PHYS 216 Assignment 3 **Due: 25 January 2017**

1. A guitar string vibrates harmonically with a frequency of 512 Hz (one octave above middle C on the musical scale). If the amplitude of oscillation of the centerpoint of the string is 0.002 m (2 mm), what are the maximum speed and the maximum acceleration at that point?

   Answer:

   The string vibrates transversely, with a displacement

   $$x = A\sin(\omega_0 t + \phi_0).$$

   The maximum displacement is $A = 0.002$ m.

   The frequency (vibrations per second) is $\nu = 512$ Hz, so

   $$\omega_0 = 2\pi\nu = 3217.0 \text{ radians/s}.$$

   The velocity is

   $$v = \dot{x} = A\omega_0 \cos(\omega_0 t + \phi_0),$$

   which has a maximum value of $A\omega_0 = 6.434$ m/s.

   The acceleration is

   $$a = \dot{v} = -A\omega_0^2 \sin(\omega_0 t + \phi_0),$$

   which has a maximum value of $A\omega_0^2 = 2.07 \times 10^4$ m/s^2.

2. Two springs having stiffness k_1 and k_2 are used to support a single object of mass m. Show a) that the angular frequency of the oscillation is $\sqrt{(k_1 + k_2)/m}$ if the springs are connected in parallel, and b) $\sqrt{k_1 k_2/(k_1 + k_2)m}$ if the springs are connected in series.

   Answer:

   If the springs are connected in parallel, they are both displaced by the same amount k and so exert forces $F_1 = k_1 x$ and $F_2 = k_2 x$ respectively. The total force acting on the mass is the sum of these forces $F = (k_1 + k_2)x$, so the system acts line a single spring of stiffness (spring constant) $k = k_1 + k_2$. The angular frequency is $\omega_0 = \sqrt{k/m} = \sqrt{(k_1 + k_2)/m}$.

   If the springs are in series, they will extend by different amounts, which add to give the total displacement. The force F on each spring is the same. Therefore,

   $$x = x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2} = F\frac{k_1 + k_2}{k_1 k_2}$$

   We see that the force is

   $$F = \frac{k_1 k_2}{k_1 + k_2} x \equiv kx$$

   so the effective spring constant of the system is now $k_1 k_2/(k_1 + k_2)$.

3. A spring of stiffness k supports a box of mass M in which is placed a block of mass m. If the system is pulled down a distance d from the equilibrium position and then released. Find the force of reaction between the block and the bottom of the box as a function of time. For what value of d does the block just begin to leave the bottom of the box at the top of the vertical oscillations.

The angular frequency will be

$$\omega_0 = \sqrt{\frac{k}{M+m}}$$

and the downward vertical displacement is given by

$$x = A\sin(\omega_0 t + \pi/2) = d\cos(\omega_0 t)$$

The constants $A = d$ and $\phi_0 = \pi/2$ are chosen to satisfy the initial conditions $x(0) = d$ and $v(0) = 0$.

The small block feels the acceleration of gravity g and also the acceleration a of the box. The force on the block is proportional to the *difference* between these accelerations (if $a = g$ the force is zero, as in a freely falling elevator).

$$F_{\text{reaction}}(t) = m(g-a) = m(g-\ddot{x}) = m\left[g + d\omega_0^2\cos(\omega_0 t)\right].$$

The maximum upward displacement occurs when $\omega_0 t = \pi$ so the block will just begin to lift of the box at that time if

$$F_{\text{reaction}}(\pi/\omega_0) = m(g - d\omega_0^2) = 0,$$

which happens when

$$d = g/\omega_0^2.$$