Chapter 3, part A

When there's an unbalanced force $\Sigma \vec{F}$ acting on an object, Newton's second law related it to it's acceleration

$$\Sigma \vec{F} = m\vec{a}$$

This can be used to find $\vec{a}$ from $\Sigma \vec{F}$, or vice-versa.

The second law holds if the measurements are made by an inertial observer: either not moving, or moving in constant velocity.

The mass is a property of the moving object, expressing it's inertia. Note that it is different from weight. This is somewhat confused in non-SI units: lb. is a unit of weight, it includes a factor of g in it's definition. kg, the SI unit, is a unit of mass.

In SI units force is measured in $\frac{kg \cdot m}{sec^2}$, which is also called Newton (N).

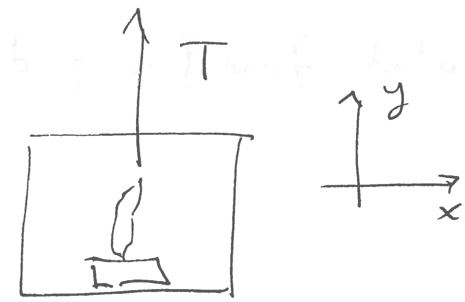
<u>Solving problems:</u>

The first part is identifying which object we are interested in, and all the forces acting on it. To do that it is useful to draw <u>free-body</u> diagram where the object is isolated, and all other objects are replaced by the forces they exert. Examples to follow.

Other crucial points:

* Implement all constraints
* Choose coordinates wisely, also make sure you use their direction consistently.
* Plan of attack first!

Sample problem 3.1



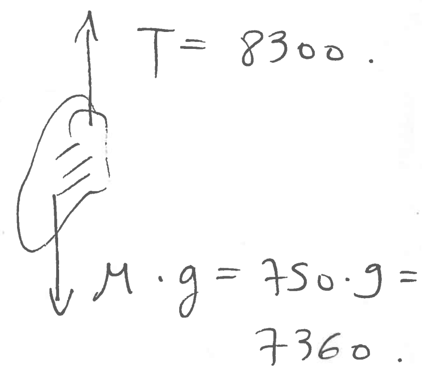
Man in an elevator standing on scale.

Tensin in cable is $T = 8300\ N$, total

mass of elevator + man + scale = 750 kg; man mass is 75 kg.

What is the reading on scale?

First, free body diagram of elevator.



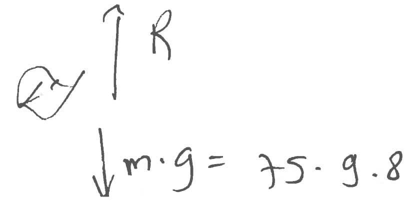
$$\Sigma F_y = 8300 - 7360 = 940 \text{ Newton} = M \cdot a_y$$

$$940 = 750 \cdot a_y \longrightarrow a_y = 1.257 \ \frac{m}{sec^2}$$

Newton's 2nd law gives acceleration from forces.

Second part: free body diagram of the man. The man applies force R on the scale (what we want to know), and the scale applies the force back (action & reaction!)

So on the man



Total force (in y-direction) on the man is

$$R - mg$$

which equals $m \cdot a_y$. We already know a_y, so

$$R - 75 \cdot g = 75 \cdot 1.257$$

$$\rightarrow R = \underline{830\,N}.$$

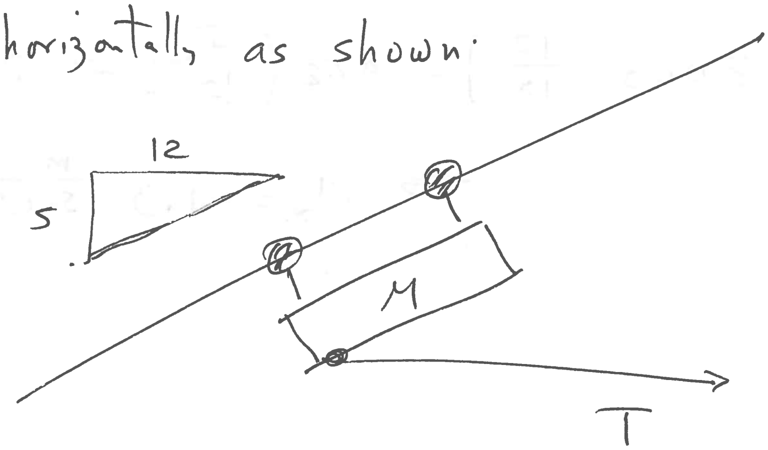
(man weighs more if elevator accelerates upwards).

Sample problem 3.2:

Cable car is pulled horizontally as shown:

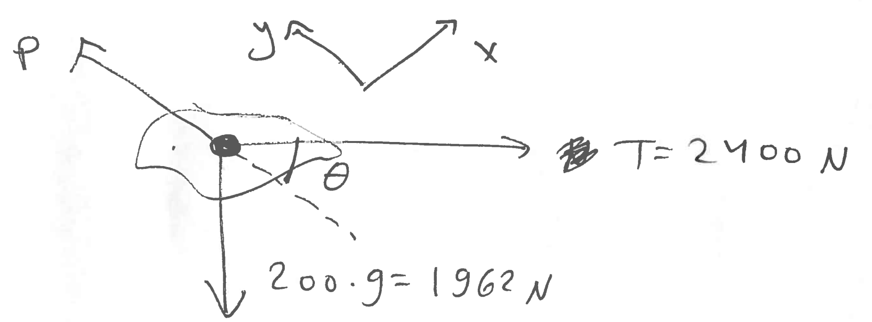
$M = 200\,kg$

$T = 2.4 \cdot 10^3 N$.



Find acceleration and force on supporting cable.

Free body diagram



Choose (x,y) as shown, because we know there's no motion in y direction. The total force in y-direction.

$$\Sigma F_y = P - 2400\left(\frac{5}{13}\right) - 1962\left(\frac{12}{13}\right) = 0 \rightarrow \boxed{P = 2730\,N}$$

cos(θ)

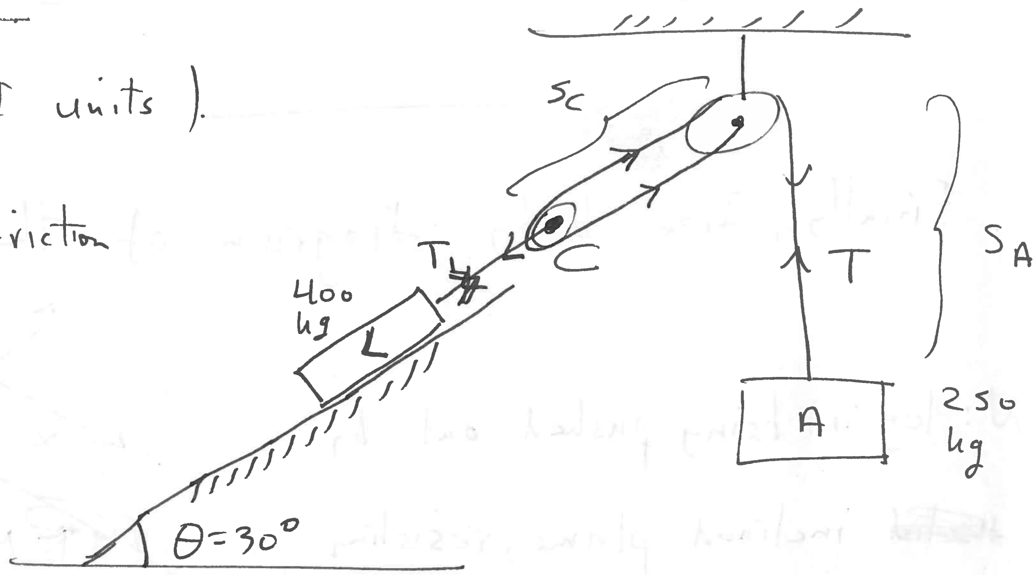
sin(θ)

Then in the x-directin

$$\Sigma F_x = 2000\left(\frac{12}{13}\right) - 1962\left(\frac{5}{13}\right) = 200 \cdot a_x$$

$$\rightarrow a_x = 7.3 \ \frac{m}{sec^2}$$

Sample problem 3.3

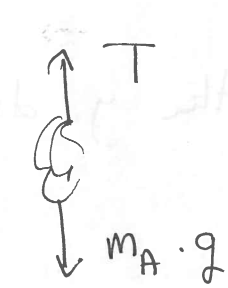


(Variation done in SI units).

Coefficient of kinetic friction is 0.5, all other information is drawn.

Find the acceleration of A and the log, and the tension T.

Start from right to Left: Free body diagram of A:



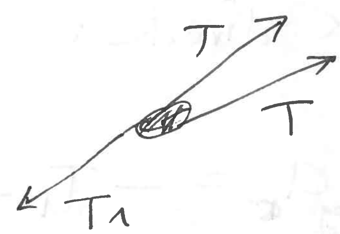
S_A increases as A goes downwards, so

$$\boxed{250 \cdot g - T = 250 \cdot a_A}$$

one relation between our unknowns.

Next: the pulley C is fixed, so

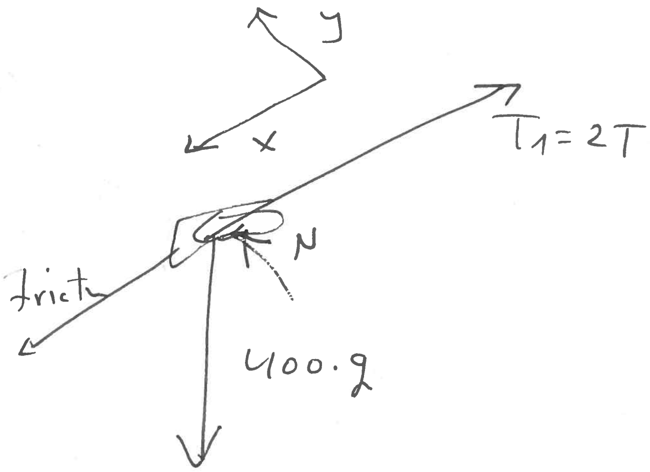
$$T_1 = 2T$$



Also the length of the cable is fixed $\rightarrow$ $S_A + 2 \cdot S_C = \text{Const.}$

$\Longrightarrow \quad a_A + 2a_C = 0$; note also $a_C = a_{log}$

Finally, free body diagram of the log.



N: log is being pushed out by ~~the~~ inclined plane, resisting the gravitational force.

By definition the horizontal force is the 0.5·N.

In the y direction there's a force balance:

$$N - 400 \cdot g \cdot \cos(30°) = 0$$

$$\longrightarrow N = 3391\,N.$$

In the x direction: (note the direction).

$$m_{log} a_{x} = -T_1 + \frac{1}{2}N + 400 \cdot g \cdot \sin(30°)$$

$$= -2T + 1695 + 1960 = -2T + 3655$$

We have now two relations between our unknowns:

(1) $2450 - T = 250 \cdot a_A$

$400 \cdot a_C = -2T + 3655$ $\qquad$ $a_C = -\frac{1}{2} a_A$

$\Rightarrow 3655 - 2T = -200\, a_A$ $\qquad$ (2).

$\rightarrow T = 2005\ N$

$a_A = 1.78\ \frac{m}{sec^2}$

Polar coordinates:

$\Sigma F_r = m a_r$ $\qquad$ $a_r = \ddot{r} - r\dot{\theta}^2$

$\Sigma F_\theta = m a_\theta$ $\qquad$ $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

Same advice applies when we choose those coordinates. Draw free body diagram, make sure you know the positive direction of the axes.

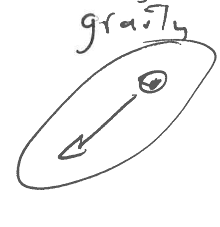
Sample problem 3.9

What is the magnitude v of the velocity of spacecraft S to maintain a circular orbit of altitude h ~~(200 km)~~ above the earth

$\Sigma F_r = m a_r$

$$G \cdot \frac{m \cdot M_{earth}}{(R+h)^2} = m a_r = m(R+h)\dot{\theta}^2$$

only force is gravity



$v_r = 0 \qquad v_\theta = r\dot{\theta} = (R+h)\dot{\theta}$

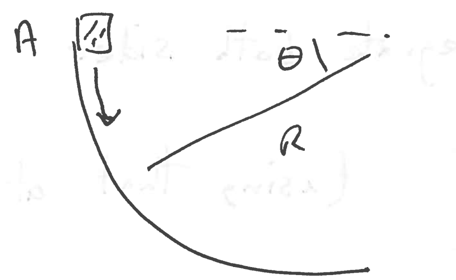
So:

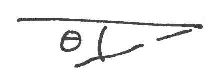
$$G \frac{\cancel{m} M_{earth}}{(R+h)^2} = \frac{\cancel{m} V_\theta^2}{(R+h)}.$$

$$v = \sqrt{\frac{G \cdot M_{earth}}{(R+h)}}$$

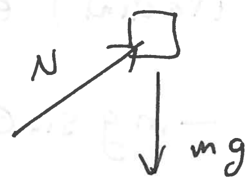
Sample problem 3.7:

An object is released from rest at A and slides as shown. For each θ find the normal force between the guide and the object, and the angular velocity ($\dot{\theta}$).





Free body diagram



For angular motion $\quad mg\cos\theta = m a_\theta = mR\ddot{\theta}$

we have $\ddot{\theta}$ as function of θ, use the followings call $\dot{\theta} = \omega$, the angular velocity:

$$\frac{d\theta}{dt} = \omega \qquad \frac{d\omega}{dt} = \ddot{\theta}$$

The above relation: $g \cos\theta = R\ddot{\theta} = R \cdot \frac{d\omega}{dt}$

$$\frac{d\theta}{dt} = \omega$$

eliminate dt $\longrightarrow$ $R\,\omega\, d\omega = g \cos\theta \, d\theta$

integrate both sides $\frac{1}{2} R \omega^2 = g \sin\theta$

(using that at initial instant $\omega = 0$ and $\theta = 0$)

$$\omega = \sqrt{\frac{2g \sin\theta}{R}}$$

The normal (radial) equation

$$-mg \sin\theta + N = a_r = R\dot{\theta}^2 = R\omega^2$$

$$N = 3mg \sin\theta$$