Relative Motion:

Make measurements in a moving reference frame, how is that related to absolute motion?

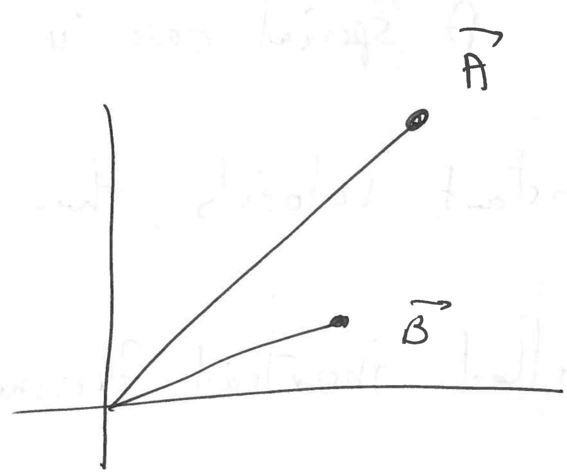
The absolute motion is described by some inertial coordinate system. Assume the moving coordinate system does not rotate, because that would be more complicated.

Assume that the motion we want to describe is

$$\vec{r}_A(t) = x_A(t)\,\hat{e}_x + y_A(t)\,\hat{e}_y$$

Now, assume a moving reference frame undergoes the motion:

$$r_B(t) = x_B(t)\,\hat{e}_x + y_B(t)$$



Then

$$\vec{r}_{A|B} = \vec{r}_A - \vec{r}_B$$

$\vec{r}_{A|B}$ is the position of A, as seen by B.

Using this basic formula, and the fact that the positions involved depend on time, we can get:

$$\vec{V}_{A|B} = \vec{V}_A - \vec{V}_B$$

$$\vec{a}_{A|B} = \vec{a}_A - \vec{a}_B$$

Or in coordinates:

$$(V_{A|B}) = (V_x^A - V_x^B)\,\hat{e}_x + (V_y^A - V_y^B)\,\hat{e}_y$$

note that $\frac{d}{dt}\hat{e}_x = 0$, which would be different if there was rotation involved.

A special case is when the moving frame is at constant velocity, then $\vec{a}_{A|B} = \vec{a}_A$; those are called inertial frames.

Sample problem 2.13

jet A flies east at $800 \frac{km}{h}$

jet B flies 45° northeast,

but appears to A as

flying in 60° angle (in northwest direction).

What is the velocity of B?

We use $\vec{V}_{B|A} = \vec{V}_B - \vec{V}_A$

Let's see what we know: $V_A = (800 \frac{km}{h}) \cdot \hat{e}_x$

We don't know $\vec{V}_B$ but we know the angle 45°:

$$\vec{V}_B = (v \cdot \cos 45°)\,\hat{e}_x + (v \sin 45°)\,\hat{e}_y$$

where v is the unknown speed we are trying to determine. This means that the relative velocity is

$$\vec{V}_{B|A} = (v \cos 45° - 800 \tfrac{km}{h})\,\hat{e}_x - v \sin 45°\,\hat{e}_y$$

we know that the angle as viewed by A is 60°:

$$\left| \frac{(V^{A|B})_y}{(V^{A|B})_x} \right| = |\tan(60^\circ)| = \frac{-V \sin 45^\circ}{V \cos 45^\circ - 800 \frac{km}{h}}$$

This gives

$$\left(V \cos 45^\circ - 800 \frac{km}{h}\right) \tan 60^\circ = -V \sin 45^\circ$$

$$\sin 45^\circ = \cos 45^\circ = 0.707$$

$$\tan(60^\circ) = 1.732$$

$$(V \cdot 0.707 \cdot 1.732) + V \cdot 0.707 = (800 \cdot 1.732) \frac{km}{h}$$

$$V \cdot 0.707 \cdot 2.732 = (800 \cdot 1.732) \frac{km}{h}$$

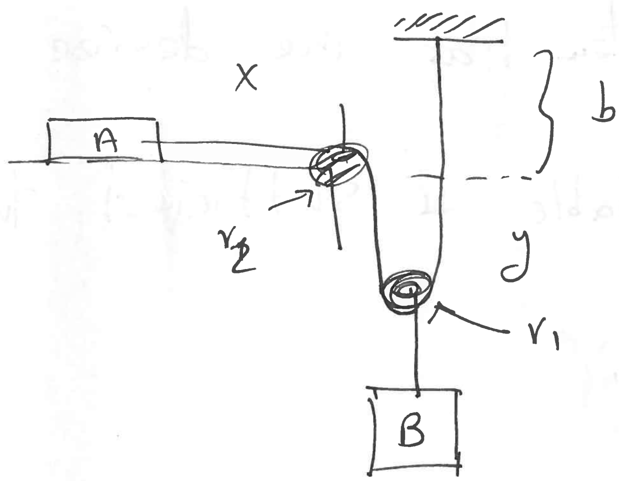
$$V = 717 \frac{km}{h}$$

Constrained Motion:

Sometimes two or more particles are connected such that their motion is not independent. We can either identify what aspect of the motion (aka "degree of freedom") is unconstrained, or use constrained variables.

A moves horizontally, its location is x.

B moves vertically, its location is y



parameters are r_1, r_2, b. The constraint is that the total length of the cable

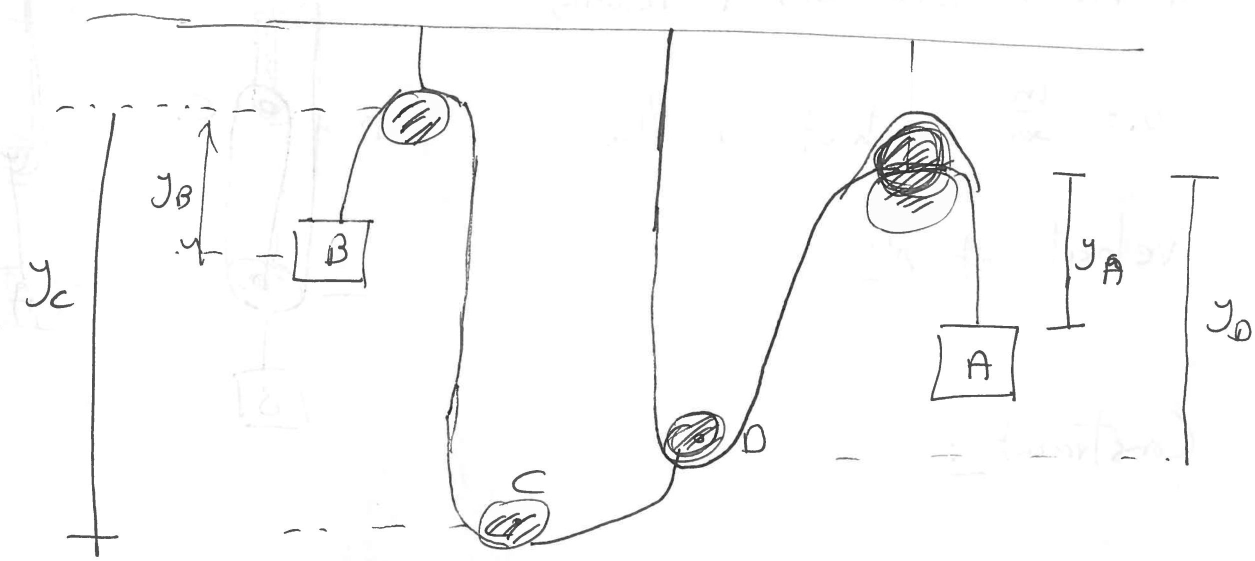
$$L = x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b = \text{Constant}.$$

One approach is to solve for x, or y, then use the constraint to obtain the other variable. Or, we can differentiate the constraint:

$$\dot{x} + 2\dot{y} = 0 \qquad \vec{V}_A + 2\vec{V}_B = 0$$

$$\ddot{x} + 2\ddot{y} = 0 \qquad \vec{a}_A + 2\vec{a}_B = 0$$

This system has one degree of freedom because one variable is sufficient to specify all the positions, Example of two degrees of freedom:



The length of the two cables:

4 positions
2 constraints

$L_A = y_A + 2y_D + \text{Const.}$ ✓ $\rightarrow$ 2 d.o.f.

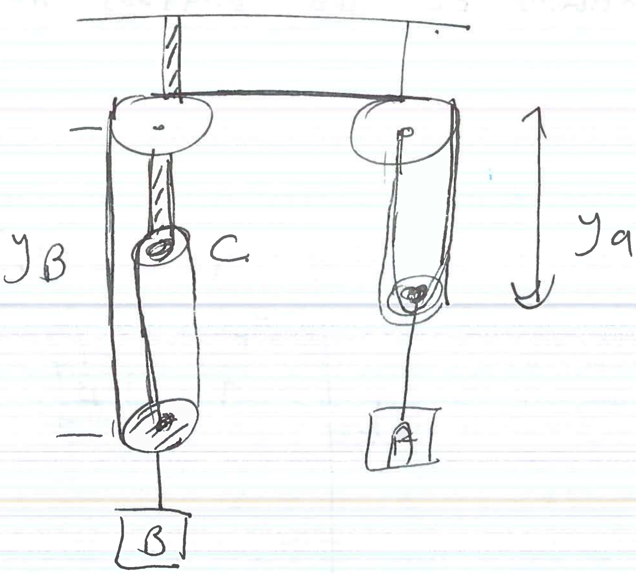
$L_B = y_B + y_B + (y_C - y_D) + \text{Const.}$ ✓

which gives $\dot{y}_A + 2\dot{y}_D = 0$ $\qquad \dot{y}_B + 2\dot{y}_C - \dot{y}_D = 0$

Useful to visualize the motion.

Sample problem 2.15

A moves downwards at velocity 0.3 $\frac{m}{sec}$; what is the velocity of B:



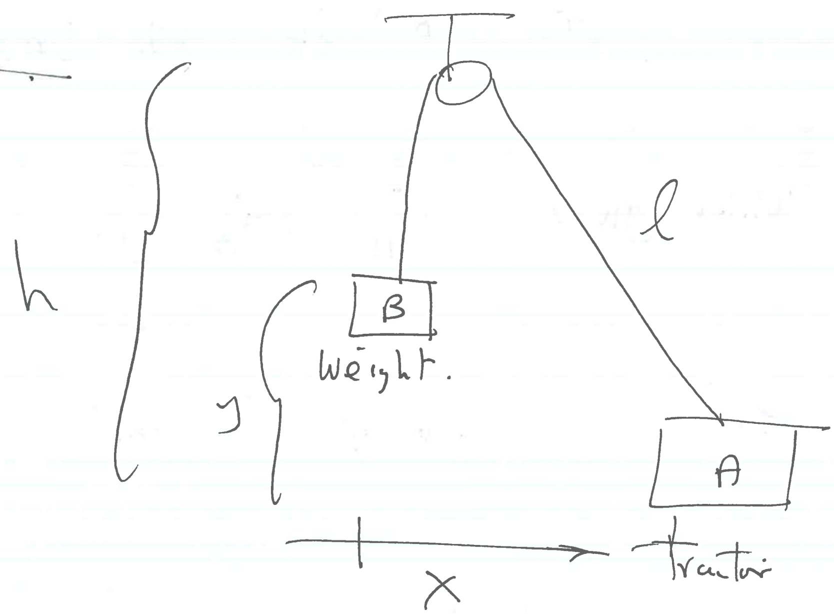
Constraint:

$$L = 3y_B + 2y_A + \text{Constants}.$$

$$3\dot{y}_B + 2\dot{y}_A = 0 \longrightarrow V_B = -0.2 \ \frac{m}{sec}.$$

Sample problem 2.16

A has forward velocity V_A, what is upward velocity V_B?



$$L = 2(h-y) + \ell =$$

$$= 2(h-y) + \sqrt{h^2 + x^2},$$

$$-2\dot{y} + \frac{x\dot{x}}{\sqrt{h^2+x^2}} = 0 \Longrightarrow V_B = \frac{1}{2}\frac{X V_a}{\sqrt{h^2+x^2}}$$