable.

Does the graph below have an Euler path?



Euler's solution:

Euler realized that in a graph with an Euler path, each vertex which is not the start or the finish must have an even degree, since every time you go to that vertex on an edge, you leave it on some other edge.



Since there are at most two vertices where we start or finish, there can be at most two vertices with odd degree in a graph with an Euler path.

So any graph with more than two vertices of odd degree cannot have an Euler path.

In some graphs, there is an Euler path that starts and ends on the same vertex. Can you argue that in these graphs, all the vertices have even degree?