Impulse and momentum:

Another consequence of Newton's laws.

$\Sigma F = ma = \frac{d}{dt}(mv)$ write $\vec{G} = m\vec{v}$ = linear momentum

(measured in N×sec).

So ΣF can be seen to change the momentum with time. Integrate with time: (similar to previously integrating over position)

$$\int_{t_1}^{t_2} (\Sigma \vec{F})\, dt = \Delta \vec{G}$$

$\int_{t_1}^{t_2} \vec{F}\, dt$ is defined to be the linear impulse of the force.

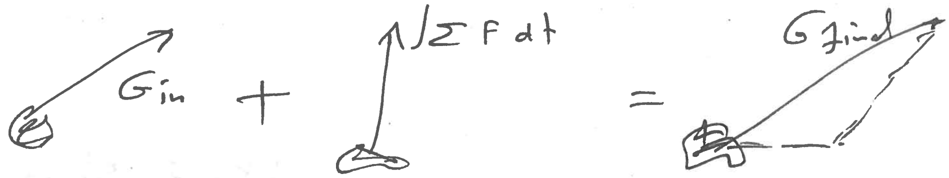
The sum over all forces gives the change in momentum.

(useful when we know $\vec{F}(t)$ instead of $\vec{F}(x)$, and it acts for a short duration)

Impulse - momentum diagram:

Shows graphically, for the body involved the equation

$$G_{initial} + \text{Impulse} = G_{final}.$$



Impulsive forces: bursts of force during short time (e.g. tennis racket). The force is so large anything else can be neglected in the short time it acts. Many times it can be treated as constant acting for a specified duration.

Conservation of momentum: if no force acts, the G is conserved (same applies to any component of G if the force in that direction is zero)

If two particles act on each other

$$\vec{F}_{ab} = -\vec{F}_{ba}$$

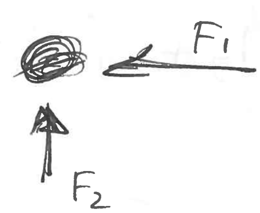
so $$\Delta \vec{G}_a = -\Delta \vec{G}_b$$

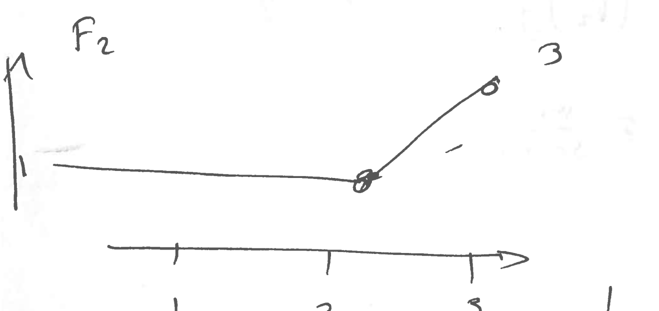
if these are the only unbalanced forces, so

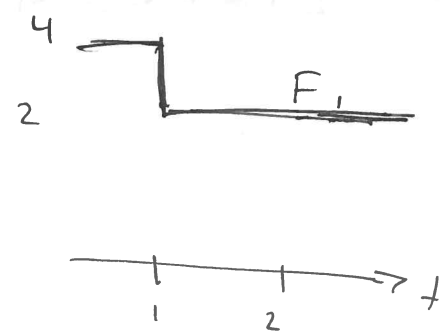
$$\Delta G_a + \Delta G_b = 0 \longrightarrow$$ conservation of momentum.

Sample problems: (3.21).

particle with mass 500 gram has velocity $10 \frac{m}{sec}$ in the X direction at $t=0$. The forces $\vec{F}_1, \vec{F}_2$ act on the particle, and their magnitude changes with time, as drawn

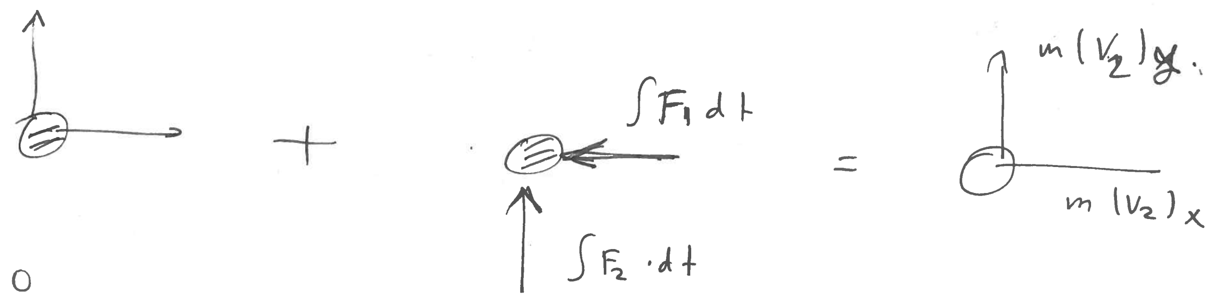






Determine the velocity V_2 at the end of the interval.

Impulse - momentum diagram:



$m(V_1)_y = 0$

$m(V_1)_x = 0.5 \cdot 10 \ \ kg \cdot \frac{m}{s}$

$$m(V_1)_x + \int (\Sigma F_x)\,dt = m(V_2)_x.$$

$$0.5 \cdot 10 - [4 \cdot 1 + 2(3-1)] = 0.5\,(V_2)_x$$

$$(V_2)_x = -6 \ \frac{m}{sec}.$$

$$m(V_1)_y + \int \Sigma F_y\,dt = m(V_2)_y.$$

$$0.5 \cdot 0 + [1 \cdot 2 + 2(3-2)] = 0.5 \cdot (V_2)_y. \quad \checkmark.$$

$$(V_2)_y = 8 \ \frac{m}{sec}.$$

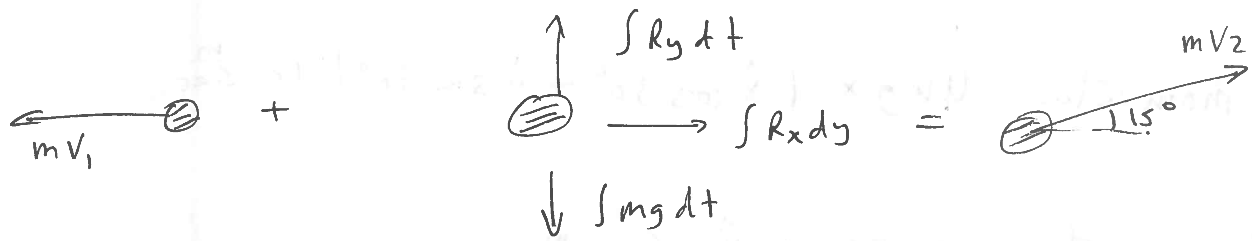
(Find force from velocity difference)

Sample problem 3-19 ; before & after impact by tennis rachet.

$V_1 = 50 \ \frac{m}{sec}$ $\quad\quad$ $V_2 = 70 \ \frac{m}{sec}$ at $15°$ angle, at the opposite direction.

$m = 200g = 0.2 \ kg.$

The duration it force is 0.02 sec, determine the average force during that Time.



$$m(V_x)_1 + \Sigma \int F_x \, dt = m(V_x)_2$$

$$-mV_1 + \overline{R_x}\,(0.02) = mV_2 \cos 15°$$

$$R_x = 1176 \text{ Newton.}$$

$$m(V_y)_1 + \Sigma \int F_y \, dt = m(V_y)_2$$

$$\overline{R_y}\,(0.02) - mg(0.02) = mV_2 \sin 15°$$

$$\overline{R_y} = 183.13 \text{ Newton.}$$

Sample problem 3.23

50 gram bullet at 600 $\frac{m}{sec}$ strikes 4kg block and is embedded in it. Determine the velocity of the block just after impact, if it was moving at 12 $\frac{m}{sec}$, at angle 30°, prior to impact.

Momentum is conserved, before the impact

block momentum $4kg \times (\hat{x}\cos 30^\circ + \hat{y}\sin 30^\circ)\cdot 12\ \frac{m}{sec}$

bullet moment $0.05kg \times \hat{y}\cdot 600\ \frac{m}{sec}$

Total momentum $41.6\,\hat{x} + 54\,\hat{y} = 4.05\,\vec{v}$

$$\vec{v} = 10.27\,\hat{x} + 13.33\,\hat{y}\ .$$

Angular impulse & momentum

Take particle of posit $\vec{r}$ and velocity $\vec{v}$, define the angular momentum (around the origin O) as

$$\vec{H}_O = m\,\vec{r} \times \vec{v}$$

$\vec{H}$ is perpendicular to both $\vec{r}$ & $\vec{v}$, and points in direction determined by the right hand rule.

In rectangular coordinates, the components of $\vec{H}$ are given by

$$\vec{H} = m \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix} = m\,(v_z y - v_y z)\,\hat{x} + \ldots$$

This has units of Newton × m × sec $= kg \cdot \frac{m^2}{sec}$

$\vec{H}$ is similar to ~~[illegible]~~ momentum in that it is conserved in the absence of forces, and is affected by angular impulse, as shown below

Rate of change:

$$H = \vec{r} \times m\vec{v}$$

$$\frac{dH}{dt} = \underset{\overset{\|}{0}}{\vec{v} \times m\vec{v}} + \vec{r} \times m\vec{a} = \underbrace{\vec{r} \times \Sigma \vec{F}}_{\Sigma M_O}$$

$\Sigma M_O \rightarrow$ sum over all moments (around the point O)

$$\boxed{\frac{dH}{dt} = \vec{r} \times \vec{M}}$$

omitting the Σ and reference to O

The most useful tool to describe angular motion (e.g rotation).

Angular impulse-momentum relation

Integrate the above relation over time, get.

$$\underbrace{\int_{t_1}^{t_2} (\Sigma M)\, dt}_{\text{angular impulse}} = \underbrace{\Delta H}_{\text{change in angular momentum}} \quad \text{or} \quad H_2 = H_1 + \int_{t_1}^{t_2} \Sigma M\, dt$$

As for the linear impulse-momentum, this is a vector equation, which has 3 components.

Motion in the plane:

If the particle is restricted to a plane, the direction of the angular momentum is fixed in the $\hat{z}$ direction: the direction perp. to the plane. In this case the above equations have only one non-trivial component.

The angular momentum is $m|\vec{r}\times\vec{v}|$ and is pointing in $\pm\hat{z}$ direction, it's magnitude is $|\vec{r}|\cdot|\vec{v}|$ (with the chosen origin), since $\vec{r},\vec{v}$ always perp. For the angular impulse $|M| = |F|\,|r| \sin\theta$

Conservation in collisions

If no moment (around some O) → angular momentum is conserved. In collision, because of action reaction

$\Delta H_a + \Delta H_b = 0$, as before

Sample problem 3.24

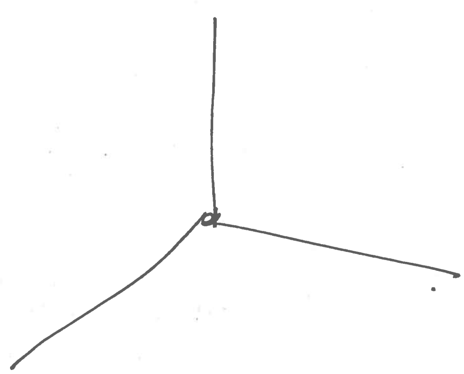
Calculate angular momentum & its change.

$m = 2kg.$

location $(x, y, z) = (3, 6, 4)\ m$

$F = 10N$ in $\hat{z}$ direction

$v = 5\ m/s$ in $\hat{y}$ direction



$$\vec{H} = \vec{r} \times m\vec{v} = (3\hat{x} + 6\hat{y} + 4\hat{z}) \times 2 \cdot (5\hat{z}) =$$

$$-40x + 30y \quad N \cdot \frac{m}{sec}$$

$$\frac{dH}{dt} = \vec{r} \times \vec{F} = (3\hat{x} + 6\hat{y} + 4\hat{z}) \times 10\,\hat{z}$$

$$= 60\,\hat{x} - 30\,\hat{y} \quad N \cdot m.$$

Sample problem 3.25.

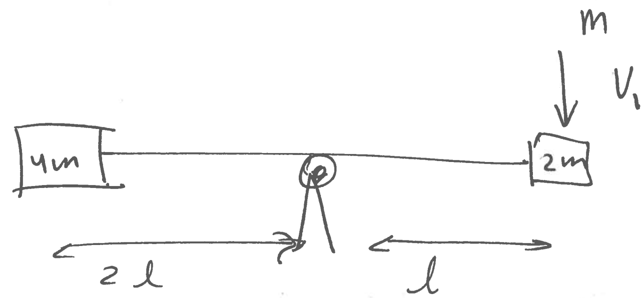
In central forces $\vec{F}, \vec{r}$ parallel, so $\vec{F} \times \vec{v} = 0$, and angular momentum is conserved. The comit has $v_A = 740 \frac{m}{sec}$ at outer edge of their trajectory ($6000 \cdot 10^9$ km), the closest approach has $75 \cdot 10^6$ km. What is the velocity then?

$$m r_A v_A = m r_B v_B = 0$$

$$v_B = \frac{r_A}{r_B} \cdot v_A = 59{,}200 \frac{m}{sec}.$$

Sample problem 3.26

The assembley is balanced, and is struck by wad of putty, embedded in the rhs. Find the angular velocity $\dot{\theta}$ just after impact.



Angular momentum is conserved. $\langle H \rangle_1 = \langle H_2 \rangle$ with the obvious origin.

Before impact $\quad H = (m V_1) \cdot l$

After impact $\quad H = 3m \cdot (l \dot{\theta}) \cdot l + 4m (2l \dot{\theta}) \cdot 2l$

$$\boxed{\dot{\theta} = \frac{V_1}{19 l}}$$