HW 11

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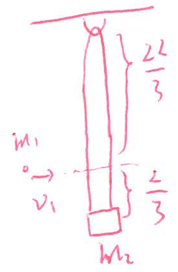

conservation of angular momentum:

$$\frac{2}{3}L \times m_1 v_1 = \frac{2}{3}L\,\omega m_1 \times \frac{2}{3}L + L\,\omega \times m_2 \times L$$

$$= \omega L^2\left(\frac{4}{9}m_1 + m_2\right)$$

$$\therefore \omega = \frac{\frac{2}{3}L m_1 v_1}{\left(\frac{4}{9}m_1 + m_2\right)L^2} = \frac{6 m_1 v_1}{(4m_1 + 9m_2)L}$$

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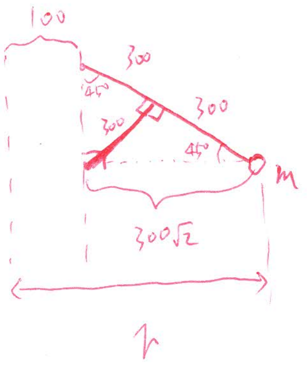

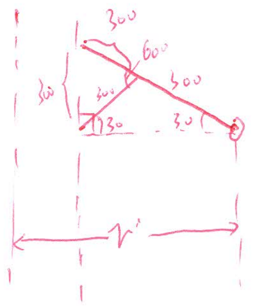

$r = 100 + 300\sqrt{2} = 524.26\,mm$

$r' = 100 + \sqrt{600^2 - 300^2} = 619.62\,mm$

conservation of angular momentum: $2m \times r \times r\omega_0 = 2m \times r' \times r'\omega'$

$$\therefore \omega' = \frac{r^2}{r'^2}\omega_0 = \frac{(100+300\sqrt{2})^2}{(100+300\sqrt{3})^2} \times \omega_0 = 28.636\ rev/min \approx 3\ rad/s$$

$U = \Delta K + \Delta G$

$$\Delta K = 2 \times \frac{1}{2}m(v'^2 - v^2) = m(r'^2\omega'^2 - r^2\omega_0^2) = mr^2\omega_0(\omega' - \omega_0)$$
$$= -6.8432\,J$$

$\Delta G = 2mg\,(300\sqrt{2}\,mm - 300\,mm) = 12.1716\,J$

$U = 5.33\,J.$

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circular orbit motion:

$$\frac{m v_0^2}{r_p} = G\frac{Mm}{r_p^2} \qquad \therefore \quad v_0^2 = \frac{GM}{r_p}$$

$E_K + U_G = 0$:

$$\frac{1}{2} m v'^2 - \frac{GMm}{r_p} = 0 \qquad \therefore \quad v'^2 = \frac{2GM}{r_p}$$

$$r_p = 6371 + 240\ km = 6611\ km$$

$$\therefore \quad \Delta v = v' - v_0 = \sqrt{\frac{GM}{r_p}}\,(\sqrt{2}-1) \qquad \frac{GM}{r_E^2} = g \quad \therefore GM = g r_E^2$$

$$\therefore \quad \Delta v = \sqrt{\frac{g r_E^2}{r_p}}\,(\sqrt{2}-1) = \sqrt{\frac{9.8 \times 6371^2 km}{6611}}\,(\sqrt{2}-1) = 3213\ m/s.$$

$$E_{tot} = \frac{1}{2} m v_{2p}^2 - \frac{GMm}{2r_p} = 0 \qquad \therefore \quad v_{2p} = \sqrt{\frac{GM}{r_p}} = \sqrt{\frac{g r_E^2}{r_p}} \simeq 7764.6\ m/s$$

as $E_{tot} = 0$, it's a parabola, thus $e = 1$

$$ed = d = \frac{h^2}{GM} \qquad h = r_p v' = \sqrt{2GMr_p} = \sqrt{2 \cdot g r_E^2 \times r_p}, \quad \frac{1}{r} = \frac{1}{d}(1+\cos\theta).$$

$$\therefore \quad d = \frac{h^2}{GM} = \frac{2GMr_p}{GM} = 2r_p$$

$$\therefore \quad \frac{1}{r} = \frac{1}{2r_p}(1+\cos\theta). \qquad \therefore \text{ for } \theta = 90^\circ,\ r = 2r_p$$