

In the country of Binland, the money comes in bills of \$1, \$2, \$4, \$8, \$16, and \$32.

① If you only have one bill of each kind, show how you can pay each dollar amount below by writing how many of each bill you would use:

Amount	\$8	\$4	\$2	\$1
\$0				
\$1				
\$2				
\$3	0	0	1	1
\$4				
\$5				
\$6				
\$7				
\$8				
\$9				
\$10				
\$11				
\$12				
\$13				
\$14				
\$15				

example

$$\begin{aligned}
 \$3 &= 0 \times \$8 \\
 &+ 0 \times \$4 \\
 &+ 1 \times \$2 \\
 &+ 1 \times \$1
 \end{aligned}$$

The numbers in the table are like a code that we can use to represent any number using 1's and 0's. For example, the code for 3 is 0011, or just 11. This is called the BINARY code.

② What is the code for the following numbers:
(if the code starts with some 0's, you don't need to write them)

a) 4 :

b) 9 :

c) 16 :

d) 61 :

③ What numbers do these binary codes represent?

a) 101

b) 1100

c) 10101

d) 100000

④ Adding in binary! For each pair of binary numbers, write their sum in binary:

a) $1 + 1 =$

b) $10 + 101 =$

c) $11 + 110 =$

d) $1 + 11111 =$

SUPER-CHALLENGE QUESTIONS:

① What is the binary code for the number one million?

② How many different numbers can be represented using binary numbers with up to 10 digits (e.g. 1011000101)

③ Compute the following sum in binary:

$$\begin{array}{r} 11011011011 \\ + 1001001001 \\ \hline \end{array}$$

Hint: $1+1=10$ in binary so you need to write 0 and carry the 1

④ Compute the following product in binary:

$$\begin{array}{r} 10101 \\ \times 11 \\ \hline \end{array}$$

Does it work to use the regular multiplication method?

BASES for number systems.

Our usual numbers are DECIMAL or BASE 10.

This means that each digit in a number counts for 10 times more than the digit to the right:

4 3 2
↑ ↑ ↑
100s 10s 1s

To write all possible numbers, we need 10 different symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

BINARY is the number system using BASE 2.

Here each digit counts for 2 times more than the digit to the right:

1 0 1
↑ ↑ ↑
4s 2s 1s

We need only 2 symbols to write numbers in binary.

We can also write numbers in any other base. For example 214 in base 8 means

$$2 \times 64 + 1 \times 8 + 4 \times 1.$$