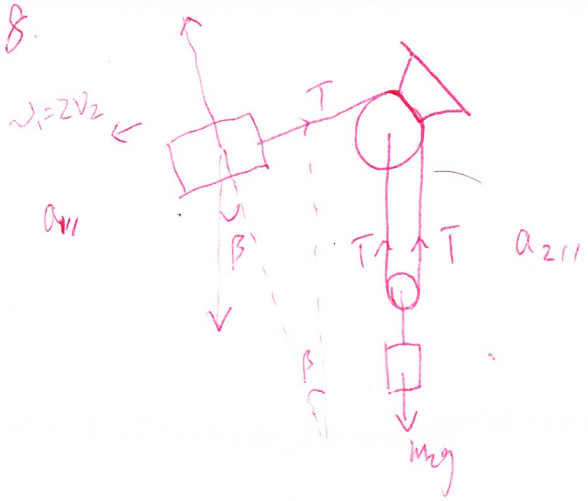
HW 6

3-68.



$v_1 = 2v_2$

$a_{1//} = 2a_{2//}$

$m_1 \frac{v_1^2}{R} = m_1 g\cos\beta - F_N$

$m_1 g\sin\beta - T = m_1 a_{1//}$ ①

$2T - m_2 g = m_2 a_{2//} = m_2 \cdot \frac{1}{2} a_{1//}$

$\Downarrow$

$T - \frac{1}{2} m_2 g = m_2 \cdot \frac{1}{4} a_{1//}$ ②

① + ② $\Rightarrow m_1 g\sin\beta - \frac{1}{2} m_2 g = m_1 a_{1//} + m_2 \cdot \frac{1}{4} a_{1//} = \left(m_1 + \frac{1}{4} m_2\right) a_{1//}$

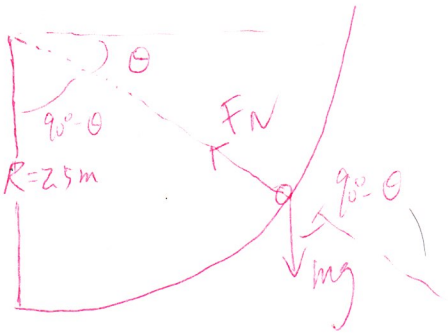
$\therefore a_{1//} = \dfrac{m_1 g\sin\beta - \frac{1}{2} m_2 g}{m_1 + \frac{1}{4} m_2}$ $= 1.78\ m/s^2$ (along slope pointing left)

$\therefore T = \frac{1}{2} m_2 g + \frac{1}{4} m_2 \dfrac{m_1 g\sin\beta - \frac{1}{2} m_2 g}{m_1 + \frac{1}{4} m_2}$ $= 2.63\ N$.

$F_N = 0$, $m_1 \frac{v_1^2}{R} = m_1 g\cos\beta$ $\therefore m_1 \frac{4v_2^2}{R} = m_1 g\cos\beta$ $\therefore v_2 = \dfrac{\sqrt{gR\cos\beta}}{2} = 1.93\ m/s$

$\therefore v_1 = 2v_2 = \sqrt{gR\cos\beta} = 3.86\ m/s$

3-84



$$F_N - mg\sin\theta = m\frac{v^2}{R}$$

$$mg\sin\theta = ma_{\parallel}$$

$mgR(\sin\theta - \sin 20^\circ) = \frac{1}{2}mv^2$ (change of gravity energy is converted into kinetic energy)

$$\therefore 2mg(\sin\theta - \sin 20^\circ) = \frac{mv^2}{R}$$

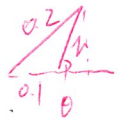
$F_N - mg\sin\theta = 2mg(\sin\theta - \sin 20^\circ)$ $\therefore F_N = 3mg\sin\theta - 2mg\sin 20^\circ$

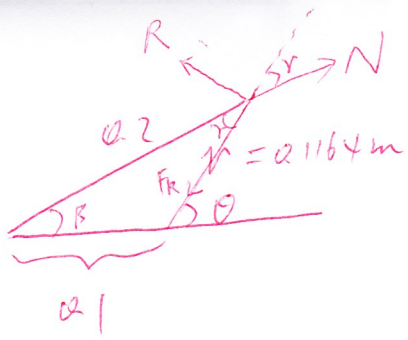
$a_{\parallel} = g\cos\theta$

$\theta = 30^\circ$, $F_N = 279.8N$ $a_{\parallel} = g\cos 30^\circ = 8.487 m/s^2$

$\theta = 90^\circ$ $F_N = 794.37N$ $a_{\parallel} = 0$

3-94.





$$(r\cos\theta+0.1)^2+(r\sin\theta)^2=0.2^2$$

$$\therefore r^2+0.2r\cos\theta-0.03=0$$

$$\therefore \frac{d}{dt}(r^2+0.2r\cos\theta-0.03)=0 \Rightarrow 2r\dot{r}+0.2\dot{r}\cos\theta-0.2r\dot{\theta}\sin\theta=0$$

plug in $r=0.1164m$, $\theta=45°$, $\dot{r}=0.66m/s$.

$$\frac{d}{dt}(2r\dot{r}+0.2\cos\theta\,\dot{r}-0.2r\dot{\theta}\sin\theta)=0 \Rightarrow 2\dot{r}^2+2r\ddot{r}+0.2\ddot{r}\cos\theta-0.2\dot{r}\dot{\theta}\sin\theta$$

$$-0.2\dot{r}\dot{\theta}\sin\theta-0.2r\ddot{\theta}\sin\theta-0.2r\dot{\theta}^2\cos\theta=0$$

plug in $r=0.1164m$ $\theta=45°$ $\dot{\theta}=15$ $\ddot{\theta}=0$ $\dot{r}=0.66m/s$

$\therefore \ddot{r}=15m/s^2$.

$$\sin\beta=\frac{r\sin\theta}{0.2} \Rightarrow \beta=24.3° \qquad \beta+\gamma=\theta, \Rightarrow \gamma=\theta-\beta=20.7°.$$

$$F_k=kx=5000\times(r-0.1)=81.9N.$$

$$N\cos\gamma-F_k=ma_r=m(\ddot{r}-r\dot{\theta}^2)=m(-11.19m/s^2) \qquad a_r=\ddot{r}-r\dot{\theta}^2=-11.19m/s^2$$

$$\therefore N=\frac{1}{\cos\gamma}(F_k+ma_r)=\underline{81.6N}.$$

$$R-N\sin\gamma=ma_\theta=m(2\dot{r}\dot{\theta}+r\ddot{\theta}) \qquad a_\theta=r\ddot{\theta}+2\dot{r}\dot{\theta}=19.8m/s^2$$

$$\therefore R=N\sin\gamma+ma_\theta=81.6N\times\sin20.7°+m\times19.8m/s^2$$

$$=38.7N.$$