Summary 1

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Abstract

This summarizes the material covered in class on approximately the first two weeks, covering some of chapter 1 and section 2.1 in the textbook.

Learning Goals

- Description of motion (kinematics) along a line.
- Velocity and acceleration.
- Integrating to convert information about velocity and acceleration into the full description of motion.

Basics

The basic structure of the course is divided to kinematics – to do with efficient description of motion– and kinetics, which describes the forces causing the motion and how they determine the motion (through Newton's laws, for example).

We will be using the SI (metric) units, where time is measured in seconds, mass in kilograms and length in meters. It is a good habit to keep tracks of the units used through the calculation, to protect against mistakes. Every quantity should come out in appropriate units, if no errors are made.

Another general advice is to visualize the problem and determine solution strategy, before starting the problem. In the course we will use various graphical representations for that purpose.

Kinematics of Particles

Basics

- Definition of a particle: when spatial extent of moving body does not matter much.
- Types of motion: Constrained versus unconstrained motion. Motion on a plane or on a line.
- Description of motion: Assigning coordinates.

  In this chapter we'll look at how motion is described, in preparation for the next chapter where we'll make predictions for motion of various bodies based on the external forces acting on them.

Motion on a line

We start with the simplest situation of particle moving on a line. An example is a sprinter. If we have a particle moving on a line, subject to some forces, our goal is to describe its motion in time.

To get the math going, we need to measure the position at any given time. Assume we are stationary and we measure the time t and the position along a line (away from the starting point) and call it x. This is the simplest example of a coordinate system: a complete description of the runners trajectory would be a function $x(t)$. Our goal in this course is to calculate the trajectory $x(t)$ for various types of motion, using the laws of physics.

Velocity

In a small time increment Δt, the runner covered a distance Δx. Their average velocity over that time period is then $\bar{v} = \frac{\Delta x}{\Delta t}$. In the limit of $\Delta t \to 0$ we get the *instantaneous* velocity. This limit defines the **derivative**:

$$v = \frac{dx}{dt} \tag{1}$$

The derivative is one of the most useful tools we will use, and one of the deepest ideas in physics: break up the problem to small parts. If we know how fast a body moves at all times, we can *integrate* to find out the resulting accumulative motion. We gain knowledge of something potentially complicated by adding up many simpler effects.

Acceleration

Now that we are familiar with derivatives, we can discuss the rate of change of the velocity, that is the acceleration:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \tag{2}$$

The acceleration also has a direction which can be the same as the velocity or opposite: for example the runner can move forward but decelerate, its forward speed decreasing with time.

Notation for derivatives: dot above the quantity differentiated.

Example: Differentiation

Sample problem 2/1 page 27.

Inverting Derivatives: Integration

We are usually given the expression for the derivatives, which are related to the forces acting on the particle, and need to find the displacement from that. The trick is to use the definitions in the way which reduced the problem to integration.

Simplest case: we are given the velocity as function of time $v(t)$, and we need to find the displacement for all times $s(t)$. For that use the definition $v(t) = \frac{ds}{dt}$ to get

$$ds = v(t)dt \tag{3}$$

Since the right hand side now depends only on t, we can integrate:

$$s(t_2) - s(t_1) = \int_{t_1}^{t_2} v(t)\, dt \tag{4}$$

if we know the acceleration $a(t)$ we can do things in stages: first find $v(t)$ by integrating once, and then find $s(t)$ by integrating again. Examples to follow.

More tricky: suppose we know the velocity not as function of time, but as function of displacement $v = v(s)$. Doing as above does not help us. Instead we use the definition to get

$$dt = ds/v(s) \tag{5}$$

which can again be integrated, giving

$$t_2 - t_1 = \int_{s_1}^{s_2} ds/v(s) \tag{6}$$

We get instead of s(t) which is familiar, t(s) which can be inverted.

Similarly, if we are given the acceleration as function of velocity, we can use the definition to get

$$dt = dv/a(v) \tag{7}$$

Finally, the most tricky situation is when we get the acceleration as function of displacement $a(s)$. We now need to use both definitions to get $vdv = ads$, which gives a relation which can be integrated on both sides

$$\frac{1}{2}(v_2^2 - v_1^2) = \int_{s_1}^{s_2} a(s)ds \tag{8}$$

This gives us v(s), a situation we already know how to deal with.

Examples: Integration

Sample problems 2.2, 2.3, problem 2.1 in the text.