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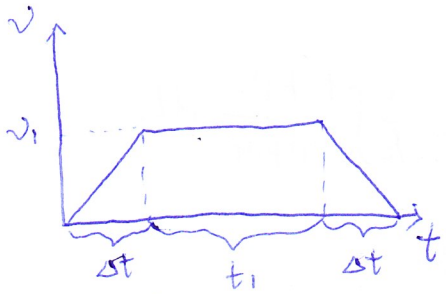

$v_1 = 22\,km/h = 6.11\,m/s$

$\frac{v_1}{\Delta t} = \frac{1}{4}g \qquad \Delta t = \frac{v_1}{\frac{1}{4}g} = \frac{6.11\,m/s}{\frac{1}{4}\times 9.8\,m/s^2} = 2.49s$

$v_1 t_1 + \frac{1}{2}\Delta t \times v_1 \times 2 = 350m \Rightarrow t_1 = 54.79s$

$\therefore t = t_1 + 2\Delta t = 59.77s.$

2.37 $\Delta v = \int a(t)\,dt.$

a). $a = kt^2 \qquad 5\,mm/s^2 = k\cdot(4s)^2 \Rightarrow k = \frac{5}{16}\,mm/s^4.$

$\therefore a = \frac{5}{16}t^2\ mm/s^2.$

$\therefore \Delta v|_{t=4s} = \int_0^{4s} a(t)\,dt = \int_0^{4s} kt^2\,dt = k\frac{t^3}{3}\Big|_0^{4s} = k\cdot\frac{64}{3}\,mm/s = \frac{5}{16}\cdot\frac{64}{3} = \frac{20}{3}\,mm/s$

$\therefore v_{t=4s} = v_0 + \Delta v|_{t=4s} = 0 + \frac{20}{3}\,mm/s = \frac{20}{3}\,mm/s.$

b). for $4s \to 8s$, $\Delta v_2 = \int_{t=0s}^{t=4s} a(t)\,dt' = \int_{0s}^{4s}\left(5+\frac{t'}{3}\right)dt' = 20 + \frac{8}{3} = \frac{68}{3}\,mm/s$

$t_0 = 4s,\ a(t') = 5 + \frac{2}{6}t'$

$t' = 0s \to 4s,\ = 5 + \frac{t'}{3}\ (mm/s)$

$\therefore v|_{t=8s} = v|_{t=4s} + \Delta v_2 = \frac{20}{3} + \frac{68}{3} = \frac{88}{3}\,mm/s$

c). $\Delta v_{8\to 12s} = \frac{20}{3} + (5+7)\times\frac{6}{2} + 2\times 2 = 46\frac{2}{3} = \frac{140}{3}\,mm/s.$

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$a = -kv^2$

$\frac{dv}{dt} = a = -kv^2$

$\therefore dv = -kv^2 dt$

$\therefore \frac{dv}{-v^2} = k\,dt$

$\therefore \int_{v_0}^{v} d\frac{1}{v} = k\int_0^t dt$

$\therefore \frac{1}{v} - \frac{1}{v_0} = kt$

$\therefore \frac{1}{v} = \frac{1}{v_0} + kt$

$\therefore v = \frac{1}{\frac{1}{v_0} + kt}$

$v = \frac{v_0}{2}, \quad t = \frac{1}{kv_0}$

$D = \int_0^t v(t)\,dt$

$= \int_0^t \frac{v_0}{1+kv_0 t}\,dt = \frac{1}{k}\int_0^t \frac{kv_0}{1+kv_0 t}\,dt.$

$= \frac{1}{k} \ln(1+kv_0 t)\Big|_0^t$

$= \frac{1}{k} \ln(1+kv_0 t).$

$t = \frac{1}{kv_0} \quad D = \frac{\ln 2}{k}.$