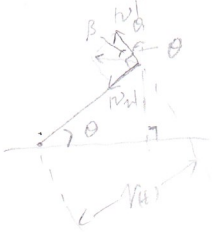
HW 3

14.3



$\dot{\theta}(t) = 0.8t - \frac{t^2}{20}$ $\quad r(t) = -0.2t + 1.6$

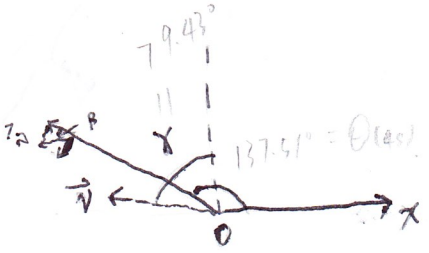
$\dot{\theta} = 0.8 - \frac{t}{10}$ $\quad \dot{r}(t) = -0.2$

$\ddot{\theta} = -\frac{1}{10}$ $\quad \ddot{r}(t) = 0$

$\theta(4s) = 137.51°$

$v_\theta = r(t)\dot{\theta}(t)$ $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$v_r = \dot{r}(t)$ $\quad a_r = \ddot{r} - r\dot{\theta}^2$



$v_\theta(4s) = r(4s) \times \dot{\theta}(4s) = 0.32\ m/s$

$v_r(4s) = \dot{r}(4s) = -0.2\ m/s$

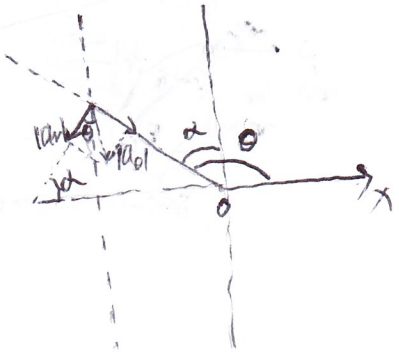
$\therefore |v(4s)| = \sqrt{v_\theta^2(4s) + v_r^2(4s)} = 0.377\ m/s$

$\cos\beta = \frac{v_\theta}{|v|} = \frac{0.32}{0.377} \Rightarrow \beta = 0.5576\,rad \approx 31.9179°$

$\theta(4s) = 2.4\ rad$

$\therefore \alpha_v = \beta + \theta = 169.4278°$

$\gamma = \alpha_v - 90° \approx 79.43°$



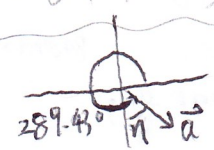
$a_r(4s) = 0 - (1.6 - 0.2\times4) \times (0.8 - \frac{4}{10})^2 = -0.128\ m/s^2$

$a_\theta(4s) = (1.6 - 0.2\times4)(-\frac{1}{10}) + 2(-0.2)(0.8 - \frac{4}{10}) = -0.24\ m/s^2$

$\therefore |a| = \sqrt{a_r^2(4s) + a_\theta^2(4s)} = 0.272\ m/s^2$

$\cos\xi = \frac{|a_r|}{|a|} = \frac{0.128}{0.272} \Rightarrow \xi \approx 61.9275°$

$\alpha_{\vec{a}} = \theta + 90° + \xi = 289.43°$



$\therefore \gamma = \alpha_{\vec{a}} - 270° = 19.43°$

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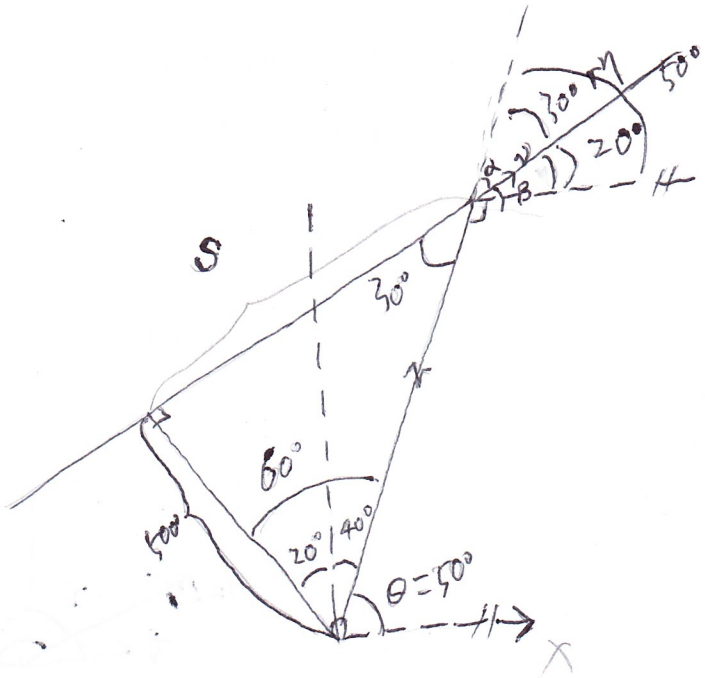
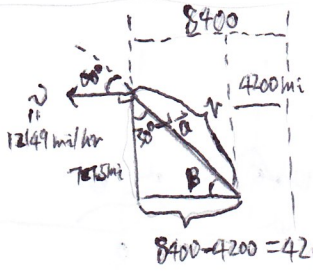


$\eta = \theta = 50°$ $\quad \eta = \alpha + 20°$ $\quad \therefore \alpha = 30°$

$\therefore v\cos\alpha = \dot{r} = 140\ ft/s \quad \therefore v = \dfrac{140\ ft/s}{\cos 30°} = 161.65\ ft/s$

$r = \dfrac{500'}{\sin\alpha} = 1000'$

$\therefore \dot{\theta} = -\left|\dfrac{v\sin\alpha}{r}\right| = -\left|\dfrac{v}{2r}\right| = -\left|\dfrac{161.65}{2\times 1000}\right| rad/s \approx -0.080825\ rad/s$



8400-4200 = 4200 mi

$\tan\beta = \frac{7275}{4200} \Rightarrow \beta = 60°$

$v = 12149\ mi/hr = 17818.5\ ft/s$

$r = 4200mi/\sin 30° \approx 8400\ mi$

$\therefore \dot{r} = v_r = v\cos 60° = v\sin 30° = 8909.27\ ft/s$

$\therefore v_\theta = \dot{\theta} = r\dot{\theta} = 8400mi \times \dot{\theta} = v\sin 60°$

$\therefore \dot{\theta} = \dfrac{17818.5\ ft/s \times \sin 60°}{8400mi} = 3.479\times 10^{-4}\ rad/s$

$v_r = \dot{r} \quad a_r = \ddot{r} - r\dot{\theta}^2$

$v_\theta = r\dot{\theta} \quad a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

$a_r = -7.159\ ft/sec^2 \quad a_\theta = 0$

$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} = 0 \quad \therefore r\ddot{\theta} = -2\dot{r}\dot{\theta} \quad \therefore \ddot{\theta} = \dfrac{-2\dot{r}\dot{\theta}}{r} = -1.398\times 10^{-7}\ rad/s$

$a_r = \ddot{r} - r\dot{\theta}^2 \quad \therefore \ddot{r} = a_r + r\dot{\theta}^2$

$= -7.159\ ft/s^2 + 8400mi \times (3.479\times 10^{-4}\ rad/s)^2$

$= -1.791\ ft/s^2$